INHOMOGENOUS FIELD WIEN FILTER DESIGN

Puneet Jain #, P. Roychowdhury, D. P. Chakravarthy, Asish K. Ray
Accelerator and Pulse Power Division, BARC, Mumbai 400085, INDIA

Abstract

The Wien velocity filter is a useful device that transports pure proton fraction from high-power ECR proton source to the RFQ. It is a deflecting device, which has crossed electrostatic, and magnetostatic fields. Both fields are perpendicular to the beam trajectory that deflects and eliminates the undesired species of ions from the main beam. A tilted-pole Wien filter [1] surpasses the classical parallel-rectangular-poles Wien filter in performance as the former eliminates the astigmatism. The present paper describes the design of an inhomogeneous field Wien filter where the equations of motion are developed and solved up to a second-order approximation for a paraxial ion beam inside an E×B mass separator without considering the space charge effects.

INTRODUCTION

A high current low emittance ECR proton source is being developed at Accelerator and Pulse Power Division (APPD), Bhabha Atomic Research Centre (BARC) for Accelerator Driven sub critical Systems (ADS) applications [2]. This source (50 keV, 30 mA) will be an injector to 3 MeV RFQ, a subsequent accelerating structure. The beam coming out of the source [3] may contain molecular hydrogen ions (H_2^+) species besides pure proton (H^+). It is desired to have more than 80% of proton fraction before it reaches RFQ. To remove the undesired species of molecular hydrogen ions, a device viz. Wien filter is used. The protons and molecular hydrogen ions in the beam have same energy (50 keV) but owing to different masses, have different velocities. The Wien filter removes the undesired molecular hydrogen ions species and hence this device is also called mass separator or Wien velocity filter (WVF). The WVF is a mass-dispersive electromagnetic optical device, having mutually perpendicular electrostatic and magnetostatic fields both being transverse to the direction of the charged particle beam. An appropriate choice of the separator's electric (E) and magnetic (B) field strengths, deflection forces sets up inside the device, which in turn cancel out for the desired beam species having axial velocity v = E/B. A classical WVF [4] consists of rectangular-parallel surfaces of electrodes and magnetic poles respectively (Figure 1). Along with spatial dispersion, the forces inside E×B separators also cause focusing of the beam. This focusing is not axisymmetric and astigmatism is introduced into the undeflected beam. A circular beam emerges out as an elliptical one out of the classical separator. This paper discusses how the astigmatism is removed up to a certain limit by introducing field gradients both in E and B.

Email : pun12j@yahoo.co.in

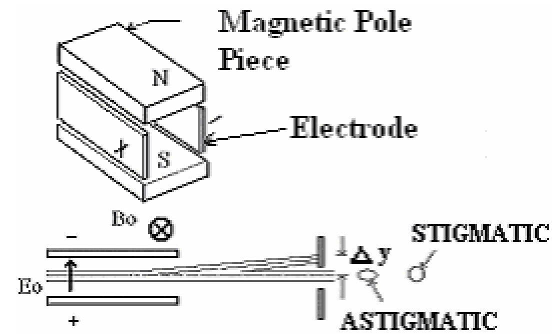

Figure 1: Schematic of classical Wien Filter

ELECTRODES GEOMETRY

We shall be using right-handed Cartesian coordinate system as shown in Figure 2.

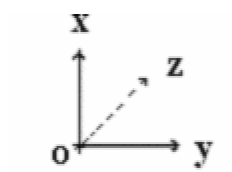

Figure 2: Coordinate system used in our analysis

For a parallel plate capacitor, one needs to choose electrodes profile as constant y lines. Obviously constant x lines shall be the electric field lines. To get an electric field gradient symmetric to y-axis we make a complex plane transformation given by w = Sin (ζ), where w (u, v) = u (x, y) + i v (x, y) and ζ (x, y) = x + i y. The equipotential curves are given by v = Cos (x) Sinh (y) and the orthogonal curves u = Sin (x) Cosh (y) shall be the corresponding field lines (Figure 3). The electrodes geometry shall therefore be according to constant v curves.

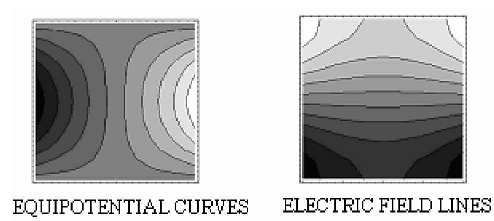

Figure 3: Electrodes profile (left figure) and the electric field lines (right figure)

MAGNETIC POLES GEOMETRY

The magnetic field used in the WVF should have a quadrupole component also besides a dipole. Such a configuration can be conceived by using tilted magnetic poles as shown in Figure 4. This field configuration is such that it gives a focusing Lorentz force for a diverging beam and a defocusing Lorentz force for a converging beam. Thus stability requirement of the beam is also maintained.

