ELECTRON GUNS AND BEAMLINES IN THE VIEW OF EMITTANCE COMPENSATION

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Abstract

Space charge effect is ever of fundamental importance for low-energy parts of accelerators. Simple and robust estimations of the emittance degradation in various electron guns were obtained analytically and numerically. Nonuniform longitudinal and transverse distribution of current and the effect of gun electrodes were taken into account. The parameters of optimal beamlines for emittance compensation were estimated.

INTRODUCTION

Emittance compensation technique has been mentioned first probably in [1]. It was explained and developed further in [2] and other papers. The two basic effects, caused by the longitudinal nonuniformity of charge density and the transverse one, and their combination in uniform and nonuniform beamlines were considered in [3] and [4], also with accelerating and bunching. The main results of the two latter works is that the charge phase advance through the beamline should be $2n\pi$ (n is integer) and the focusing should be optimal. Then the normalized emittance dilution is well estimated as

$$\varepsilon_n \cong \xi r \sqrt{\frac{I}{I_0 \beta\gamma}}, \qquad (1)$$

where r is the rms size of the beam at the entrance; I is the peak current; $I_0 = 4\pi \cdot mc^2 / Z_0 |e|$, ≈ 17.045 kA for electrons; $\beta = v/c$; $\gamma = 1/\sqrt{1-\beta^2}$; v is the longitudinal velocity; and ξ is the dimensionless coefficient depended on the type of the beamline.

In this paper we consider electron guns in the same view. We take into account only macroscopic space charge effect and neglect thermal and grid emittance. There are at least three significant differences from the previous cases:

- Metallic electrodes always exist near the emitter. Its charge depends on the one of the beam and generates comparable fields.
- A bunch ever starts at very low energy, so its velocity and length are small enough. If the length of a bunch in the moving frame is comparable to or less than its radius, the interaction between its slices can't be neglected.
- The head and the tail of a bunch are in different conditions. If we consider them at the same position, the bunch will have lower energy in the first case and the transverse force will be smaller due to nonlocal interaction.

We use a steady-state code that takes into account the first effect, but not the two others.

EMITTANCE DILUTION IN GUNS

Phenomena and Basic Scaling

If the emitter is round and the beam is homogeneous and stationary, the gun geometry can be optimized so that the space charge effect doesn't affect the emittance, as in the well known Pierce gun [5]. If the beam is not uniform in the longitudinal direction, the transverse phase portraits of its slices differ and their emittances are not equal to zero. Let's consider these phenomena and estimate the total emittance.

Particle motion in the same gun is similar if its voltage and current meet Child-Langmuir law $I \propto U^{3/2}$. In this case the emittance (not normalized!) doesn't depend on the current. The quality factor of a gun

$$\frac{\varepsilon}{r\sqrt{\frac{I}{I_0(\beta\gamma)^3}}} \equiv \frac{\varepsilon}{r\sqrt{j}}, \qquad (2)$$

where r is the emitter radius, also doesn't depend on the current. At the same time, brightness $I/\varepsilon_n^2 \propto \sqrt{U}$. If all the dimensions of a gun are changed proportionally, its quality factor preserves while its brightness is $\propto \sqrt{U}/r^2$. Thus, one should find the quality factor and the optimal compensation beamline for any gun.

Charge Amplitude and Phase

Consider round beams and small deviations δ from the principal trajectories in the transverse phase space as in [3] and [4]. Then the motion is described by a second order differential equation with variable coefficients and a transform matrix exists for each beamline

$$\begin{pmatrix} \delta \\ \delta' \end{pmatrix} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \cdot \begin{pmatrix} \delta_0 \\ \delta'_0 \end{pmatrix}. \qquad (3)$$

$\delta' = 0$ (see [3] (3)) at the emitter and we should zero it at some point after the gun to minimize the emittance. Thus, only C and C' are significant in our case and one can define the charge phase advance as

$$\varphi = \arctan\left(\frac{-C'x}{C\sqrt{j}}\right), \qquad (4)$$

where x is the rms-size of a slice. The quadrant should be taken so that the signs of $\cos\varphi$ and $-\sin\varphi$ coincide to ones of C and C' respectively. This definition possesses a critical property: if a uniform beamline with the phase advance $\pi - \varphi$ and x, j, and $\beta\gamma$ equal to ones at the exit of an arbitrary beamline with the phase advance φ, the total phase advance is π.

