EVALUATION OF EXTERNAL Q USING KROLL-YU METHOD WITH MICROWAVE STUDIO

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Abstract

Design and development of a superconducting cavity has been taken up as a part of Accelerator Driven Subcritical project(ADSS). An input coupler is designed for the same using Kroll-Yu method[1]. The evaluation procedure is optimised and the method has been successfully implemented for the evaluation of high external Q [2]. The validity of the Kroll-Yu method is tested with the external Q calculations by P. Balleyguier method for the Benchmark cavtity which is a pillbox cavity of diameter ~200 mm, length ~150 mm with semi-rigid coaxial line[3]. It is found that the careful choice of data points provide accurate results over wide range.

INTRODUCTION

BARC is involved in the development of technology for the Accelerator Driven Sub-Critical System (ADSS) that will be mainly utilized for the transmutation of nuclear waste and enrichment of U^{233}. The application demands a proton beam (30mA, 1 GeV) that will be generated by various stages of accelerator structures. Up to 100 MeV proton beam will be generated by the normal conducting accelerator structures whereas from 100 MeV to 1 GeV superconducting structure will be more efficient. It will be a coupled cavity structure made up of niobium cavities of elliptical shape. As an initial step single elliptical cavities operating at 700 MHz and 1.056 GHz were designed. [4, 5]

A co-axial type of input coupler has been designed for the same cavity. The power will be transferred efficiently from the coupler to the cavity only when the impedance of the coupler matches with the cavity impedance i.e. the coupling factor between cavity and transmission line is unity. Coupling factor, β is defined as,

$$\beta = Q_0 / Q_{ext} \quad \text{--(1)}$$

where, Q_0 and Q_{ext} are the quality factors of the cavity and the transmission line respectively. Q_{ext} is the factor that depends solely on the geometry and can be evaluated by different computational methods with the help of 3-D electromagnetic codes.

COMPUTATION OF EXTERNAL Q

A number of computer programs viz., MAFIA, SUPERFISH, URMEL have been developed for the design of cavity resonators which give information about the field distribution, shunt impedance, Q-value but cannot calculate Q-value due to external coupling due to waveguide/ co-axial line. Thus, Kroll-Yu in 1990 developed a procedure to calculate external Q by using these codes. As this method is based on the frequency resonance differences, it was thought that it is better suited for the computation of low Q_{ext}. In 1997, Pascal Balleyguier developed a method based on the field calculations for the computation of high external Q. [3,6]

This paper presents results of the external Q calculation using both the methods with the help of computer code Microwave Studio[7]. The results indicate that Kroll-Yu method is also suitable for the high external Q calculation provided the data points near the resonant frequency of the cavity are chosen.

Kroll-Yu Method

Kroll-Yu suggested a method [1] that is easy to implement. A (cavity + waveguide) system terminated in a short (E_t=0, E_t----tangential electric field component) can be treated as two coupled resonators, cavity and waveguide.

The formulation deals with frequency, ω, as a function of phase change, ψ along the guide length that is given by $2\pi D/\lambda_g$ where λ_g is the guide wavelength and D is the guide length. A quantity $G \equiv -(1/2)\,(d\psi/d\omega)$ plotted as a function of ω exhibits a typical resonance curve with peak at the resonant frequency of (cavity + waveguide) system feeding into the matched load. When multiplied by the resonant frequency, the height of the curve is equal to Q_{ext}. The eigen modes of this system are complex in nature viz, u + i v. Also,

$$G = (1/2)\,(v/((w-u)^2 + v^2)) + (1/2)\,\chi'(u) \quad \text{----------(2)}$$

This is a resonant curve at frequency ω=u and $Q_{ext}=(1/2)(u/v) + (1/2)\chi'(u)$. The exact relation between ψ and ω is given by 4 – parameter formula,

$$\psi(\omega) = \tan^{-1}[v/(\omega-u)] - \chi(\omega) + n\pi \quad \text{---------(3)}$$

where , $\chi(\omega) \cong \chi(u) + \chi'(u)\,(\omega-u)$

By using simulation codes like MAFIA, Microwave Studio, HFSS one can obtain different ψ-ω pairs (data points) for different values of waveguide length, D. The essence of the method is to fit data points by 4 -parameter formula. The fitted values of u and v give information about the resonant frequency and external Q value.

