IDEAL COIL-SHAPE FOR PERFECT FIELD IN SUPERCONDUCTING SEXTUPOLE MAGNETS

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Abstract

Superconducting magnets have become very essential components in high energy accelerators. The field in coil-dominated superconducting magnets depend mostly on the shape of the current-carrying coil. A coil shape generated by two overlapping ellipses produces the perfect dipole magnetic field. Similarly, a perfect quadrupole field can be produced with a coil shape generated by two perpendicular and concentric intersecting ellipses. For a sextupole, however, there is no mention of such coil shape in the literature. We have given here a coil shape for generating the perfect sextupolar magnetic field.

INTRODUCTION

In high energy accelerators like LHC at CERN [1,2], VEPP at Novosibirsk [3] etc. superconducting sextupole magnets are used as correctors of second order beam aberrations.

In these magnets the field quality depends on the coil shape. A $\cos\theta$ azimuthal current distribution on a cylindrical surface produces a perfect dipole magnetic field inside. Similarly, a $\cos(3\theta)$ azimuthal current distribution produces a perfect sextupole field. This, however, is not a practical way as one has to use a large number of power supplies for achieving the required current distribution. The practical way is to have a constant current pass through a coil winding whose number of turns varies azimuthally.
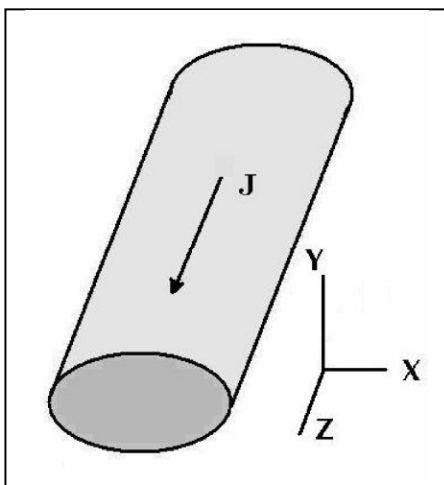

Figure 1. Coil cross-section and current direction.

It is well-known that a constant current flowing through a coil, whose shape is generated by two displaced overlapping ellipses, produces the perfect dipole field. Similarly, two perpendicularly intersecting and concentric ellipses produces a coil shape which generates the perfect quadrupole field [4-9]. However, for a superconducting sextupole no such ideal coil shape is available in the literature. In this work, we have worked out such an ideal coil shape for generating the perfect sextupolar field.

FIELD DUE TO LONG COIL

For a current I flowing along the z-direction through a point (x,y) in a thin strand of conductor of infinite length, the field B at a point (x_0,y_0) whose distance is r from the conductor, is given by the Biot-Savart law as

$$B(x_0, y_0) = \frac{\mu_0 I}{2\pi r} \tag{1}$$

where μ_0 is the permeability in air. The field B is perpendicular to both the radius vector r and the conductor. The field component in the y-direction is

$$B_y(x_0, y_0) = \frac{\mu_0 I}{2\pi} \frac{(x - x_0)}{(x - x_0)^2 + (y - y_0)^2} \tag{2}$$

For a coil of extended dimension the field is calculated by integrating the above over the cross-section of the coil. For a uniform current density J flowing along the length (z-direction) of the coil, i.e., perpendicular to the cross-section of the coil (Fig. 1), the field $B_y(x_0,y_0)$ is

$$B_y(x_0, y_0) = \frac{\mu_0 J}{2\pi} \iint_{y\ x} \frac{(x - x_0)}{(x - x_0)^2 + (y - y_0)^2} dx dy$$

$$= \frac{\mu_0 J}{4\pi} \oint \ln\left[(x(y) - x_0)^2 + (y - y_0)^2\right] dy \tag{3}$$

where $x(y)$ defines the boundary of the coil as a function of y, and integration is done for the closed path along the coil boundary. The x-component of the field is similarly

$$B_x(x_0, y_0) = \frac{\mu_0 J}{4\pi} \oint \ln\left[(x - x_0)^2 + (y(x) - y_0)^2\right] dy \tag{4}$$

The coil profile $x(y)$ (or conversely $y(x)$) defines the field type and its quality. For an elliptic shape with semi-axes a and b, and defined by the parametric equations

$$x(\varphi) = a\cos\phi, \qquad y(\varphi) = b\sin(\varphi) \tag{5}$$

eqs.(3) and (4) reduce to

$$B_y(x_0, y_0) = \mu_0 J \frac{bx_0}{(a+b)} \tag{6}$$

$$B_x(x_0, y_0) = -\mu_0 J \frac{ay_0}{(a+b)} \tag{7}$$

Thus there is a constant gradient of the field B_y along the x-axis, and of B_x along the y-axis. Actually, along any radial direction, the gradient of the field component perpendicular to that radius vector is constant.