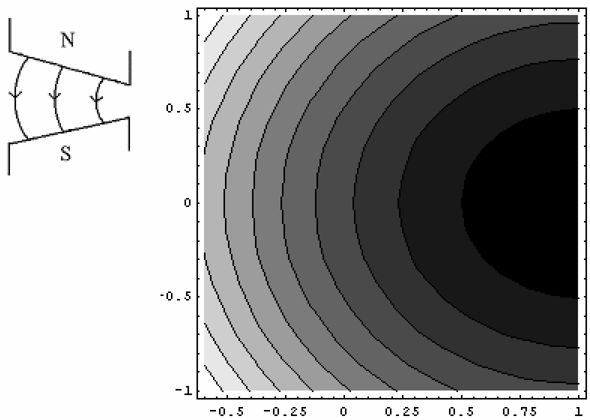

Figure 4: Schematic of Magnet pole pieces (left figure) and the corresponding magnetic field lines (right figure)

EQUATIONS OF MOTION

The following assumptions were made before setting up the equations of motion.

Assumptions

- The effect of space charge has been neglected. The formalism is developed for a single charged particle.
- The fields have no components or variations in the z direction. Employing shorting rings can reduce the width of the axial fringe fields at the entrance and exit of the separator.
- The natural length scale to which transverse displacements are compared, r_0, is the cyclotron radius m_0v_0/eB_0 of a particle of velocity v_0, in a magnetic field B_0. A second expression for r_0, found by evoking the balance condition $E_0 = v_0B_0$, is $r_0=2V_0/E_0$

eV_0 is the kinetic energy of the beam due to the extractor potential. In a paraxial approximation, the terms up to second order viz. $(x/r_0)^2$, $(y/r_0)^2$, (xy/r_0^2) are retained.

E and B fields

Following the coordinate system as shown in figure 2, the E and B fields can be written as:

$$E_x = - E_0 (xy/r_0^2)$$

$$E_y = E_0 [1 + ½ (x/r_0)^2 + ½ (y/r_0)^2]$$

$$B_x = - B_0 [1 - ½ (y/r_0)]$$

$$B_y = ½ B_0 (y/r_0)$$

$$v_z = v_0 [1 + y/r_0 + ½ (x/r_0)^2 + ½ xy/r_0^2]$$

From the Lorentz equation, $m_0\mathbf{a} = e (\mathbf{E_0} + \mathbf{v_0} \times \mathbf{B_0})$ we get the following equations of motion in x and y directions:

$$(d^2/dt^2)\, x = - ½ (ev_0B_0/m_0r_0)\, x – ½ (ev_0B_0/m_0r_0^2)\, 3xy$$

$$(d^2/dt^2)\, y = - ½ (ev_0B_0/m_0r_0)\, y – ½ (ev_0B_0/m_0r_0^2)\, xy$$

Noting the fact that $d^2/dt^2 = v_0^2 (d^2/dz^2)$, the same may be rewritten as:

$$(d^2/dz^2)\, x = - ½ (eB_0/m_0 v_0)^2 x – ½ (eB_0/m_0 v_0)^2 (3xy/r_0) \quad \text{………….. (1)}$$

$$(d^2/dz^2)\, y = - ½ (eB_0/m_0 v_0)^2 y – ½ (eB_0/m_0 v_0)^2 (xy/r_0) \quad \text{………….. (2)}$$

The above-mentioned pair of equations [eqn. (1) and eqn. (2)] is a coupled oscillator problem.

Solutions

For a 50 keV proton beam $v_0 = 3.0 \times 10^6$ m/s. For a weak separator the typical value of the focal length is ~ 100 cm, and cyclotron radius r_0 ~ 25 cm. The corresponding values of fields are: E_0 ~ 4 kV/cm and B_0 ~ 0.133 T

Plugging these numbers back in the equations (1) and (2), we get

$$(d^2/dz^2)\, x = - 8.5\, x – 102\, xy \quad \text{………….. (1a)}$$

$$(d^2/dz^2)\, y = - 8.5\, y – 34\, xy \quad \text{………….. (2a)}$$

It is clearly seen that the focusing forces strengths are same in both the transverse directions. Thus astigmatic problem is resolved. An interesting feature to be noted is that the coupling term is stronger than the focusing strengths.

If we denote (d/dz) x = x' and likewise for other variables then it may be further expressed as:

$$x'' = - C\, x – 4.24\, C^{3/2} xy \quad \text{………….. (1b)}$$

$$y'' = - C\, y – 1.41\, C^{3/2} xy \quad \text{………….. (2b)}$$

where $C = ½ (eB_0/m_0 v_0)^2 = 8.5\ m^{-2}$ corresponding to the same values chosen previously.

