# OPTIMIZATION OF THE BEPCII LATTICE WITH FREQUENCY MAP ANALYSIS\*

Y. Jiao<sup>#</sup>, J.Q. Wang, D.M. Zhou, Q. Qin, G. Xu, S.X. Fang Institute of High Energy Physics, Beijing 100049, China.

## Abstract

The method of frequency map analysis (FMA) is used to analyze and optimize the lattice of the BEPCII storage ring. With the longitudinal oscillation and radiation being considered, both on- and off-momentum frequency maps are studied to have a complete picture of beam dynamics. The synchro-betatron resonances are found to be responsible for one limit of the dynamic aperture, and some efforts are made to reduce the effect of these resonances.

### **INTRODUCTION**

BEPCII is the upgrade project of the Beijing Electron Positron Collider (BEPC) being built at IHEP. To enhance its luminosity, a double-ring design is adopted to house the micro- $\beta$  scheme and multi-bunch collision. For the colliding beams the design luminosity is  $1 \times 10^{33}$  cm<sup>-2</sup>s<sup>-1</sup> optimized at beam energy of 1.89 GeV, which is about two orders higher than that of the BEPC<sup>[1]</sup>. According to the beam-beam simulations with Cai's code<sup>[2]</sup> and Zhang's code<sup>[3]</sup>, the working point is chose to be close to the half integer resonance, which thus calls for elaborate analysis and optimization of the beam dynamics.

Frequency map analysis (FMA) is a very useful tool to study the single particle dynamics by constructing a oneto-one relationship between the space of initial conditions (x, y, x' = y' = 0) and the tune space  $(Q_x, Q_y)^{[4]}$ . With this method, we can obtain the dynamic aperture (DA) and the corresponding frequency map (FM) at the same time, and study the effects and features of only one resonance out of others. This method has been used among many light sources (ALS, NSLS, SPEAR3, SOLEIL, ESRF, SSRF, etc) <sup>[5-10]</sup> and other machines (LHC, SNS) <sup>[11, 12]</sup>. It is the first time that we systematically apply the FMA to the lattice of the BEPCII storage ring.

The paper is organized as follows. In Sec. 2, we present the FMA results of BEPCII lattices and study the effect of synchro-betatron resonances on the single beam dynamics in detail. Several ways to the optimize the lattices are discussed in Sec. 3. We summarize and conclude our work in Sec. 4.

# FMA ON THE BEPCII LATTICES

# **BEPCII** collision mode

The BEPCII lattice loses the 4-fold symmetry of the BEPC, due to the double-ring geometric structure <sup>[1]</sup>. The vertical beta function at the IP is squeezed by a pair of

\*Work supported by National Natural Science Foundation of China (10375076) #ijaovi@iben.ac.cn

#jiaoyi@ihep.ac.cn

defocusing superconducting quadrupoles. The dispersion functions are free at the IP, the RF region (outer ring) and the injection point. The beta functions are required to be less than 15 m at the RF cavities in both vertical and horizontal planes and largger than 20 m at the injection point in horizontal plane to save the kicker strength.

A recently designed collision mode lattice  $^{[13]}$  with the working point of (6.51, 5.58) is put into analysis using FMA. Table 1 gives the main parameters of the collision mode and the injection mode which will be referred in the following text.

Table 1: Main parameters of the BEPCII lattices

Parameter	Unit	Collision	Injection
		mode	mode
Beam energy	GeV	1.89	1.89
Circumference	m	237.53	237.53
RF voltage	MV	1.5	1.5
$Q_x/Q_y/Q_s$		6.51/5.58/	6.58/5.62/
		0.034	0.034
Natural chromatity		-10.7/-21.0	-10.5/-12.1
Horizontal natural	nm∙	141	125
emittance	rad	141	155
$\boldsymbol{\beta}_{x}^{*} / \boldsymbol{\beta}_{y}^{*}$ (IP)	m	1/0.015	2.8/0.04

For the horizontal working point is much close to the half integer resonance, the nonlinear optimization of the lattice with 4 families of sextupoles by MAD <sup>[14]</sup>, is not perfect. As shown in Fig 1, the average off-momentum DAs with errors (20 random seeds) from SAD <sup>[15]</sup> are only slightly largger than the required aperture for collision which is  $10\sigma_x \times 10\sigma_y$  and labeled as the solid square, but smaller than the requirements for injection, i.e.,  $13.5\sigma_x \times 10\sigma_y$ , the dashed square. The minimum DAs with errors (among 20 random seeds) for different momentum deviations are even smaller (for more details, refer to [13]).