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$$\begin{pmatrix}\cos(\pi-\varphi) & \dfrac{x}{\sqrt{j}}\sin(\pi-\varphi)\\ -\dfrac{\sqrt{j}}{x}\sin(\pi-\varphi) & \cos(\pi-\varphi)\end{pmatrix}\cdot\begin{pmatrix} a\cos\varphi & * \\ -a\dfrac{\sqrt{j}}{x}\sin\varphi & *\end{pmatrix}=\begin{pmatrix}-a & *\\ 0 & *\end{pmatrix}$$

The same situation occurs with 2π. Then one can define the relative charge amplitude as

$$a=\sqrt{C'^2+\left(C\sqrt{j}/x\right)^2}\,. \tag{5}$$

Basic Gun

A diode gun similar to used in [6] has been simulated first. Its geometry is shown in fig. 1. The emitter radius was 5 mm, the distance between the electrodes was 123 mm, while the beam was observed at 200 mm from the cathode. The optimal current was 2 A at 300 kV. SAM simulation code [7] was used to calculate beam motion in the gun. As usually for emittance compensation, a bunch has been divided by slices, and each slice was considered independently as a steady-state beam. The current density at the cathode was always homogeneous.

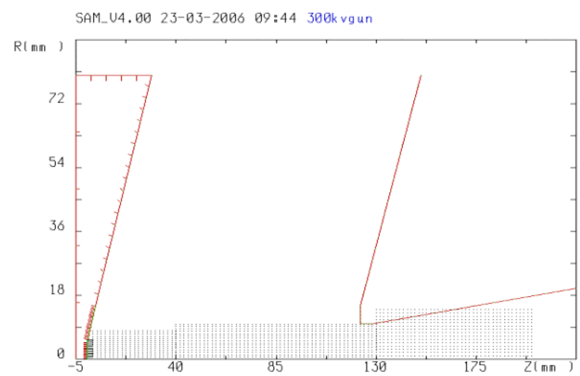

Figure 1: The geometry of the basic gun, red solid lines are electrodes.

The calculated beam parameters depending on the beam current are depicted in fig. 2. They were calculated by the following formulae:

$$\begin{aligned} x&=\sqrt{<x^2>},\\ x'&=<xx'>/\sqrt{<x^2>},\\ \varepsilon&=\sqrt{<x^2><x'^2>-<xx'>^2}\,. \end{aligned} \tag{6}$$

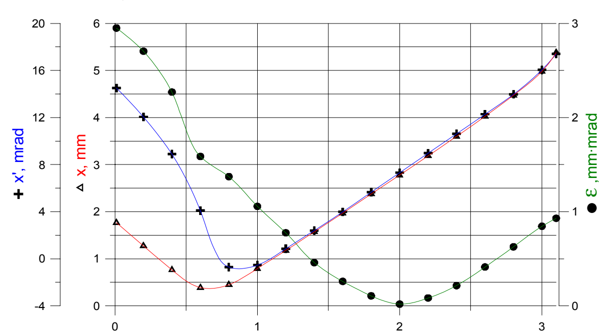

Figure 2: Beam parameters vs. beam current: rms-size, its derivative and emittance.

Let's calculate the charge phase and the relative charge amplitude from these data now. The beam size at the cathode preserves, but its "proportional" size $x\propto\sqrt{I}$ [3], so that

$$\frac{\partial x}{\partial I}=\frac{1}{2}\frac{x}{I}\,. \tag{7}$$

Than the initial charge vibration amplitude is

$$\delta x=-\delta I\frac{\partial x}{\partial I}=-\frac{1}{2}x\frac{\delta I}{I}\ \Rightarrow\ \frac{\partial\delta_0}{\partial I}=-\frac{1}{2I}\,. \tag{8}$$

