ION-ELECTRON INSTABILITY IN ELECTRON COOLING*

A. Burov, FNAL, Batavia, IL 60510, USA

Abstract

The drift motion of cooling electrons makes them able to respond to transverse perturbations of a cooled ion beam. This response may lead to transverse instabilities at specific longitudinal wave numbers. While the dipole instabilities can be suppressed by a combination of the Landau damping, machine impedance, and the active damper, the quadrupole and higher order modes can lead to either emittance growth, or lifetime degradation, or both. The growth rates of these instabilities are strongly determined by the machine coupling. Thus, tuning out of the coupling resonance and / or reduction of the machine coupling can be an efficient remedy.

INTRODUCTION

Being able to make beams brighter, electron cooling brings specific problems for the cooled beams as well.

First, electron cooling, as any cooling, makes the cooled beam less stable against any kind of coherent motion, would it be driven by an impedance of the chamber, or by a stochastic cooling system, or by a structure resonance of space-charge shifted envelope modes [1].

Second, interaction of the individual ions with the macroscopic field of the electron beam can lead to lifetime degradation and emittance growth of the cooled particles. A weak-strong beam-beam effect [2] and an excitation by the noise of the electron beam at betatron harmonics are phenomena of that sort.

Third, coherent beam-beam interaction could be a deteriorating factor as well [3].

Coherent ion-electron interaction was theoretically considered in a model of transversely immobile, totally magnetized electrons [4]. This interaction appeared to be too weak for realistic parameters of electron coolers, indicating that the electron drift mobility should be taken into account. That approach was first attempted in two simultaneous papers, [5] and [6]. Being linear and local, the electron response can be described as a perturbation of the ion's revolution matrix. At first order, this non-symplectic perturbation matrix is proportional to a product of the electron and ion currents. The perturbation of the cooler matrix was calculated in Ref. [5] for arbitrary ion-electron and electron-ion phase advances, and neglecting the Larmor phase advance of ions. The analysis was limited there by the determinant calculation. In Ref. [6], the perturbed cooler matrix was calculated assuming the electron-ion, ion-electron and ion Larmor phase advances being small. Based on analysis of the perturbed revolution matrix, it was shown that horizontal-vertical coupling of the unperturbed ion motion is essential for the ion-electron instability. Slightly later, in Ref. [7], the eigenvalue analysis for the perturbed revolution matrix was performed in a case when the zero-current revolution matrix is block-diagonal, with identical blocks, and a beam waist in a middle of the cooler. The instability growth rate was found analytically for these conditions in the same order as in Ref. [6]. This result, obtained without any coupling assumption, seemed to contradict to Ref. [6]. It is shown below how this contradiction is untangled.

BEAM-BEAM INTERACTION

Electron motion is predominantly a drift in a crossing electric field of the ions and the homogeneous magnetic field of the cooler. It can be shown that for any electrostatic field, the electron beam drifts as an incompressible liquid, $dn_e / dt = 0$. Thus, for a rectangular profile of the electron beam, all the density perturbations are on its border: $\partial n_e / \partial t \propto \delta(r - a_e)$.

From here, two consequences follow. First, dipole ion-electron motion does not depend on the ion emittance as soon as the ions are mainly inside the e-beam. The effective ion density n_i is determined by its linear density λ_i and the electron beam radius, $n_i \equiv \lambda_i / (\pi a_e^2)$. Second, a quadrupole ion-electron growth rate $\Lambda^{(2)}$ relates to the dipole rate $\Lambda^{(1)}$ in the same way as for the conventional impedance [8]: $\Lambda^{(2)} = 2(a_i / a_e)^2 \Lambda^{(1)}$, where a_i^2 is the rms ion beam radius.

