NUMERICAL STUDY OF FIELD ERRORS DUE TO MECHANICAL TOLERANCES IN SUPERCONDUCTING MINIUNDULATORS

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Abstract

Using a wire model, analytical formulae are derived to describe the spatial distribution of the magnetic field of a superconducting miniundulator (supramini) as determined by the errors in positioning the wires. Semi-analytical numerical simulations are performed to estimate the tolerances of various errors required for a satisfactory function of the supramini, including the effects of systematic errors such as pitch, yaw and roll of a whole supramini coil, and random errors of the wire positions. These results can be used to help assess the minimal required mechanical tolerances.

INTRODUCTION

Superconducting miniundulators (supraminis) are believed to be a key component of 4th generation sources including FELs and Energy Recovery Linacs (ERL). Furthermore, they are expected to play an important role in the upgrading of 3rd generation sources [1-4].

In this paper, we evaluate the influence of the different potential mechanical errors on the quality of the magnetic field and specify acceptable tolerances. Based on a wire model introduced earlier [5], we derive analytical formulae to calculate the magnetic field including the mechanical errors. The effects of systematic errors are treated analytically, random errors are simulated numerically. The effects of these errors on the quality of the field are then graphically analyzed.

MODEL AND PROCEDURE

According to [5], the magnetic field that is produced by an array of wire pairs (wp) and that includes the effect of compensation coils can be written as

$$B_y(x,y,z) = B_y^u(x,y,z) + B_y^l(x,y,z) + B_y^c(x,y,z) \quad (1)$$

where B_y^u is the magnetic field produced by the upper part of coil, B_y^l is the magnetic field produced by the lower part of coil, and B_y^c is the magnetic field produced by the compensation coils. Explicit expressions for the first field contribution will be derived in the following while the other two will be taken from [5].

In our analysis of the mechanical errors of the supramini, we include systematic as well as random errors. To define coordinates, we call (x,y,z) the space frame where the (x,z)-plane is the midplane of the supramini without positional errors and z the direction of the electron beam. Coordinates (X,Y,Z) represent the body frame of the upper coil in which the (X,Z)-plane contains the wire axis and the current flow, X runs either along the axis of the central wire in the case of an odd number of wire pairs or along the equivalent straight line at half-distance between the two central wires in the case of an even number of wire pairs, Y is normal to the (X, Z)-plane through the center of the wire arrangement which is the origin of the coordinate frame as well.

Starting from the ideal position of the upper and lower coils, we construct the displaced error-causing position by translating and rotating the upper coil only while keeping the lower one in its ideal position. This is no restriction of the general validity. Translations are carried out along the space axes (x,y,z) and rotations according to the Euler angles as defined in fig. 1. We define, as shown in fig. 1,

- the pitch error when the upper coil is rotated by an angle θ about the x' axis,
- the yaw error when the upper coil is rotated by an angle α about the y_1 axis, and
- the roll error when the upper coil is rotated by an angle φ about the z_2 axis.

All rotations are in a counter-clockwise sense, i.e., right-handed in the mathematically positive sense. The transition from the space frame (x,y,z) to the body frame of the upper coil (X,Y,Z) is achieved by an initial translation along y by the distance from a wire centre to the midplane, three subsequent translations along x, y, z, and three rotations as defined in fig.1 where we adopted a convention as given in [6].

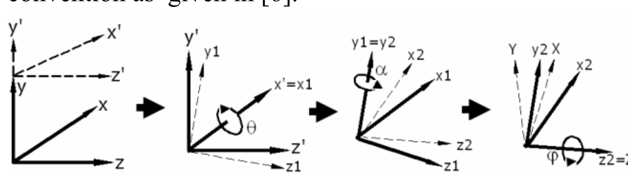

Figure 1: Coordinate transformations leading from the space frame to the body frame of the upper coil with positional errors. Translations of the (x',y',z') frame are not shown.

All error rotation angles θ, α, φ and translational errors $\bar{x}, \bar{y}, \bar{z}$ are supposed to be <<1 and <<g which is the gap, respectively. The magnetic field components produced by the upper coil in the coordinates X, Y and Z are written as $B_X^u(X,Y,Z)$, $B_Y^u(X,Y,Z)$ and $B_Z^u(X,Y,Z)$.

Assuming $B_X^u(X,Y,Z) = 0$, we obtain the magnetic field of the upper coil in the coordinates x, y, and z as

$$B_y^u(x,y,z) = B_Y^u(X,Y,Z)(\cos\varphi\cos\theta - \sin\theta\sin\alpha\sin\varphi) - B_Z^u(X,Y,Z)\cos\alpha\sin\theta$$

with

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x-\bar{x} \\ y-\overline{G}-\bar{y} \\ z-\bar{z} \end{bmatrix}$$

where $\overline{G}$ is the distance from a wire centre to the midplane.

