INJECTION OF BEAM SHAPED LOCALLY WITH NONLINEAR OPTICS*

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Abstract

We discuss nonlinear beam shaping by octupole and sextupole to fold the tails of a Gaussian beam into its core, for the purpose of improving betatron injection in storage rings by significantly reducing the beam width at the injection septum and thus reducing beam centroid offset from the stored beam. Necessary conditions as well as challenges for such nonlinear injections are explored.

INTRODUCTION

Injection is an important (yet often problematic) process in circular accelerators. Usually fast and strong kicks are used to merge the incoming beam with the stored beam (noticeable exceptions are charge-exchange H^- injection with a stripping foil and International Linear Collider damping ring injection where no stored beam exists during injection). Due to Liouville's theorem, certain phase-space separation of the incoming and stored beams are unavoidable. Here we consider the transverse injection scheme where the two beams are separated in the transverse phase space, say the horizontal plane. This injection scheme is commonly used in lepton machines where the two separated beams can be radiation damped into one. It is important to minimize the phase-space separation so that both beams can stay within the acceptance of the circular machine. This becomes more and more critical in modern storage-ring-based light sources such as the Advanced Photon Source, where acceptance is sacrificed for small emittance and narrow-gap insertion devices. On the other hand, the sizes of the stored and injected beams as well as the physical existence of a (septum) kicker requires certain clearance from the beams and thus limits the minimum separation reachable by the two beams. Therefore, the area close to the septum becomes the bottleneck of the injection process, which may result in demanding requirements for the kicker and incoming beam emittance. This note will explore possibilities to ease this bottleneck by properly folding the long tails of the Gaussian phase-space distribution of an incoming beam locally with nonlinear optics such that the incoming beam can come much closer to the septum and the stored beam.

NONLINEAR BEAM SHAPING

It is well-known that octupoles can be used to fold the tails of a Gaussian beam onto its core and make a more uniform distribution with sharper edges [1-4]. This technique is commonly used to make uniform illumination on targets. There are also proposals of using octupole beam shaping in linear colliders to make a cylindrical beam lens for final focusing or to make a nonlinear collimation system. To explore nonlinear beam shaping for injection purposes, we consider using a sextupole ($n = 2$) or octupole ($n = 3$).

To simplify the discussion we assume the nonlinear element is thin and write the beam transport from the nonlinear element to the injection point as

$$\begin{bmatrix} x \\ p \end{bmatrix} = R \begin{bmatrix} x_0 \\ p_0 \end{bmatrix} + kR \begin{bmatrix} 0 \\ x_0^n \end{bmatrix}, \qquad (1)$$

where R is the linear transfer matrix between the initial and final phase-space points $\{x_0, p_0\}$ and $\{x, p\}$, respectively; and k is the integrated strength of the nonlinear element. Using the Twiss parameters at the ends and the phase-advance Δ between them, the R-matrix can be written in the well-known form

$$R = \begin{bmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{bmatrix} \begin{bmatrix} \cos\Delta & \sin\Delta \\ -\sin\Delta & \cos\Delta \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{\beta_0}} & 0 \\ \frac{\alpha_0}{\sqrt{\beta_0}} & \sqrt{\beta_0} \end{bmatrix}. \qquad (2)$$

Since our main concern is the transverse dimension of the beam at the injection point, we express the transverse position as

$$x = \sqrt{2\beta J_0}\,[\cos(\theta_0 + \Delta) + 2\,\bar{a}\sin\Delta\cos^n\theta_0], \qquad (3)$$

where J_0 and θ_0 are the action-angle variables in the initial phase space and the parameter $\bar{a} \equiv 2^{\frac{n-3}{2}} k\, \beta_0^{\frac{n+1}{2}}\, J_0^{\frac{n-1}{2}}$. Our focus is on the beam edge given by the 3σ contour with $J_0 = 9\epsilon/2$, where ϵ is the emittance of injected beam. The first term yields the unperturbed position and the second term gives the correction due to the nonlinear kick. For a given $\bar{a}$, extreme position x_m will be reached when $\partial_{\theta_0} x = \partial_\Delta x = 0$, i.e.,

$$\begin{aligned} \sin(\theta_0 + \Delta) &= -2\bar{a}\sin\Delta\; n\cos^{n-1}\theta_0 \sin\theta_0 \\ &= 2\bar{a}\cos\Delta\; \cos^n\theta_0. \end{aligned} \qquad (4)$$