This field is inside a conductor which is solid, a particle beam can not pass throught it. One requires a vacuum space for the beam to pass through. Superposition of positive and negative current distributions through two overlapping coil shapes creates a vacuum space and modifies the field inside that space. As has already been mentioned, two displaced elliptic current distributions produce the perfect dipole field, and two intersecting perpendicular elliptic coil shapes produces the perfect quadrupole field.

We point out that intersecting perpendicular elliptic shape is not the only shape which produces pure quadrupole field. Recently we showed that there exist coil shapes different from the above, which give perfect quadrupole field [10,11].

COIL FOR PERFECT SEXTUPOLE FIELD

Unlike a quadrupolar field, which is linear in form, the sextupolar field has a nonlinear nature along any direction. Since the field inside an elliptic coil produces linear field, one cannot use this field to produce a sextupolar field. One needs to produce a field which has second order spatial variation by using a suitable coil shape. Obviously, the parametric equations for x and y will have to be different from those for an ellipse. The following shapes for $x(y)$ and $y(x)$ have been tried.

$$x(\varphi) = a \cos\phi - c \sin(2\phi)$$
$$y(\phi) = a \sin\phi - c \cos(2\varphi) \tag{8}$$

where c is a constant which modifies the ellipse. With this shape eqs.(3) and (4) yield

$$B_y(x_0, y_0) = \mu_0 J \left[\frac{x_0}{2} + \frac{c}{a^2} x_0 y_0 \right] \tag{9}$$

and

$$B_x(x_0, y_0) = -\mu_0 J \left[\frac{y_0}{2} + \frac{c}{2a^2} (x_0^2 - y_0^2) \right] \tag{10}$$

These fields are mixtures of a sextupolar field ($\propto x_0 y_0$, $x_0^2 - y_0^2$) and a gradient field (x_0, y_0). Fig. 2 shows a typical coil shape given by eq.(8). Unlike the two-fold symmetric elliptic shape, this has a three-fold symmetry.
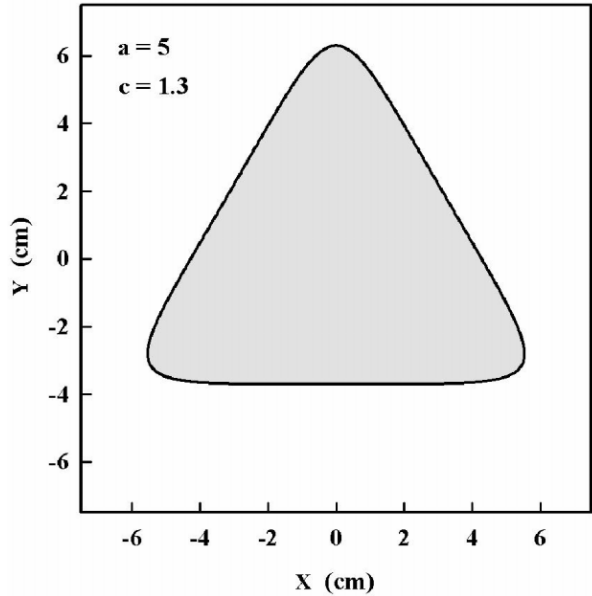

Figure 2. A three-fold symmetric coil shape which produces a second order magnetic field variation.

In order to produce a pure sextupolar field (and, of course, create a vacuum space) one has to superimpose two different current distributions.

$$B_y(x_0, y_0) = B_y(x_0, y_0, c, J) + B_y(x_0, y_0, -c, -J)$$
$$= \frac{2\mu_0 J}{a^2} x_0 y_0$$