Also it is observed that for Q_{ext} >20, choice of $\chi'(u)$ equal to zero is excellent [1]. Thus, for determination of u and Q_{ext} just three points are sufficient (3-parameter fit).

The implementation formulae are given by ,

$$u = \omega_2 + A\,B_{12} + \omega_1 A^2/(1+A^2) \quad \text{----------(4)}$$

$$v = (u-\omega_1)\,A - B_{12} \quad \text{--------(5)}$$

$$\chi(u) = \tan^{-1}[v/(\omega_i-u)] - \psi_i - \chi'(u)\,(\omega_i-u) \quad \text{----(6)}$$

Where, A, B_{12} and B_{23} are the quantities dependent on ω_i .

Setting $\chi'(u)$ to zero, three parameter fit of the resonance curve is obtained. Fitted values of frequency and Q_{ext} are calculated by using above implementation formulae for the given set of data points (ψ_i, ω_i). In the present analysis, external Q is obtained by 3- parameter fit of the data points.

Figure(1) shows the behaviour of (cavity + coupler) system as a function of position of the SHORT. As seen in the figure, the cavity mode (TM_{010}) frequency does not vary much whereas the coupler mode (TEM) frequency changes appreciably as a function of position of the SHORT. It was observed that the values of fitted parameters (f and Q_{ext}) were greatly affected by the choice of the data points.[2] It was observed that, the choice of data points in the vicinity of a point where cavity resonance frequency and the coupler frequency were nearly same, provided consistent results. Thus for Q_{ext} evaluation, points in the shaded portion indicated in the figure(1) were chosen.[2]

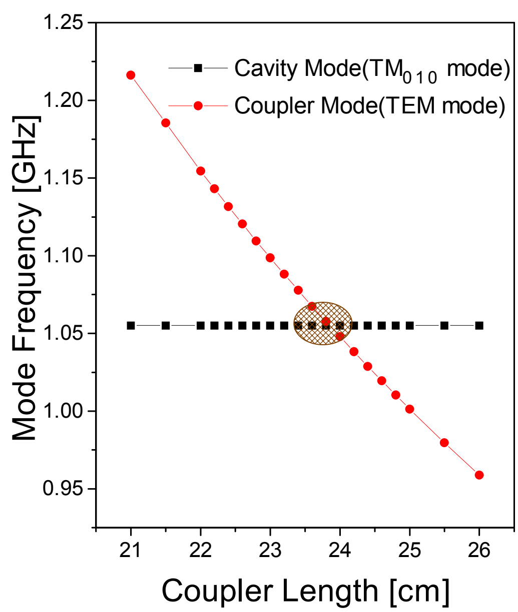

Figure 1: Study of Mode frequencies as a function of position of SHORT

Validation of Kroll-Yu Method

External Q was computed for the cavity operating at 1.056 GHz having beam pipe of radius 4.0 cm and a coaxial coupler of ID:ϕ 3.478 cm ; OD : ϕ 7.92 cm placed at a distance of 12 cm from the cavity centre using Kroll-Yu method and P. Balleyguier method.[3,6,8] (See Table 1).

Similar calculations were carried out for a cavity designed at a resonant frequency of 700 MHz, beam pipe radius of 5.0 cm having coaxial coupler of ID :ϕ 4.348cm; OD :ϕ 10cm connected at a distance of 12 cm from the cavity centre.(See Table 2)

Table 1

Penetration Depth (cm)	External Q, Q_{ext}	
	P.Balleyguier Method	Kroll-Yu Method
-2.0	3.69×10^7	5.13×10^7
-1.0	8.66×10^6	1.81×10^7
0.0	3.04×10^6	2.68×10^6
1.0	1.08×10^6	1.91×10^6
2.0	4.86×10^5	7.19×10^5

Table 2

Penetration Depth (cm)	External Q, Q_{ext}	
	P.Balleyguier Method	Kroll-Yu Method
-2.0	4.3×10^6	4.04×10^6
-1.0	1.31×10^6	1.45×10^6
0.0	5.95×10^5	6.37×10^5
1.0	4.0×10^5	2.72×10^5
2.0	2.21×10^5	1.06×10^5

To check the validity of Kroll-Yu method over the wide range of Q_{ext} values, the calculations were carried out for the Bench mark cavity that is a pill box cavity operating at 1146 MHz having diameter ~200mm, length ~150mm with semi-rigid coaxial line (ϕ in/out=1.65/5.35 mm). The penetration depth was varied from 0 to 5.5 cm [9]. Table 3 shows the calculated results.

