STUDY OF SPACE CHARGE COMPENSATION IN LEBT

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Abstract

A 20 MeV, 30 mA CW proton accelerator is being built in BARC which consists of 50 keV ECR ion-source, LEBT, 3 MeV RFQ, MEBT and 20 MeV DTL. In designing low energy beam transport (LEBT) line, which matches the beam from ion-source to RFQ, the expansion of the proton beam is a severe problem. As the energy of the beam is 50 keV, Coulomb repulsion is enormous and for minimization of this repulsion, space charge compensation is done. To simulate the beam dynamics part, a PIC code is written, which allows beam of different distributions like KV, Parabolic and Waterbag. This is an electrostatic code, which can also take care of external magnetic fields. A Monte Carlo collision scheme is being implemented for the ionization of the background gas. In this paper, we are presenting the simulation of space charge compensation of the 30 mA proton beam at 50 keV.

INTRODUCTION

Newly proposed accelerators with application to nuclear waste transmutation, subcritical nuclear reactors, neutron spallation sources require high intensity linacs. A 20 MeV, 30 mA CW proton accelerator is being built in BARC which consists of 50 keV ECR ion-source, LEBT, 3 MeV RFQ, MEBT and 20 MeV DTL [1]. In the low energy section of such accelerators beams of tens of mA are strongly subjected to the Coulombian repulsion. The transport of such space-charge dominated beams are challenging task because of the same reason. Apart from applying the usual magnetic fields based structure for focussing; the method of space-charge neutralization has also been tried for the same. In this process, a gas is introduced in the beam pipe, which gets ionized by the beam. The produced electrons are trapped in the beam potential and reduce the repulsive space charge forces. A better understanding of the kinetics of the process will make it more efficient and may find different application in other fields. To study this kind of situation, we need to solve the full Poisson-Vlasov model including the different kind of collisions. This makes PIC-MCC simulation technique as a competitive candidate.

BASICS OF A PARTICLE-IN-CELL/ MONTE CARLO MODEL

In the PIC method, so-called ‘‘superparticles’’ move in the simulation region through an artificial grid on a timestep basis. Each of these superparticles represents typically about 10^8 real particles. Only charged particles are simulated with these superparticles; neutrals are assumed to form a continuum. In the beginning of the simulation, every charged particle is assigned to a specific position on the grid, leading to a self-generated electric field. The particles move in response to both the applied and self-generated fields, according to Newton’s laws. This gives rise to new positions for the particles, changing the self-generated field, and hence changing the force acting on the particles. Mathematically, this is done every timestep by first *weighting* the positions of the particles to the grid, yielding the charge densities on the grid points. The potential and electric field on the grid points are then determined from the calculated charges, by Poisson’s equation. A weighting procedure is applied again, to obtain the forces on the positions of the particles from the previously obtained field on the grid points. From the force on the positions of the particles, first, the velocity of every particle is calculated and, from the velocity, the position is determined, using a leap-frog algorithm [2]. After the particles are placed in their positions, a Monte Carlo algorithm is used to simulate collisions between particles. This procedure is repeated for many timesteps, until convergence is reached.

In the MC module, a random number between 0 and 1 is chosen to determine for every particle whether a collision occurs or not. If a collision takes place, a second random number is generated to determine the collision type. The energy and direction of the particles after the collision are determined, depending on the collision type, again using random numbers. We make use of the ‘‘null-collision’’ method [3]. In this approach, a fictitious collision process (null-collision) is introduced, with a collision frequency such that when it is added to the sum of the collision frequencies of the real collision processes, a constant total collision frequency over position and energy is obtained. In this way, the maximum fraction of the total number of particles in the simulation that undergo a collision (either a real or a null collision) during a timestep Δt, is given by

$$P_{null} = 1 - \exp(-v'\Delta t) \quad (1)$$

where

$$v' = \max_{x,E}(n_t \sigma_T v) \quad (2)$$

In Eq.(2), x denotes the position, E is the energy of the incident particle, n_t is the density of target particles at particle position, σ_T is the total cross section for every species and ν is the velocity of the incident particle. Typically the target particles are assumed to be uniformly

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distributed in the system as a background species with a constant n_t. However, in the case of electron-ion collision n_t is also a function of position. The more details of PIC-MC are discussed by Vahedi [3].

