BEAM DYNAMICS EFFECTS WITH INSERTION DEVICES FOR THE PROPOSED 3 GEV RING IN TAIWAN

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Abstract

The effects of insertion devices on beam dynamics of storage rings were studied. We will focus on the changes of the emittance and the energy spread in the presence of insertion devices. Formulas for the beam emittance and the energy spread with insertion devices are also derived, in which an intrinsic parameter depending on the design of the lattice is introduced in the calculations. Simulation results and comparisons of achromatic and non-achromatic cases in the proposed 3 GeV synchrotron radiation light source in Taiwan are shown.

INTRODUCTION

A new 3 GeV third generation synchrotron radiation light source is proposed by National Synchrotron Radiation Research Center (NSRRC) in Taiwan[1]. The circumference of the storage ring is 486 m with natural emittance 1.7 nm-rad for non-achromatic case and 5.2 nm-rad for achromatic case. The twiss functions of non-achromatic case are shown in Figure 1. The lattice has a structure of 24 cells of DBA with 6-fold symmetry. It has 24 straights including 6 long straights (10.9 m) and 18 standard straights (5.8 m) for injection or insertion devices (IDs).

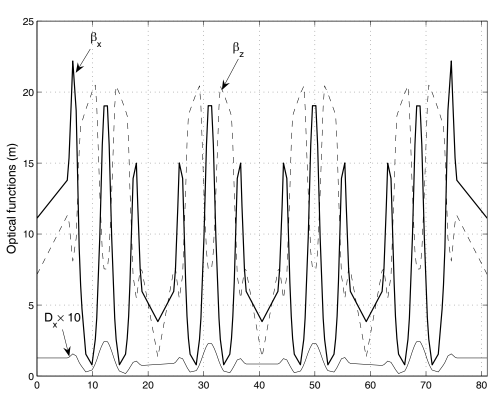

Figure 1: Optical functions of one superperiod (non-achromat lattice)

IDs are often used to produce partial coherient light or to shift energy ranges of synchrotron radiation light source. In the meanwhile, they bring some effects on electron beam parameters. First consider the synchrotron radiation integrals defined by

$$I_2 = \oint \frac{1}{\rho^2} ds \tag{1}$$

$$I_3 = \oint \frac{1}{|\rho|^3} ds \tag{2}$$

$$I_4 = \oint \frac{D_x}{\rho} (\frac{1}{\rho^2} + 2K) ds \tag{3}$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho|^3} ds, \text{ where } \mathcal{H} = \frac{D_x^2 + (\alpha_x D_x + \beta_x D_x')^2}{\beta_x} \tag{4}$$

The integrals are taken over the whole ring. K is the focusing function. β_x is the horizontal betatron amplitude funcion and $\alpha = -\beta_x'/2$. D_x is the horizontal dispersion function and D_x' is its differential.

Some important parameters of beam dynamics can be characterized by synchrotron radiation integrals.

- Energy loss per turn

$$U_0 = C_\gamma \frac{\beta^3}{2\pi} E^4 I_2 \approx \frac{C_\gamma E^4 I_2}{2\pi} \tag{5}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_0}{(mc^2)^3} = 8.85 \times 10^{-5} \text{ m/(GeV)}^3$$

- Energy spread

$$(\frac{\sigma_E}{E})^2 = C_q \gamma^2 \frac{I_3}{2I_2 + I_4} \tag{6}$$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} = 3.83 \times 10^{-13} \text{ m}$$

- Natural emittance

$$\varepsilon_{x0} = C_q \gamma^2 \frac{I_5}{I_2 - I_4}\ , \tag{7}$$

where r_0 and m are the radius and the mass of the particle (here electron); γ is the Lorentz factor; c is the speed of light in vaccum; E is the particle energy; and $\hbar$ is Planck's constant.

EFFECTS OF INSERTION DEVICES

Adding IDs brings additional magnitudes of synchrotron radiation integrals. Taking this effect into account the

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following relations can be derived.

$$\Delta U_w = \frac{C_\gamma E^4 \Delta I_2}{2\pi} \tag{8}$$

$$\sigma_E = \sigma_E^0 \sqrt{\frac{1+\frac{\Delta I_3}{I_3^0}}{1+\frac{2\Delta I_2+\Delta I_4}{2I_2^0+I_4^0}}} \tag{9}$$

$$\varepsilon_x = \varepsilon_x^0 \left(\frac{1+\frac{\Delta I_5}{I_5^0}}{1+\frac{\Delta I_2-\Delta I_4}{I_2^0-I_4^0}}\right), \tag{10}$$

where the superscript 0 denotes that the value is evaluated for the case of no IDs added. There is no approximation made so far. For separate function accelerators, we can omit I_4 because I_4 is much smaller than I_2. Similarly ΔI_4 is also small.