Table 3

Penetration Depth (mm)	External Q, Q_{ext}	
	P.Balleyguier Method [9]	Kroll-Yu Method
0	6068050	3553030
2.5	407113	269016
5	81493	74439
7.5	23841	28111
10	10536	12086.1
12.5	5361	4449.1
15	2790	2660.16
17.5	1682	1709.18
20	1064	1061.46
25	455	482.99
30	207	232.46
35	106	118.52
40	56.5	61.87
45	33	34.15
50	19.9	20.36
55	13.1	13.87

For the penetration depth of 0 to 2.0 cm, data points near the resonance frequency of the cavity (as shown in Figure 1) were used whereas for the penetration depth from 2.5 cm to 5.5 cm points far away from the cavity resonance frequency give accurate answer.

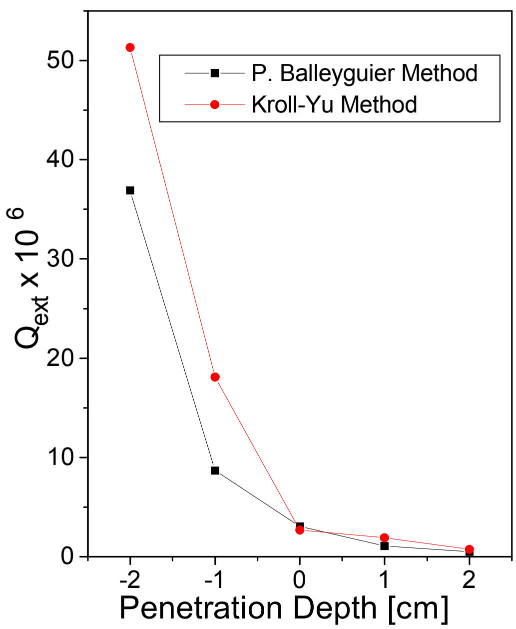

Figure 2 : Plot of Q_{ext} as a function of penetration Depth

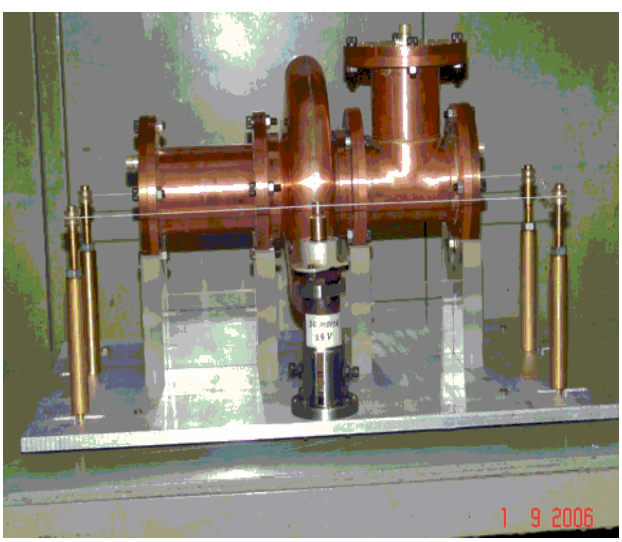

Figure 3 : Experimental Set-up (Copper mock-up)

CONCLUSION

Figure (2) shows the variation of Q_{ext} as a function of penetration depth calculated for the cavity operating at 1.056 GHz using P.Balleyguier as well as Kroll-Yu method. There is a discrepancy in the results at very high Q_{ext} ($> 10^7$) values while at lower values it is matching well.

From the results listed in Table 3 it is clear that Kroll-Yu method is also a suitable tool for the evaluation of external Q provided the data points are chosen carefully.

A comparison between the two techniques shows that P.Balleyguier method is very simple, straightforward and can be used efficiently for low as well as high external Q.

To get better insight of the subject a copper mock-up of the SC cavity with coaxial input coupler is fabricated at MDPDS Workshop, BARC, Mumbai (See Figure 3). The experimental study is under progress.

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