The proton beam emerging from the final extractor electrode is expected to have 8 mm diameter and 0.2 pi mm-mrad emittance.

To solve the second order coupled ordinary differential equations the following initial conditions were chosen:

x (0) = 4 mm, x'(0) = 40 mrad, and,

y (0) = 4 mm, y'(0) = 40 mrad

ANALYSIS OF SOLUTIONS

The phase space plot between x vs. x' and y vs. y' for C=8.5 is shown in Figure 5. The phase space plot is closed and hence the stable solutions exist.

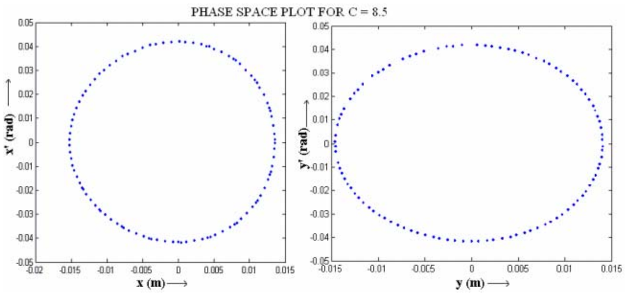

Figure 5: Phase space plots in x and y planes for C =8.5 m^{-2}

Next we examine the range of E_0 and B_0 that gives the stable solutions maintaining the Wien filter condition viz. $E_0 = v_0B_0$. Since v_0 is fixed for a given beam, we therefore need to consider only one variable say B_0.

As $B_0 \propto \sqrt{C}$, we shall see the solutions in phase space with C as a variable parameter. It is evident from the equations of motion (equations 1b and 2b) that as the parameter C is decreased there is no problem in the stability of the solutions but the focal length increases as $f \propto C^{-1}$. This will increase the length of the beam line and hence the cost of the system goes up. We therefore examine the stability of the solutions with increased values of C. The phase space plot between x vs. x' and y vs. y' for C increased to ten times of its actual value. (Figure 6)

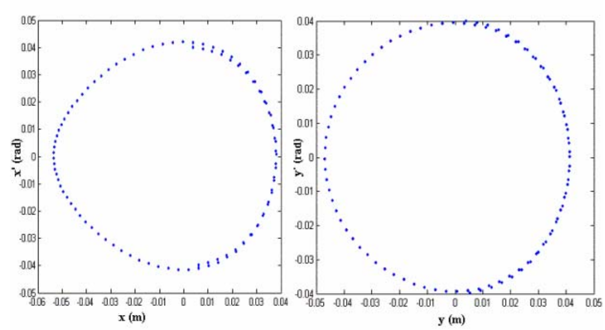

Figure 6: Phase space plots in x and y planes for C =85 m^{-2}

The ten times value of C corresponds to approximately thrice the value of B_0. The phase space plot in x-x' plane shows that the motion is slightly unbounded. The motion in y-y' plane, however, remains bounded.

Further increasing C to say fifteen times (corresponding to approximately four times B_0) the phase space plot in y-y' plane shows that the motion is slightly unbounded, however in x-x' plane the motion is completely unbounded. (figure 7)

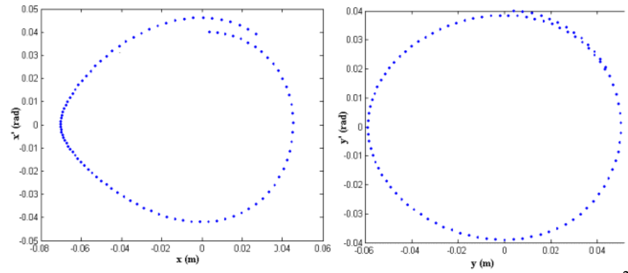

Figure 7: Phase space plots in x and y planes for C =127.5m^{-2}

The equations of motion can also be expressed by an effective Hamiltonian with nonlinear terms as:

$$d/dz\,(C\,x^2 + x'^2 + 12\,C\,x^2y) - 12\,C\,x^2y' = 0$$

$$d/dz\,(C\,y^2 + y'^2 + 4\,C\,y^2x) - 4\,C\,y^2x' = 0$$

The nonlinear terms viz. x^2y' and y^2x' have the same effect as the Hamiltonian given by:

$$H(x,x',y,y') = C(x^2 + y^2) + (x'^2 + y'^2) + (12C\,x^2y + 4C\,y^2x)$$

With this Hamiltonian, one can further explore the beam dynamics.

CONCLUSIONS

- Inhomogeneity in electric and magnetic fields does help in removing astigmatism problem.
- Coupling terms appear only due to inhomogeneous characteristics of electric and magnetic fields.
- To achieve focusing at relatively smaller lengths, the electric and magnetic fields should be large, however at large values, the solutions do not remain stable. For our system (50 keV proton beam) there is a bound on minimum focal length, which is $f \geq 10$ cm.

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