Let θ_m and Δ_m be a solution set, then we have the condition $\cos(\theta_m + \Delta_m) = -(n+1)\sin\theta_m \sin\Delta_m$ and the extreme value

$$x_m = \sqrt{2\beta J_0}[-(n+1)\sin\theta_m + 2\bar{a}\cos^n\theta_m]\sin\Delta_m. \qquad (5)$$

Therefore, it is desirable to have the phase advance Δ_m close to a multiple of π (note that there is no solution for $\Delta_m = \mathrm{integer}\cdot\pi$). A more geometric view of this is that, under such a condition, the peaks of the two terms in Eq. (3)

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are synchronized for more effective cancellation. Because of this and an important reason discussed below, it is preferable to choose a phase advance Δ close to but less than π.

With a phase advance $\Delta \simeq \pi$, the transverse position becomes

$$x \simeq \sqrt{2\beta J_0}\,[-\cos\theta_0 + 2\,\bar{a}\sin\Delta\cos^n\theta_0]\,. \quad (6)$$

For a sextupole, Eq. (6) can be written as

$$x^{\rm sex} = \sqrt{2\beta J_0}\,2\bar{a}\sin\Delta\left[\left(\cos\theta_0 - \frac{1}{4\bar{a}\sin\Delta}\right)^2 - \left(\frac{1}{4\bar{a}\sin\Delta}\right)^2\right]. \quad (7)$$

Note that the mapping due to a sextupole is asymmetric. The last term in the above equation gives the minimum width $\sqrt{2\beta J_0}/|8\bar{a}\sin\Delta|$ provided that $4\bar{a}\sin\Delta \geq 1$. If we choose $\bar{a}\sin\Delta = \pm 1/2$, a beam will be compressed by a factor of 4 on one side while enlarged by a factor of 2 on the other side. For an octupole, Eq. (6) has the extreme values

$$x_{m1} = \pm\sqrt{2\beta J_0}\,(1 - 2\,\bar{a}\sin\Delta), \quad (8)$$

$$x_{m2} = \pm\sqrt{2\beta J_0}\,\sqrt{\frac{4}{27}\frac{1}{2\bar{a}\sin\Delta}}, \quad (9)$$

at $\theta_0 = \text{integer}\cdot\pi$ and $\cos^2\theta_0 = 1/6\,\bar{a}\sin\Delta$, respectively. Setting $|x_{m1}| = |x_{m2}|$ yields the optimum solution

$$x^{\rm oct}_{\rm opt} = \sqrt{2\beta J_0}\,\frac{\cos(3\theta_0)}{3}, \quad \text{with } \bar{a}\sin\Delta = \frac{2}{3}, \quad (10)$$

where a factor 3 reduction in amplitude can be obtained, which is consistent with [5].

SYMMETRIC FOLDING BY OCTUPOLE

To illustrate the beam shaping effects, we show some examples with the parameters $\beta = \beta_0 = 20\,m$, $\alpha = \alpha_0 = 0$, $\epsilon = 90\,nmrad$, which are close to the injected beam parameters at the final septum kicker in the Advanced Photon Source. Two cases are shown here. One with a phase advance $\Delta = 0.98\pi$, and $k = 6.55\times10^4\,m^{-3}$, which gives $\bar{a}\sin\Delta \simeq 2/3$ for 3σ contour. The other with a phase advance $\Delta = 0.9\pi$, and $k = 1.2\times10^4\,m^{-3}$, which gives $\bar{a}\sin\Delta = 0.6$. In Fig. 1a and Fig. 1b the phase-space contours are plotted for the two cases together with the 3σ-contours for the original beam and a Gaussian beam with 3 times smaller emittance. In Fig. 1c beam distributions across the axis are plotted for the two cases together with the original distribution as well as an unshaped distribution with a 3 times smaller emittance. Clearly octupole beam-shaping is symmetric, which is important for many applications (and thus more commonly used than sextupoles).