For random errors, we will introduce the mechanical errors of the position of the wires. We assume a random distribution of positional errors of the wires where $\Delta_z(1, i)$ and $\Delta_z(2, i)$ denote the positional error of the ith wire in the z direction for the upper and the lower part of the undulator, and $\Delta_y(1, i)$ and $\Delta_y(2, i)$ accordingly for the y direction. As introduced above, the translational errors of the upper coil are $\overline{x}, \overline{y}, \overline{z}$ in the x, y and z directions, respectively.

Then, we will calculate the spatial distribution of the magnetic field from the zero-error wire model (see formula (1)) in combination with the mechanical errors as described above (we suppose the compensation field without mechanical error).

The random errors are assumed to be non-correlated, so that they can be treated separately. For a hypothetical supramini, a set of 2p (p is the number of wire pairs) random values was computer generated and then selected so that they followed a Gaussian distribution with standard deviation σ_1 for $\Delta_z(1, i)$ and $\Delta_z(2, i)$. Another set of 2p random values was generated following a Gaussian distribution with standard deviation σ_2 and for $\Delta_y(1, i)$ and $\Delta_y(2, i)$.

The random errors require many simulations with different seeds to get a statistical analysis. Careful selection of the simulation times is important for the analysis. In our statistical analysis, we will use 200 sets of random error distributions for each given standard deviation.

ERROR ANALYSIS

Next, we will focus on the effects of the field in the midplane due to the mechanical error on four parameters, namely, phase error, field zero, peak field, and the integral field. In the simulation, we use specifications of the prototype supramini at SSLS which are λ_u = 14 mm, N=50, I = 16 mm^2×1000 A/mm^2, g = 5 mm, a = 27 mm, and $\overline{x} = \overline{y} = \overline{z} = 0$.

Influence of integral coil positional errors

We will discuss the most general situation as one coil ideally aligned (in the simulation, we assume the lower one) and the other one rotated with respect to it.

From the simulation, we derive the influence of the pitch, yaw and roll errors on the magnetic field. The pitch error has a significant influence on the phase error, for example, the phase error will be larger than 8.2° when the pitch error is 0.1mrad. If we want to keep the phase error smaller than 1°, the pitch error should be restricted to less than 10 μrad. The effects on the integral field and the field zeros are not too strong.

For the yaw error, the effects on phase error, peak field, integral field and field zeros are not too strong. Assuming for example a yaw error of 200 μrad, the phase error equals 0.115° and the maximum change of field zeros is 0.01 μm.

The effects caused by the roll error only are not strong as the maximum change of field zeros is just 0.02 μm when the roll error reaches 200 μrad.

Taking pitch, yaw and roll errors together, obviously, the effects are mainly, i.e., to more than 90%, caused by the pitch error.

Influence of positional errors of wire windings

We will use 200 sets of random error distributions to do our statistical simulation. Every set of 2p random values was computer generated following a Gaussian distribution with standard deviation σ_1 for $\Delta_z(1, i)$ and $\Delta_z(2, i)$. Another set of 2p random values was generated with standard deviation σ_2 and for $\Delta_y(1, i)$ and $\Delta_y(2, i)$.

In the analysis of the effects on the phase error, peak field and the integral field, for a given standard deviation, we will derive results by averaging the data calculated by the 200 sets of random error distributions computer generated by 200 different seeds. For the field zero, there are many field zeros in one supramini (for example, there are 101 fields zeros for the supramini we used in the simulation). We will calculate the RMS for each field zero change caused by the errors over 200 distributions for a given standard deviation, and then average all RMS to get the effect on the field zero caused by the error with a given standard deviation.

The change of field zeros

Fig. 2 shows the change of field zeros versus the RMS errors in both, y and z direction. It can be seen that the change of field zeros grows almost linearly with the errors as described by the following approximate formulae.

Change by the error in z direction:

$$\delta z_0[\mu m] = 0.733385\sigma_1[\mu m]$$

Change by the error in y direction:

$$\delta z_0[\mu m] = 0.1926\sigma_2[\mu m]$$

The effect of errors in z direction is 3.8 times larger than in y direction.

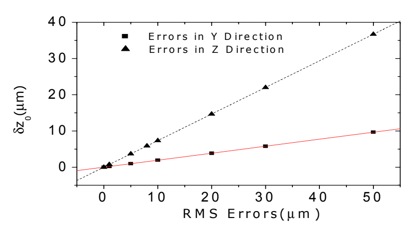

Figure 2: Change of field zeros versus positional error

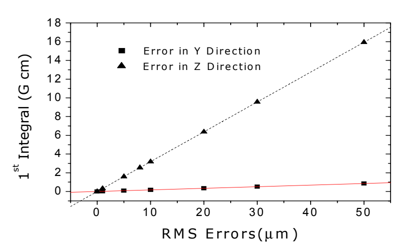

Figure 3: 1st field integral versus positional error

The change of field integrals

As shown in figs. 3 and 4, the field integrals also depend linearly on the RMS positional errors. The fit yields