From the definition of synchrotron radiation integrals, the following relations holds: $I_2^0 = \frac{2\pi}{\rho_0}$ and $I_3^0 = \frac{2\pi}{\rho_0^2}$, where ρ_0 is the bending radius of dipole magnets with field strength B_0. For the integral $I_5^0 = \oint \frac{\mathcal{H}}{|\rho|^3} ds$, $\frac{1}{\rho}$ is nonzero only in dipole magnets and we can tear the integral into a product of total dipole length and the averaged $\mathcal{H}$ in the dipole magnets.

$$I_5^0 = \oint_{\text{dipoles}} \frac{\mathcal{H}(s)}{|\rho_0|^3} ds = \frac{1}{\rho_0^3} \times 2\pi\rho_0 \langle\mathcal{H}\rangle = \frac{2\pi}{\rho_0^2}\langle\mathcal{H}\rangle, \tag{11}$$

where the bracket denotes the average operator only in the dipole magnets. The $\langle\mathcal{H}\rangle$ can be evaluated by equation (7)

$$\varepsilon_{x0} \approx C_q\gamma^2 \frac{I_5}{I_2} = C_q\gamma^2 \frac{2\pi\langle\mathcal{H}\rangle/\rho_0^2}{2\pi/\rho_0} = C_q\gamma^2 \frac{\langle\mathcal{H}\rangle}{\rho_0} \tag{12}$$

if we know the natural emittance of the lattice.

Now we try to estimate the perturbed quantities ΔI_2, ΔI_3, and ΔI_5. Suppose B_w is the peak field, λ_w is the period length, and N is the number of periods of the ID. The length of the ID is therefore $L_w = N\lambda_w$. A simple model of magnetic field of the ID can be written as

$$B_z(s) = B_w \cos(\frac{2\pi s}{\lambda_w}) \tag{13}$$

Let ρ_w be the bending radius corresponding to the B_w, ρ is also a function of s.

$$\frac{1}{\rho(s)} = \frac{1}{\rho_w}\cos(\frac{2\pi s}{\lambda_w}) \tag{14}$$

ΔI_2 and ΔI_3 are independent of the dispersion function and can be evaluated through (1) and (2) directly by changing the range of the integral from the overall ring to only the range of that ID.

$$\begin{aligned} \Delta I_2 &= N\int_0^{\lambda_w} \frac{1}{\rho^2} ds = \frac{N}{\rho_w^2}\int_0^{\lambda_w} \cos^2(\frac{2\pi s}{\lambda_w}) ds \\ &= \frac{\lambda_w N}{2\rho_w^2} = \frac{L_w}{2\rho_w^2} \end{aligned} \tag{15}$$

$$\begin{aligned} \Delta I_3 &= N\int_0^{\lambda_w} \frac{1}{|\rho|^3} ds = \frac{N}{\rho_w^3}\int_0^{\lambda_w} |\cos(\frac{2\pi s}{\lambda_w})|^3 ds \\ &= \frac{4}{3\pi}\frac{\lambda_w N}{\rho_w^3} = \frac{4L_w}{3\pi\rho_w^3} \end{aligned} \tag{16}$$

The change of $\mathcal{H}$-function due to the ID is small. $\mathcal{H}$ keeps nearly a constant in the range of ID and can be carried outside the integral

$$\begin{aligned} \Delta I_5 &= \int_w \frac{\mathcal{H}}{|\rho|^3} ds = \frac{1}{\rho_w^3}\int_w \mathcal{H}(s)|\cos^3(\frac{2\pi s}{\lambda_w})| ds \\ &= \frac{\hat{\mathcal{H}}}{\rho_w^3}\int_w |\cos^3(\frac{2\pi s}{\lambda_w})| ds = \frac{4\hat{\mathcal{H}} L_w}{3\pi\rho_w^3}, \end{aligned} \tag{17}$$

where $\hat{\mathcal{H}}$ is the $\mathcal{H}$ value in straight section without IDs.