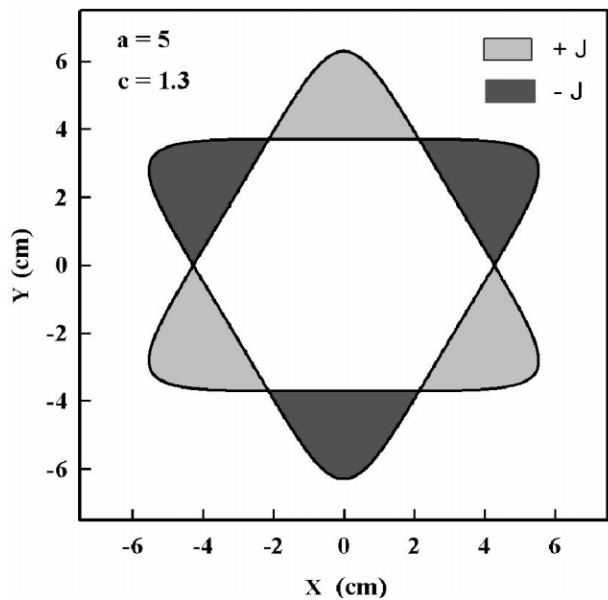$$B_x(x_0, y_0) = -\frac{\mu_0 Jc}{a^2} (x_0^2 - y_0^2)$$

Figure 3. Superimposition of two opposite current distributions generates a perfect sextupole field.

These field values are valid within the overlapping region between the two current distributions. Fig. 3 shows the superimposed coil shape. This has a six-fold geometrical symmetry, the electrical symmetry is three-fold though. Fig.4 shows the lines of force obtained by POISSON calculation in this sextupole.
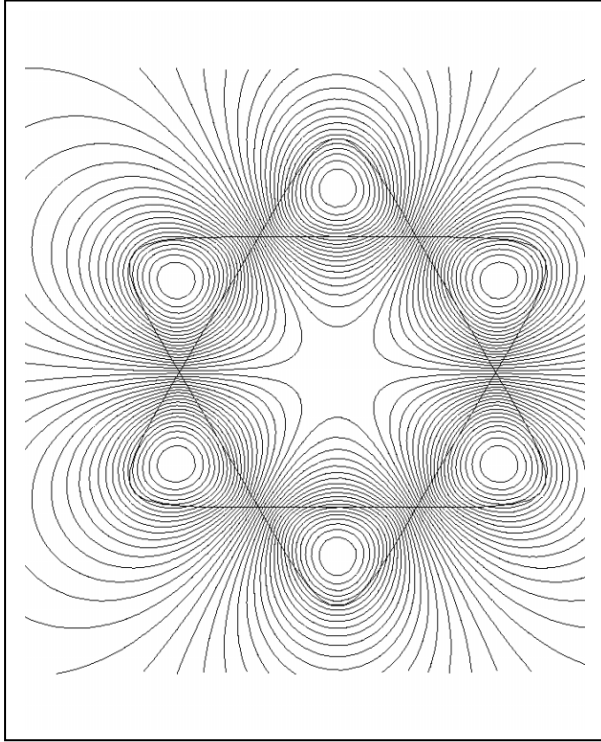

Figure 4. Lines of force in a sextupole of ideal coil shape.

CONCLUSIONS

It has been shown by us that just as coil shapes exist for producing perfect dipole and quadrupole magnetic fields, for a sextupole also there exists an ideal coil shape. This shape is generated by the superimposition of two opposite three-fold symmetric uniform current distributions given by eq.(8). This finding should be of help in designing practical superconducting sextupole magnets.

For canceling the linear part of the field it is not necessary that the two current distributions be of the same size (i.e., same a and c). For two shapes with a_1, c_1 and a_2, $-c_2$ as the parameters, the superposed field components are

$$B_y(x_0, y_0) = \mu_0 J \left[\frac{c_1}{a_1^2} + \frac{c_2}{a_2^2} \right] x_0 y_0 \qquad (11)$$

and

$$B_x(x_0, y_0) = -\mu_0 J \left[\frac{c_1}{2a_1^2} + \frac{c_2}{2a_2^2} \right] (x_0^2 - y_0^2) \quad (12)$$

If the second distribution has a circular shape (i.e., a circular inner aperture for the sextupole) we have $c_2=0$, $c_1=c$ and therefore

$$B_y(x_0, y_0) = \frac{\mu_0 Jc}{a^2} x_0 y_0 \qquad (13)$$

and

$$B_y(x_0, y_0) = -\frac{\mu_0 Jc}{2a^2} (x_0^2 - y_0^2) \qquad (14)$$

A current distribution completely within the other will generate a shape where only three separate coils will be required. One can have an inner aperture of his choice by this method. This type of coil shape will be useful in constructing twin sextupoles in a common-coil configuration.

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