Figure 1: The average DAs with  $\Delta p/p \in (-10\sigma_e, 10\sigma_e)$  from SAD, with multipolar magnetic errors<sup>[13]</sup>.

For FMA analysis, tracking is done with AT <sup>[16]</sup>. Starting from the RF cavity, the particles with different

initial conditions ( $x_0$ ,  $y_0$ ,  $x_0'=y_0'=0$ , l=0,  $\delta=0$  &  $\pm 0.006$ ) are tracked for 2000 turns with RF cavity and radiation on. The tunes over the first 1000 turns and the last 1000 turns as well as the diffusion rate of particle motion are computed with the FMA method.

The DAs and FMs are given in Fig. 2. There exists great disparity between the on- and off-momentum DAs and FMs. That is, the off-momentum FMs are not folded like the on-momentum case and the tune footprints cover the range  $Q_r \in (6.514, 6.516)$  with a higher diffusion rate or even particle loss. The great disparity can be explained as follows. According to the theory of super-periodic structural resonances analysis (SSRA)<sup>[10,17]</sup>, the horizontal working point is very close to the half integer resonance, which is expected to be a second order super-periodic structural resonance with strong nonlinearity. The chromaticity correction with sextupoles does not have enough effective action range of momentum deviations, leading to a great difference between the on- and offmomentum beam dynamics. The off-momentum beam dynamics could be affected by the synchro-betatron resonance  $2Q_x - Q_s = 13$ , thus the DA is much smaller than the on-momentum DA especially in vertical plane.



Figure 2: The FMs with  $\Delta p/p=0$  (a), -0.006 (b) and 0.006 (c), respectively. (d) shows the FMs of three cases together. The colour scale corresponds to the orbit diffusion from regular (blue) to chaotic (red) motion.

# **BEPCII** injection mode

The injection mode relaxes the constraints on the low vertical beta function at the IP and the working point close to half integer resonance, but calls for larger DA and better beam dynamics. We choose the working point as (6.58, 5.62), which is located in a relatively clean area in tune space.

The tracking (with RF cavity and radiation on) is performed over 2000 turns without errors, with the last 1000 turns being devoted to the diffusion computation. The on- and off-momentum beam dynamics is similar, thus only the on-momentum case is analyzed (Fig 3). The DA is large, namely,  $[-30, 45]_{y=0} \times [0, 42]_{x=0}$  mm. From the one-to-one correspondence between the DA and FM, we find that the horizontal DA is limited by the transverse coupling resonance  $Q_x - Q_y = 1$ . Furthermore, we find the synchro-betatron resonance  $2Q_x + 3Q_y + Q_s = 30$  locates at the border of the DA, i.e., it limits the stability area. To confirm the existence of the synchro-betatron resonance, A particle is launched with initial conditions (x, x', y, y'),  $\delta_{l}(l) = (30 \text{ mm}, 0, 15.5 \text{ mm}, 0.3\%, 0)$  and tracked using AT. The voltage of the RF cavity is the designed value of 1.5 MV. The particle lost in 400 turns, with the coordinates of the particle plotted turn by turn in Fig 4. The vertical amplitudes increase rapidly and exceed the vacuum height as  $\pm 26$ mm. The tunes abstracted from the last 200 turns  $(Q_x, Q_y, Q_s) = (6.5807, 5.6058, 0.0327)$  are just on the resonance. For comparison, we make the tracking with the same initial coordinates under two cases: 1) RF cavity and radiation are off; 2) the voltage of the RF cavity is 3 MV (the  $Q_s$  will change). Both of the particles are stable.



Figure 3: The DA and FM of the injection mode lattice with  $\Delta p/p=0$  (a); FMs with  $\Delta p/p=0, \pm 0.006$  together (b).



Figure 4: Tracking with longitudinal oscillations and damping. The dashed dot lines indicate the height of the vacuum chamber  $\pm 26$ mm.

# **OPTIMIZATING THE BEPCII LATTICE**

#### BEPCII collision mode

That we could not move the working point away, or the luminosity will decrease, increases the difficulty of optimization. A big effort is made to find a set of sextupoles with smaller anharmonicities, changing the  $(dQ_x/dx^2, dQ_x/dy^2, dQ_y/dy^2)$  from (73, 77, 208) to (70, 68,

198), aiming to make the tunes reach the dangerous resonance at bigger amplitude. As shown in Fig 5(a), the average DAs with errors (20 random seeds) are slightly bigger than that before optimization and satisfy the requirement for beam injection.