ASYMMETRIC FOLDING BY SEXTUPOLE

Similar to the octupole examples above, Fig. 2 shows beam shaping with a sextupole. For the $\Delta = 0.98\pi$ example, a sextupole with $k = 200\,m^{-2}$ ($\bar{a}\sin\Delta \simeq 0.5$) is used. For the $\Delta = 0.9\pi$ example, a sextupole with $k = 27\,m^{-2}$ ($\bar{a}\sin\Delta \simeq 0.34$) is used. Unlike using an octupole, sextupole beam shaping is asymmetric. It can effectively fold one side of the Gaussian tail back into its core, but on the other side, the tail becomes longer. Since the main congestion during injection is only on the side close to the septum, sextupole beam shaping can be used to ease injection. Note that the minimum width in Fig. 2c is smaller than that in Fig. 1c.

The illustrations in Figs. 1 and 2 demonstrate that an octupole or sextupole can effectively fold the Gaussian tail back into its core and significantly reduce the effective beam width. For the moderately shaped beams in Figs. 1b and 2b, it is equivalent to reducing the injected beam emittance threefold, while the required field strengths are feasible.

LOCALIZING NONLINEAR DISTORTION

Although nonlinear beam shaping can significantly reduce beam width and thus ease the injection bottleneck around the final septum, overall the beam will occupy a much larger phase-space area after strong nonlinear distortions. Therefore, it is important to make such a distortion as local as possible. An obvious solution is to compensate the initial kick with another nonlinear kick after a π phase advance. If the Twiss parameters at both kicks are the same, we have the common “-I” correction where kicks of the same strength will cancel each other and leave no nonlinear effects afterward. However, the map between the opposite kicks does not have to be a “-I” transformer, as long as it is linear with a π phase advance. Under such conditions, the beam at the compensating element (with Twiss parameters β_c and α_c) is given by

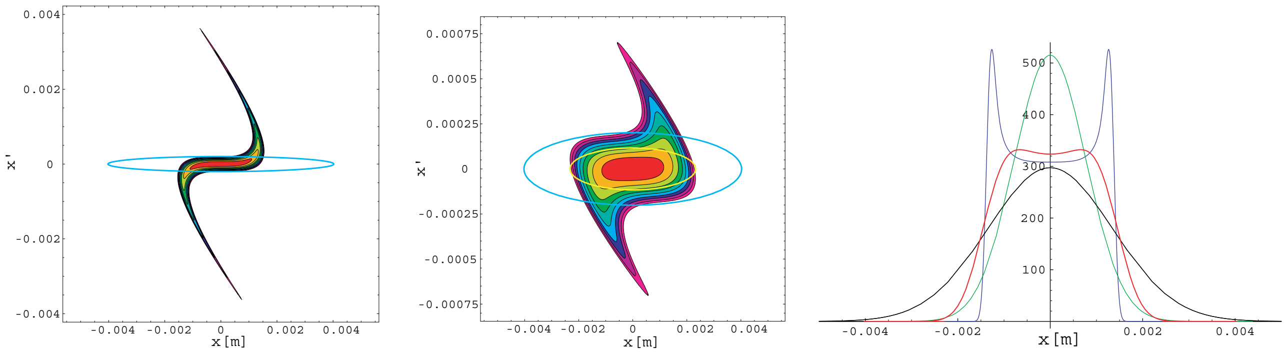
$$\begin{bmatrix} x \\ p \end{bmatrix} = -\begin{bmatrix} \sqrt{\beta_c/\beta_0} & 0 \\ \frac{\alpha_0-\alpha_c}{\sqrt{\beta_c\beta_0}} & \sqrt{\beta_0/\beta_c} \end{bmatrix}\begin{bmatrix} x_0 \\ p_0 \end{bmatrix} - k\sqrt{\frac{\beta_0}{\beta_c}}\begin{bmatrix} 0 \\ x_0^n \end{bmatrix}, \quad (11)$$