Linearity and axial symmetry allow expression of ion-electron equations of motion in terms of circular variables $\xi = x + iy$:

$$\begin{aligned} &\xi_i'' - k_{ie}^2 \xi_e + i k_{iL} \xi_i' = 0 ; \\ &\xi_e' - i k_{ed} (\xi_i - \xi_e) = 0 . \end{aligned} \tag{1}$$

Here the coefficients *k* describe mutual influence of the beams and a perturbation of the ion motion by the magnetic field. In practice, all the phase advances over the cooler length $\psi = kl$ are small, and Eqs. (1) can be solved by a perturbation method, assuming zero initial condition for the electron offset. The resulting 4D cooler matrix can be presented as follows:

$$\mathbf{C} = \mathbf{C}_0 + \alpha \mathbf{S}_f^{-1} \cdot \mathbf{M} \cdot \mathbf{S}_f . \tag{2}$$

Here $\mathbf{C}_0$ is a matrix of the pure solenoid, $\mathbf{S}_f$ is the fringe field matrix, $\alpha = \psi_{ie}^2 \psi_{ed}$ is the beam-beam interaction parameter, and **M** describes the matrix structure of the beam-beam perturbation:

$$\mathbf{M}=\begin{pmatrix}\mathsf{M}_{\rm d} & -\mathsf{M}_{\rm c}\\ \mathsf{M}_{\rm c} & \mathsf{M}_{\rm d}\end{pmatrix};\quad \mathsf{M}_{\rm c}=\frac{1}{2}\begin{pmatrix}1/3 & l/12\\ 1/l & 1/3\end{pmatrix};$$
$$\mathsf{M}_{\rm d}=\frac{\psi_{iL}}{6}\begin{pmatrix}1/4 & l/10\\ 1/l & 1/2\end{pmatrix}+\frac{\psi_{ed}}{6}\begin{pmatrix}1/4 & l/20\\ 1/l & 1/2\end{pmatrix} \quad (3)$$

The block structure of the perturbation matrix **M** is common for all rotation-invariant matrices. Note that the diagonal block of **M** contains additional small phase factors. The coupling block and the Larmor part of the diagonal block have been found in Ref. [6]; the coupling block and the electron-drift part of the diagonal block have been obtained in Ref. [5].

Taking a reference point in a middle of the cooler, an entire 4×4 revolution matrix can be expressed as

$$\mathbf{R}=(\mathbf{I}+\mathbf{P})\cdot\mathbf{R}^{(0)};$$
$$\mathbf{P}=\alpha\mathbf{C}_0^{-1/2}\cdot\mathbf{S}_f^{-1}\cdot\mathbf{M}\cdot\mathbf{S}_f\cdot\mathbf{C}_0^{-1/2}\ , \quad (4)$$

where **I** is the identity matrix, and **P** will be referred as a normalized perturbation matrix. The normalized perturbation is dominated by its coupling part: $\mathsf{P}_{\rm d}/\mathsf{P}_{\rm c}\cong 0.1\psi_{iL,ed}$. Typically, this ratio is ~$10^{-2}-10^{-3}$.

PERTURBATION THEORY

To find out how the perturbation **P** shifts eigenvalues of the revolution matrix, a perturbation approach can be developed. For the unperturbed eigenvectors, the Lebedev-Bogacz form can be used [9]:

$\mathbf{R}^{(0)}\mathbf{V}_m^{(0)}=\exp(-i\mu_m)\mathbf{V}_m^{(0)}$; $m=1,2,-1,-2$; $\mathbf{V}_{-m}^{(0)}=\mathbf{V}_m^{(0)*}$

$$\mathbf{V}_1^{(0)}=\left(\sqrt{\beta_{1x}},\ \frac{i(u-1)-\alpha_{1x}}{\sqrt{\beta_{1x}}},\ \sqrt{\beta_{1y}}e^{i\nu_1},\ \frac{-iu-\alpha_{1y}}{\sqrt{\beta_{1y}}}e^{i\nu_1}\right)^{\rm T}$$

$$\mathbf{V}_2^{(0)}=\left(\sqrt{\beta_{2x}}e^{i\nu_2},\ \frac{-iu-\alpha_{2x}}{\sqrt{\beta_{2x}}}e^{i\nu_2},\ \sqrt{\beta_{2y}},\ \frac{i(u-1)-\alpha_{2y}}{\sqrt{\beta_{2y}}}\right)^{\rm T}$$