- Change of the first field integral
  Errors in z direction: $I_z[G\,cm] = 0.31877\sigma_1[\mu m]$
  Errors in y direction: $I_y[G\,cm] = 0.01703\sigma_2[\mu m]$
- Change of the second field integral
  Errors in z direction: $II_z[G\,cm^2] = 22.98\sigma_1[\mu m]$
  Errors in y direction: $II_y[G\,cm^2] = 9.78\sigma_2[\mu m]$

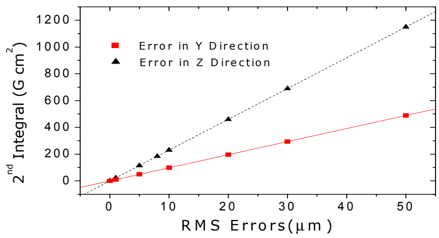

Figure 4: 2nd field integral versus positional error

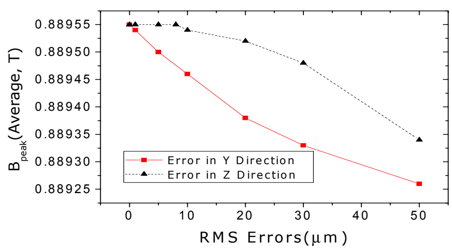

Figure 5: Average peak field versus positional error

The change of phase error and peak field

The average peak field and the RMS field error are also investigated, as in Fig. 5 and 6. In Fig. 5, $\sigma_{\Delta B} / B_{peak}(\%) = 0.42$ even for vanishing RMS error because of the influence of the finite length and of the end field structures on the field as discussed in [5].

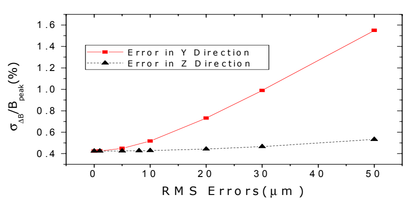

Figure 6: $\sigma_{\Delta B} / B_{peak}(\%)$ versus positional error

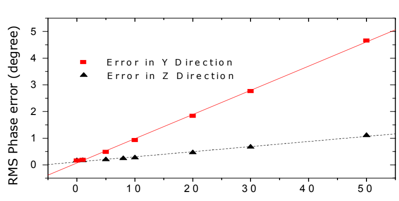

Figure 7: Phase error versus positional error

Fig. 7 gives the RMS phase error as a function of the error in y and z directions.

Errors in z direction: $\sigma_\varphi[\deg] = 0.16 + 0.0187\sigma_1[\mu m]$

Errors in y direction: $\sigma_\varphi[\deg] = 0.16 + 0.0889\sigma_2[\mu m]$

Discussion

From the above error study, the pitch error is found the most critical among the systematic errors pitch, yaw, and roll, As for the errors in wire positions, errors in z direction affect the field zeros and integrals more than those in y direction, while the phase error depends more strongly on errors in the y direction. These results can be used to help assess the minimal required positional tolerances, for example, if we want to construct an undulator with a phase error < 1.5°, the maximum change of field zero < 10 μm, $\sigma_{\Delta B} / B_{peak}$ < 1%, the required tolerances are a pitch error < 5 μrad, yaw error and roll error < 200 μrad, and positional errors of the wire < 4 μm in both directions.

CONCLUSION

In this paper, the influences of mechanical errors on the quality of the field of supraminis are investigated. Using analytical formulae including the effect of the errors in positioning the wires, semi-analytical numerical simulations are performed to estimate the tolerances of various errors. The pitch error was found the most critical error. Although the simulations are for a special specification of SSLS' prototype supramini, this method can be applied generally on other cases of such superconducting undulators.

REFERENCES

[1] H.O. Moser, B. Krevet, H. Holzapfel, "Mikroundulator", German patent P 41 01 094.9-33, Jan. 16, 1991.

[2] S. Casalbuoni, M. Hagelstein, B. Kostka, and R. Rossmanith, " Generation of X-ray radiation in a storage ring by a superconductive cold-bore in-vacuum undulator", Physical Review ST-Accelerator and Beams 9, 010702 (2006)

[3] Mikael Eriksson, Lars-Johan Lindgren, Erik Wallen, Sverker Werin, "A cascaded optical klystron on an energy recovery linac – race track microtron" Nucl. Instr. Meth. A 507 (2003) 470–474.

[4] G. Ingold , I. Ben-Zvi, L. Solomon, M. Woodle, "Fabrication of a high-field short-period superconducting undulator", Nucl. Instr. Meth. A375 (1996) 451-455

[5] H.O. Moser and C.Z. Diao, "Finite-length field error and its compensation in superconducting miniundulators". Nucl. Instr. Meth. A535 (2004) 606-613

[6] Olivier Chocron, Handout No 2, Course 2.05, October 2000. http://web.mit.edu/2.05/www/Handout/HO2.PDF