thus, a compensating kick of strength

$$\bar{k} = \left(\frac{\beta_0}{\beta_c}\right)^{\frac{n+1}{2}} k \quad (12)$$

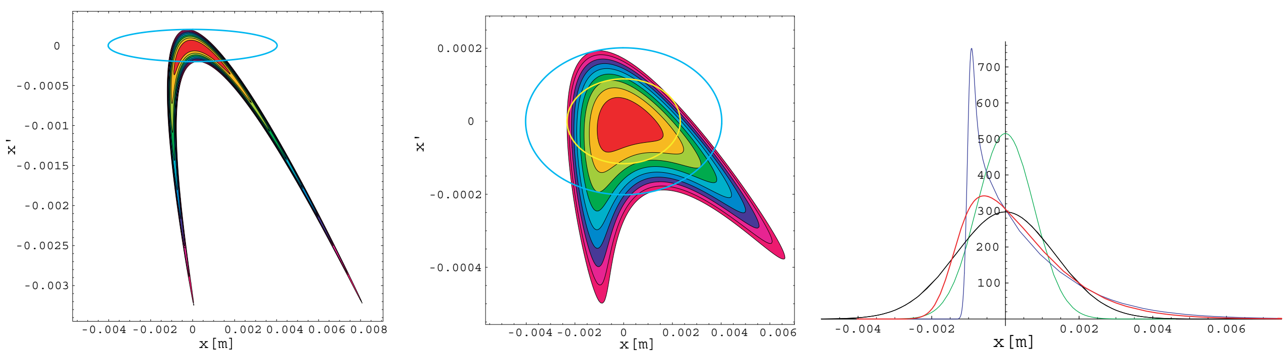
will remove the nonlinear term. The β-factor could be used to reduce the strength of the compensating nonlinear element that is located inside the storage ring. For example, $\beta_c = 30\,m$ will reduce the compensating sextupole strength from $k = 27$ in Fig. 2b to $\bar{k} = 15$.

As we discussed before, the phase advance between the beam-shaping element to the injection point is close to but less than π, thus the compensating nonlinear element will be fairly close to the injection point with a small phase advance between to make up the difference from π. Therefore, the storage ring will see little of the nonlinear manipulation of the injected beam. However, the compensating nonlinear element in the storage ring will usually be strong and need certain upstream nonlinear elements in the ring



(a) $\Delta = 0.98\pi$, $\bar{a}\sin\Delta \simeq 2/3$ (b) $\Delta = 0.9\pi$, $\bar{a}\sin\Delta \simeq 0.6$ (c) beam distributions across the axis

Figure 1: Illustration of octupole beam shaping: (a) contour plots of phase-space distributions, up to 3σ contour for a shaped beam and 3σ contour only for the original beam; (b) similar to (a) but with additional 3σ contour (yellow) of a three times smaller emittance; (c) beam distributions across the axis for the original beam (black), shaped beam (blue) with $\Delta = 0.98\pi$, shaped beam (red) with $\Delta = 0.9\pi$, and unshaped beam (green) with a three times smaller emittance.



(a) $\Delta = 0.98\pi$, $\bar{a}\sin\Delta \simeq 1/2$ (b) $\Delta = 0.9\pi$, $\bar{a}\sin\Delta \simeq 0.34$ (c) beam distributions across the axis

Figure 2: Illustration of sextupole beam shaping. See the caption of Fig. 1 for explanation.

to reduce its effect on the stored beam such that most of the ring will be sufficiently linear with adequate dynamic aperture. Though nontrivial, it seems possible to accommodate such a nonlinear injection in a storage ring, especially in new designs (whether it is worthwhile is another issue). Note that the strength of the compensating element

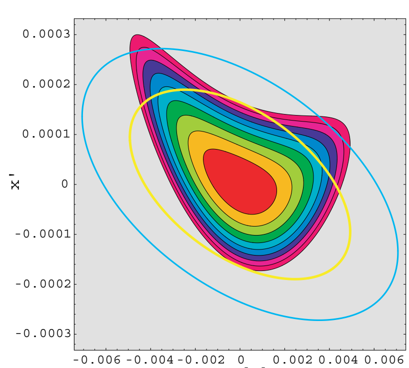


Figure 3: Phase-space contours after an incomplete compensation with half of the required strength. Inner (outer) ellipse is the undistorted 3σ (4.3σ) contour.

can be somewhat smaller than $\bar{k}$, because non-perfect correction will only leave a small amount of nonlinear distortion in the injected beam, which is tolerable in the ring. As an example, Fig. 3 plots the phase space after a compensating sextupole with only half the $\bar{k}$ required by Eq. (12). Another possibility is to cut off the nonlinear tails before injection by collimation. Further feasibility study is necessary, including the effects of possible centroid offsets in the compensating element.

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