Furthermore, we try to reduce the effect of resonance  $2Q_x - Q_s = 13$ . The growth time of resonance is <sup>[18]</sup>

$$\frac{1}{\tau} = f_0 \hat{\delta} H = f_0 \hat{\delta} \left| \sum_n k_{sn} D_{xn} \beta_{xn} e^{2i\phi_{xn}} \right|$$

where  $\hat{\delta}$  is the energy oscillation amplitude,  $f_0$  is the revolution frequency,  $k_s$ ,  $D_x$ ,  $\beta_x$ ,  $\phi$  are the integrated sextupole strength, horizontal beta function, horizontal dispersion, and azimuth angle, respectively. We use 9 groups of sextupoles instead of 4 groups and minimize the value of H by fine-tuning sextupole strength. For example, we change the H from 19.7 to 3.1 for  $\hat{\delta} = 0.003$ . The following tracking using SAD shows that the offmomentum DAs with errors (20 random seeds) have an obvious increase in horizontal plane (see Fig 5(b)).



Figure 5: The optimized DAs from SAD with errors and  $\Delta p/p \in (-10\sigma_e, 10\sigma_e)$ , (a): reduce the anharmonicities, (b): reduce the growth time of the resonance  $2Q_x - Q_s = 13$ .

### BEPCII injection mode

If we keep the tune footprints away from the resonance  $2Q_x + 3Q_y + Q_s = 30$  and  $Q_x - Q_y = 1$ , the limit of the dynamics will vanish and the DA will be larger. Optimizations based on this idea are performed. The working point is moved to (6.57, 5.61), and the amplitude tune shift slope at origin is reduced from  $(dQ_x/dx^2)_{x=0}=760$  to 133. As shown in Fig 6, the optimized DA (without errors) increases to be  $[-40, 50]_{y=0} \times [0, 45]_{x=0}$  mm. The DA (with errors) obtained with SAD is much bigger than the requirements of the injection mode, which confirms our optimization.



Figure 6: The DA and FM without errors from AT (a); the average DAs with errors and  $\Delta p/p \in (-10\sigma_e, 10\sigma_e)$  from SAD (b).

## CONCLUSION

The analysis and optimizations of the BEPCII collision and injection modes with FMA are described in this paper. FMA turns out to be a good and effective tool to look into the beam dynamics and the resonance structure in a global way, thus enabling us to avoid dangerous areas in tune space and to further improve the beam dynamics.

The RF cavity and synchrotron radiation are turned on during the tracking, that is, the longitudinal oscillation and the damping process are taken into account. The synchro-betatron resonances may be exited and affected the single particle dynamics. In the case of the BEPCII injection mode lattice with the working point of (6.58, 5.62), such a resonance is responsible for the limit of the DA. When we move the working point away from the resonance, the DA significantly increases.

Compared to the injection mode, the limiting factor for the lattice is more severe, the optimization consequently becomes more complicated and difficult. Reducing the nonlinear anharmonicity terms and minimizing the effect of the synchro-betatron resonance  $2Q_x - Q_s = 13$ , are proved to be beneficial to the improvement of the beam dynamics.

# REFERENCES

- [1] IHEP-BEPCII-SB-03-3, 2003.
- [2] G. Xu, IHEP-URAP-Note/2005-06, 2005.
- [3] Y. Zhang, IHEP Ph. D thesis, 2005.
- [4] L. Nadolski and J. Laskar, Phys. Rev. ST Accel. Beams 6, 114801 (2003).
- [5] C. Steier et al, EPAC 2002, pp. 1077, & D. Robin et al, Phys. Rev. Lett. 85, p. 558 (2000).
- [6] M. Belgroune et al, EPAC 2002, p. 1229.
- [7] Y. Papaphilippou et al, PAC 2003, p. 3189, & Y. Papaphilippou et al, EPAC 2003, p. 2050.
- [8] Y. Papaphilippou. PAC 2001, p. 462.
- [9] A. Terebilo et al, PAC 1998, p. 1457.
- [10] Y. Jiao et al. NIM A 566 (2006) p.270.
- [11] Y. Papaphilippou, Phys. Rev. ST Accel. Beams 2, 104001 (1999).
- [12] Y. Papaphilippou, PAC2001, p. 462.
- [13] Q. Qin, IHEP-AC-AP-Note/ 2006-01, 2006.
- [14] MAD code homepage, http:// mad.web.cern.ch/mad/.
- [15] SAD code homepage, http://acc-physics.kek.jp/SAD/sad.html.
- [16] AT code homepage, http:// www-ssrl.slac.stanford.edu/at /welcome.html.
- [17] S. X. Fang, Q. Qin. HEP & NP, 2006, 9: 880.
- [18] A. Piwiniski, DESY 93-187 (1993).