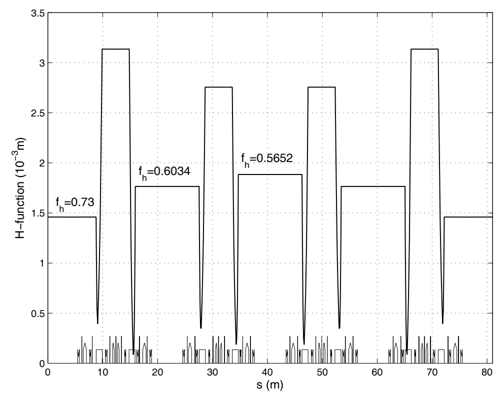

Figure 2: $\mathcal{H}$-function of one superperiod (non-achromat lattice)

Defining f_h by $\langle\mathcal{H}\rangle = f_h\hat{\mathcal{H}}$, f_h is a factor depending on the design of the lattice and the position of IDs. Figure 2 shows the $\mathcal{H}$-function of one superperiod of the non-achromat lattice calculated by MAD. The value of f_h at three kinds of straights are 0.7300, 0.6034, 0.5652, respectively. Putting equations (15-10) into equations (8-10), we get the final formulas for non-achromat lattices.

$$\Delta U_w = \frac{C_\gamma E^4 L_w}{4\pi\rho_w^2} \tag{18}$$

$$\sigma_E = \sigma_E^0 \sqrt{\frac{1+\frac{2L_w\rho_0^2}{3\pi^2\rho_w^3}}{1+\frac{L_w\rho_0}{4\pi\rho_w^2}}} \tag{19}$$

$$\varepsilon_x = \varepsilon_x^0 \left(\frac{1+\frac{2L_w\rho_0^2}{3\pi^2 f_h\rho_w^3}}{1+\frac{L_w\rho_0}{4\pi\rho_w^2}}\right) \tag{20}$$

One should note that the emittance will decrease slightly for $B_w < \frac{3\pi f_h B_0}{8}$ and grow rapidly when $B_w \geq \frac{3\pi f_h B_0}{8}$. Similary case for the energy spread where the critical field $B_w = \frac{3\pi B_0}{8}$.

For achromat lattices, the dispersion function vanishes in straight sections and the perturbed dispersion function caused by the ID itself is small and can be omitted. Therefore ΔI_5 is approximately zero by definition and the equation of the emittance change can be written as

$$\varepsilon_x = \varepsilon_x^0\left(\frac{1}{1+\frac{\Delta I_2}{I_2}}\right) = \varepsilon_x^0\left(\frac{1}{1+\frac{L_w\rho_0}{4\pi\rho_w^2}}\right) \tag{21}$$

For more than one IDs, the total effects on the synchrotron integrals are accumulated from the contribution of each ID individually. The change of the energy spread and the emittance can be generalized as

$$\sigma_E = \sigma_E^0 \sqrt{\frac{1+\sum_w \frac{2L_w\rho_0^2}{3\pi^2\rho_w^3}}{1+\sum_w \frac{L_w\rho_0}{4\pi\rho_w^2}}} \tag{22}$$

$$\varepsilon_x = \varepsilon_x^0 \left(\frac{1+\sum_w \frac{2L_w\rho_0^2}{3\pi^2 f_{h,w}\rho_w^3}}{1+\sum_w \frac{L_w\rho_0}{4\pi\rho_w^2}} \right) \tag{23}$$

For ideal helical undulators, $\Delta I_2 = \frac{L_w}{\rho_w^2}$, $\Delta I_3 = \frac{L_w}{\rho_w^3}$, and $\Delta I_5 = \frac{\hat{\mathcal{H}}L_w}{\rho_w^3}$. The two equations above can be modified as

$$\sigma_E = \sigma_E^0 \sqrt{\frac{1+\sum_w \frac{L_w\rho_0^2}{4\pi\rho_w^3}}{1+\sum_w \frac{L_w\rho_0}{4\pi\rho_w^2}}} \tag{24}$$

$$\varepsilon_x = \varepsilon_x^0 \left(\frac{1+\sum_w \frac{L_w\rho_0^2}{4\pi f_{h,w}\rho_w^3}}{1+\sum_w \frac{L_w\rho_0}{4\pi\rho_w^2}} \right) \tag{25}$$

RESULTS AND DISCUSSIONS

The parameters of the IDs proposed by NSRRC are listed in Table 1. During the simulation only vertical field components are considered for the elliptically polarizing undulators. Randomly distributing all the IDs in the ring as shown in Figure 3, good agreements of the results of ID effects calculated by MAD and predicted by this formula are shown in Table 2.