PIC- MCC MODEL FOR SPACE-CHARGE COMPENSATION OF CW H^+ BEAM

To simulate the process, a 2D(x-y) PIC-MCC code is being developed which will be able to do the beam dynamics of the CW beam in a system consisting of drift space, solenoid and quadrupoles with or without presence of neutralizing gas. The very first need of a PIC code is to get different particle distributions, to represent the beam. These simulations are highly dependent on the representation of the beam in phase space, which necessitates the generation of distributions in accordance to different models like KV, Waterbag, parabolic and Gaussian normally used in the literature.

Phase space distributions

Consider the distributions in 4Dphase space (x, x′, y, y′) as n(x, x′, y, y′) = dN/dxdx′dydy′ which depends on total emittance ε:

$$n(x,x',y,y') = n(\varepsilon) \qquad \varepsilon = A_x^2 + cA_y^2 \tag{3}$$

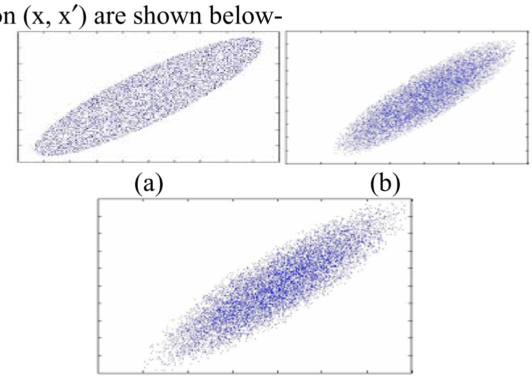
Where

$$A_x^2 = \left(\sqrt{\beta_x}\,x' + \frac{\alpha_x x}{\sqrt{\beta_x}}\right)^2 + \left(\frac{x}{\sqrt{\beta_x}}\right)^2 \tag{4}$$

$$A_y^2 = \left(\sqrt{\beta_y}\,y' + \frac{\alpha_y y}{\sqrt{\beta_y}}\right)^2 + \left(\frac{y}{\sqrt{\beta_y}}\right)^2$$

c is ratio of emittances in x and y plane [4].

The equation ε = const describes a hyperellipsoid surface in phase space x, x′, y, y′. As distribution function depends on ε, the phase space density, n, will be constant on one hyperellipsoid surface while it will vary from one surface to another. A numerical algorithm is written to generate the different distribution like KV, Waterbag and parabolic [5]. The projections of the different distributions on (x, x′) are shown below-



(a) (b) (c)

Figure 1: Projections of 4D phase space distributions on (x, x′) plane.(a) KV (b) Waterbag (c) Parabolic

Poisson Solver

As the particles are in non-relativistic regime, we need not to solve the full wave-equation. This provides enough reason to write an electrostatic PIC code rather than attempting for an electromagnetic one. To find the potential and in turn the electric field, we need to solve the Poisson equation. We have used the direct method to solve the Poisson equation. This method is called spectral method, since it involves the expansion of potential and source terms as truncated Fourier series. In one direction, the boundary conditions may be Diritchlet or Neumann boundary condition while in the other direction the provision is for mixed boundary conditions.

$$\alpha_l u(x,y) + \beta_l \frac{\partial u(x,y)}{\partial x} = \gamma_l(y) \tag{5}$$

at $x = x_l$ and

$$\alpha_h u(x,y) + \beta_h \frac{\partial u(x,y)}{\partial x} = \gamma_h(y) \tag{6}$$

at $x = x_h$.

$$u(x,0) = u(x,l) = 0 \quad or \quad \frac{\partial u(x,y=0)}{\partial y} = \frac{\partial u(x,y=l)}{\partial y} = 0 \tag{7}$$

Integrator of particle trajectories

In the most general case of integration of particle trajectories in this code are done in following steps-

- In the first stage, the particle performs a half step acceleration in the electric field
- After that, the vector of the particle velocity accomplishes rotation in the magnetic field, utilizing the Boris scheme.
- In the third stage, the particle performs again half step acceleration in electric field (if applicable MC module will take over).
- And at the final stage particles are advanced with this velocity.

Now let's consider the different component of the lattice.

Drift

In drift, magnetic field and external electric field are absent so the particle moves in its own self field.

Solenoid

In case of solenoids, focussing fringe fields are calculated upto the linear terms and given by following.

$$B_r = -\frac{r'}{2} B'(z) \tag{8}$$

This term is responsible for the end effects of the solenoid which otherwise have flat magnetic field profile. In our PIC code, we have implemented it at the beginning and end points. This model is known as hard edge model of solenoids.