The deviation from the "proportional" trajectory at the gun exit is

$$\delta=\frac{\delta x-\frac{1}{2}x\frac{\delta I}{I}}{x}=\left(\frac{1}{x}\frac{\partial x}{\partial I}-\frac{1}{2I}\right)\delta I\ \Rightarrow\ \frac{\partial\delta}{\partial I}=\frac{1}{x}\frac{\partial x}{\partial I}-\frac{1}{2I}\,, \tag{9}$$

and its derivative by the longitudinal coordinate is

$$\delta'\equiv\left(\frac{\delta x}{x}\right)'=\frac{\delta x'}{x}-x'\frac{\delta x}{x^2}\ \Rightarrow\ \frac{\partial\delta'}{\partial I}=\frac{1}{x}\frac{\partial x'}{\partial I}-\frac{x'}{x^2}\frac{\partial x}{\partial I}\,. \tag{10}$$

The significant matrix elements are

$$C\propto\frac{\partial\delta}{\partial I},C'\propto\frac{\partial\delta'}{\partial I}\,, \tag{11}$$

and the charge phase at the gun exit is

$$\varphi=\arctan\left(\frac{-C'x}{C\sqrt{j}}\right)=\arctan\left(\frac{\dfrac{\partial x'}{\partial I}-\dfrac{x'}{x}\dfrac{\partial x}{\partial I}}{\left(\dfrac{1}{2I}-\dfrac{1}{x}\dfrac{\partial x}{\partial I}\right)\sqrt{j}}\right). \tag{12}$$

To calculate the relative charge amplitude one should compare $\partial\delta/\partial I$ with the same derivative for a beam starting at the same point with fixed initial conditions, that is $\partial\delta/\partial I=-1/2I$ and $\partial\delta'/\partial I=-1/2I\cdot\sqrt{j}/x$. The root of the sum of the squares of these ratios gives the relative amplitude:

$$A=\sqrt{\left(\frac{2I}{x}\frac{\partial x}{\partial I}-1\right)^2+j\left(2I\left(\frac{\partial x'}{\partial I}-\frac{x'}{x}\frac{\partial x}{\partial I}\right)\right)^2}\,. \tag{13}$$

The dependencies of the phase and the amplitude on the current for the mentioned gun are shown in fig. 3.

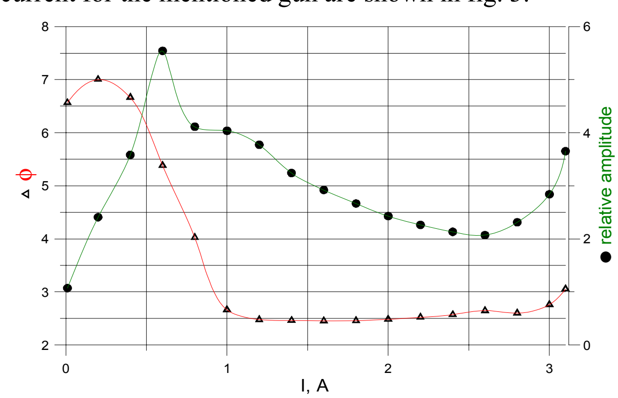

Figure 3: Charge phase and relative amplitude vs. beam current.

It is seen that the phase is almost constant within current limits from 1 to 3 A and its value is $\approx 2.5\approx 0.8\pi$. Thus, if an ideal uniform beamline (where the phase advance doesn't depend on the amplitude) with the phase advance $\approx 1.2\pi$ is placed after the gun, one should expect the minimal emittance. The following questions are still left: (i) what peak current of a bunch gives the minimum emittance in this system, (ii) which slice should be matched to the compensation beamline, and (iii) what is the optimal phase advance of the latter. The dependencies of the non-compensated emittance and the $\varepsilon/\sqrt{j}$ ($r = \text{const}$) on the peak current of a Gaussian bunch are shown in fig. 4. The compensated values one can find in fig. 5.

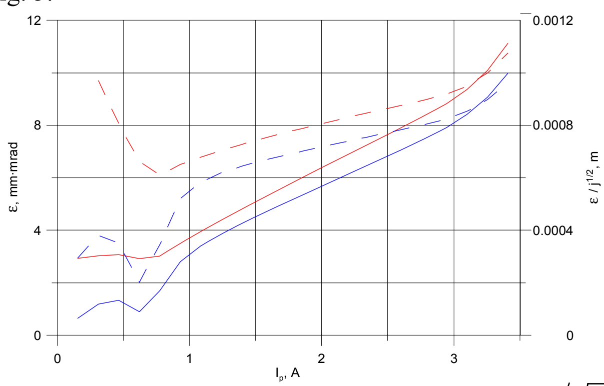

Figure 4: Non-compensated emittance (solid) and $\varepsilon/\sqrt{j}$ (dashed) vs. peak current of Gaussian bunch. Upper (red) curves take slice emittances into account.