Table 1: Parameters of IDs of TPS

ID Name	Length (m)	Peak Field (Bz/Bx) (Tesla)	Num of ID
EPU10	3.7	1.0	1
EPU7	3.6	1.0/0.77	2
SW6	2.0	3.5	1
EPU6	3.7	0.9/0.7	2
EPU4.6	3.7	0.76/0.49	2
IVU2.8	3.7	0.9	10
SU1.5	2.0	1.4	1
CU1.8	3.7	1.29	4

Take a look at equation (21) for achromatic case, it can be rewritten as $\varepsilon_x = \varepsilon_x^0 \left(\frac{1}{1+\frac{\Delta U_w}{U_0}} \right)$ by equations (6) and (8). The theorical minimal emittance of DBA lattice is 3 times larger than that of DB lattice. To reduce the emittance by a factor of 3, the required radiated energy from IDs (ΔU_w) must be twice than the energy radiated from the dipole (U_0, here is 0.8598 MeV/turn). But the total energy loss is too high for the rf cavity to compensate. The dipole field must be designed to be weaker so that the rf voltage be in a resonable range. On the other hand, strong bending dipole fields are preferred to hold the growth of the emittance for non-achromatic case because the emittance grows when $B_w \geq \frac{3\pi f_h B_0}{8}$.

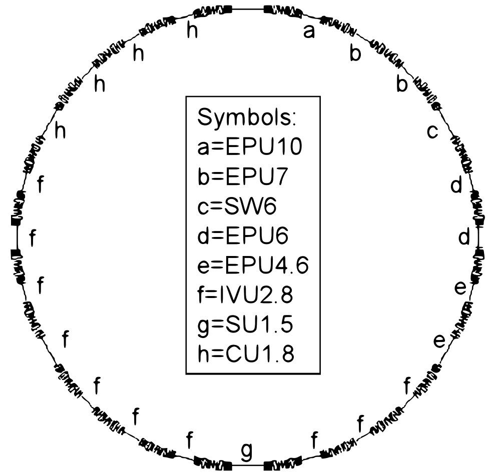

Figure 3: The locations of IDs

Table 2: Comparison of Effects of IDs on Beam Dynamics calculated by MAD8 and by the formula (non-achromat lattice)

ID Name	Relative Energy Spread (10^{-4})		Emittance (nm-rad)	
	MAD8	Formula	MAD8	Formula
w/o ID	8.8598	8.8598	1.6748	1.6748
EPU10	8.8294	8.8291	1.6818	1.6821
EPU7	8.8011	8.8006	1.6915	1.6920
SW6	9.7174	9.7392	2.2394	2.2392
EPU6	8.7996	8.7991	1.6808	1.6811
EPU4.6	8.8040	8.8036	1.6664	1.6665
IVU2.8	8.5919	8.5911	1.6924	1.6933
SU1.5	8.8595	8.8596	1.6903	1.6904
CU1.7	8.8092	8.8094	1.7881	1.7889
ALL ID	9.1251	9.1403	2.1758	2.1770

Under the situation of all proposed IDs installed, the total energy lost from all IDs are 0.5914 MeV per turn and the energy spread is 0.0914%. The natural emittance for non-achromatic and achromatic cases are 2.1770 and 3.1122 nm-rad respectively. For non-achromatic case the presence of dispersion in straights must be taken in account.

The two dimensional effective emittance is defined by $\varepsilon_{\text{eff}} = \sqrt{\varepsilon_x^2 + H\sigma_\delta^2\varepsilon_x}$, where σ_δ represents the relative energy spread $\frac{\sigma_E}{E}$. The effective emittance at three kinds of straights are respectively 2.7189, 2.8194, and 2.8576 nm-rad, which are smaller than 3.1122 nm-rad of the achromat lattice.

ACKNOWLEDGEMENTS

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REFERENCES

[1] See “NSRRC TPS Proposal”.

[2] S. Y. Lee, “Accelerator Physics”, World Scientific, 2004.