Quadrupole

The fields of quadrupoles are given by the following equation-

$$B_x = B_0 \frac{x}{a}, B_y = B_0 \frac{y}{a} \tag{9}$$

The particles trajectories are evaluated in the above prescribed manner.

We have completed the implementation till this point in code and results are described in the next section. The MC module is still under development. However, the methodology has been worked out and in brief, it is discussed below.

In the beginning, we will start with the ionization process only. The ionization cross-section of various gases by proton at different energies can be described by the empirical scaling law [4]. As we are interested in the detail kinetics of the space charge compensation process the ionization by energetic electrons will also be taken care of. This requires the energy and angular distribution of produced electron in the ionization process by proton. There are number of experimental data available and we are making use of that by fitting some empirical formulas.

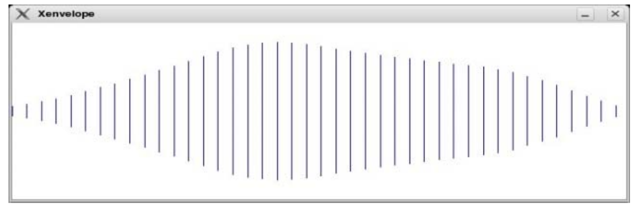
RESULTS

As we have stated earlier that PIC part of this code has already been implemented. We have used this code to simulate the low energy beam transport section for the 20 MeV, 30 mA proton accelerator being built in BARC. A comparison between the parameters obtained by TRACE2D and our code is presented.

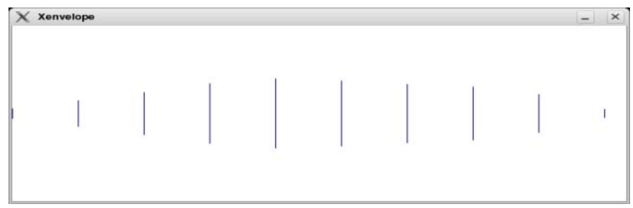
Table 1:LEBT Parameters

ELEMENT	LENGTH	STRENGTH
Drift	60 cm	
Solenoid	30 cm	1.903 kG
Drift	50 cm	
Solenoid	30 cm	2.113 kG
Drift	15 cm	
Total Length	185 cm	

The LEBT is designed using TRACE2D with the following input Twiss parameters [6], $\varepsilon = 0.02\pi$ cm mrad, $\beta_x=\beta_y=24.768$ cm/rad, $\gamma_x=\gamma_y=0.171$ rad/cm and $\alpha_x=\alpha_y= -1.8$. The output parameters are found to be as $\varepsilon = 0.02\pi$ cm mrad, $\alpha_x=\alpha_y= 1.8$, $\beta_x=\beta_y=6.43$ cm/rad. As TRACE2D is a envelope tracking code, so constant emittance is obvious. With the same input and lattice when we tried with our code, we got the output values as $\varepsilon = 0.0225\pi$ cm mrad, $\alpha_x=1.834$, $\alpha_y=1.90$, $\beta_x=6.61$ cm/rad, $\beta_y=6.87$ cm/rad. The particle trajectories are shown in the Fig.2



(a)



(b)

Figure 2: The trajectories of particle through LEBT line for (a) 30 mA, (b) 3 mA.

We also tried to see the maximum beam size as a function of space charge compensation (reduced current). We found that with 90% compensation the maximum beam size reduces from 6.1 cm to 3.40 cm.

CONCLUSION

The development of PIC part of the code is already completed and results has been compared with TRACE2D calculations and also in some cases solving analytical KV equation. The results are in good agreement. The difference in some cases is expected as ours is a PIC code while others are envelope tracking codes. The MC module implementation is in progress.

REFERENCES

[1] P. Singh, “Status of the LEHIPA Project at BARC”, InPAC-2006, p 50.

[2] C.K. Birdsall and A.B. Langdon, “Plasma Physics via Computer Simulation” (IOP, Bristol) 1991.

[3] V.Vahedi and M. Surendra, Comput. Phys. Commun, **87**, 179(1995).

[4] M. Reiser, “Theory and Design of Charged Particle Beams” (John Wiley & Sons, New York) 1994.

[5] S.C.L. Srivastava, S.V.L.S. Rao, P. Singh, “Generation of different particle distributions in 4d phase space”, InPAC-2006, p 209.

[6] Rajni Pande *et al.,* “Effect of Space Charge Compensation on LEBT Design”, Proc. DAE Symp. Nucl. Phys.(India), 46(B), 466(2003).