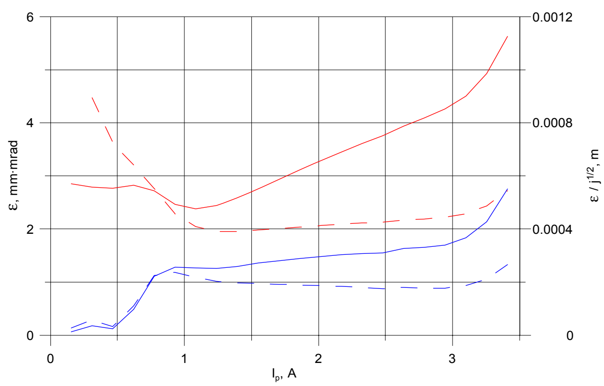

Figure 5: Compensated emittance (solid) and quality factor (dashed) vs. peak current of Gaussian bunch. Upper (red) curves take slice emittances into account.

One can see that $\varepsilon/\sqrt{j}$ in this case is almost independent on the current and equals $\approx 2\cdot10^{-4}$ m without slice emittances and $\approx 4\cdot10^{-4}$ m with them. A non-ideal compensation beamline increases the quality factor by ([3]: (5), (15), Table 1)

$$0.023\frac{x_1}{x}a^3\frac{\varphi}{2\pi} \approx 0.27\,. \tag{14}$$

Thus $\varepsilon/\sqrt{j}$ will be $\approx 4\cdot10^{-4} + 0.27\cdot2.5\cdot10^{-3} \approx 1\cdot10^{-3}$ m.

Other Guns

Four other guns have been simulated in the same way to investigate the influence of the gun geometry. The emitter radius was the same while the length was varied. The electrodes were shaped to make perfect electric field. Additional electrodes were added to the guns "Short 2" and "Long 2" to equalize their perveance to the primary one. The optimal current in all the cases was ≈ 2 A. The results are placed in table 1. The values in parentheses in the second column mean the observation points. The last column considers the slice emittances.

Table 1: Guns parameters

Gun	Length, mm	U, kV	φ	$\varepsilon/\sqrt{j}$, m	$\varepsilon/\sqrt{j}$ (slices), m
Basic	123 (200)	300	2.5	$2\cdot10^{-4}$	$4\cdot10^{-4}$
Short	61.5 (100)	150	2.2	$7.5\cdot10^{-5}$	$4.8\cdot10^{-4}$
Short 2	61.5 (100)	300	2	$2.5\cdot10^{-4}$	$5\cdot10^{-4}$
Long	246 (400)	850	2.5	$4\cdot10^{-4}$	$5.4\cdot10^{-4}$
Long 2	246 (400)	300	3.1	$1.2\cdot10^{-4}$	$4.6\cdot10^{-4}$

One can see the charge phase is ever 2.5±0.5 and $\varepsilon/\sqrt{j} = (4.7 \pm 0.7)\cdot10^{-4}\,\text{m}$.

CONCLUSIONS

- Emittance compensation applied to an electron gun always improves emittance by several times.
- The expected compensated normalized emittance of a well-designed gun with an ideal compensation beamline is

$$\varepsilon_n \approx 0.2x_e\sqrt{\frac{I}{I_0\beta\gamma}} = 0.1r_e\sqrt{\frac{I}{I_0\beta\gamma}}\,, \tag{15}$$

where r_e is the emitter radius and x_e is the rms beam size at the emitter.

- A non-ideal optimal compensation beamline worsens this value to

$$\varepsilon_n \approx 0.45x_e\sqrt{\frac{I}{I_0\beta\gamma}} = 0.225r_e\sqrt{\frac{I}{I_0\beta\gamma}}\,. \tag{16}$$

- The charge phase advance of the compensation beamline should be 1.05…1.35π.
- The compensation beamline should be matched to the 0.5…0.75 of the peak current.

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