These 4 eigenvectors are symplectic-orthogonal:

$$\mathbf{V}_m^{(0)+}\cdot\mathbf{U}\cdot\mathbf{V}_n^{(0)}=-2i\delta_{mn}\,\mathrm{sgn}(m)\ , \quad (5)$$

where **U** is the symplectic unit matrix. The superscripts *, $^{\rm T}$ and $^+$ stand for the complex conjugation, transposition and Hermit conjugation.

The perturbation formalism is developed here similar to the same problem in Quantum Mechanics [10], leading to the following result for the growth rates:

$$\Lambda_n=-\mathrm{Im}\left(\mathbf{V}_n^{(0)+}\cdot\mathbf{U}\cdot\mathbf{P}\cdot\mathbf{V}_n^{(0)}\right)/(2T_0);\ n=1,2$$
$$\Lambda_1+\Lambda_2=\mathrm{tr}(\mathbf{P})/(2T_0) \quad (6)$$

with T_0 as the revolution time.

In case of degeneration, $\mu_1=\mu_2\ [\mathrm{mod}\,2\pi]$, a choice for the unperturbed eigenvectors is not unique. Again, as for the similar Quantum Mechanical problem, the correct linear combinations are those, making the perturbation diagonal within the sub-space of degeneration.

GROWTH RATES

Neglecting the small diagonal part, the growth rate is calculated as

$$\Lambda_{1,2}^{\rm c}=\pm\alpha\kappa_{xy}/(2T_0);$$
$$\kappa_{xy}=\sqrt{\beta_{1x}\beta_{1y}/l^2}\,\sin(\nu_1)=\sqrt{\beta_{2x}\beta_{2y}/l^2}\,\sin(\nu_2) \quad (7)$$

Without coupling, only the small diagonal part works, yielding the following growth rate:

$$\Lambda^{\rm d}=\alpha(\psi_{iL}/24-\psi_{ed}/12)/(2T_0)\ . \quad (8)$$

The coupling is so important because the electron drift is orthogonal to the ion offset. Thus, for planar (uncoupled) modes, the force, acting back on the ions, is orthogonal to the ion velocity. The resulted work is zero, and thus the rate is zero too (at the lowest order over the small phases).

A specific case of a block-diagonal revolution matrix with identical *x* and *y* blocks, and with the envelope waist in the cooler center was treated in Ref. [7]. For that case, the growth rate was calculated:

$$\Lambda=\pm\alpha\beta_0/(4T_0l) \quad (9)$$

with β_0 as 2D beta-function in the cooler's center. This result follows also from the perturbation formula (6), applied to this degenerate case. Indeed, due to the rotation invariance, the correct eigenvectors are the circular modes:

$$\mathbf{V}_1=(\mathbf{V}_x+i\mathbf{V}_y)/\sqrt{2};\quad \mathbf{V}_2=(i\mathbf{V}_x+\mathbf{V}_y)/\sqrt{2}\ ,$$

where $\mathbf{V}_{x,y}$ are uncoupled (planar) eigenvectors. So, even without the machine coupling, there is an area around the coupling resonance, where even a small ion-electron interaction makes the optics 100% coupled. The width of this degenerate area is here $\mu_x-\mu_y\cong\Lambda T_0$. For practical cases, this width is as small as ~$10^{-4}-10^{-5}$. Thus, without machine coupling, the growth rate is zero everywhere, apart of the separate punctured point of the coupling resonance. It was not realized in Ref. [7], that even a tiny step out of that special point makes the rate strongly suppressed.

More specific result can be obtained for a following form of the revolution matrix:

$$\mathbf{R}=\mathbf{D}^{-1/2}\cdot\mathbf{R}_D\cdot\mathbf{D}^{-1/2}\cdot\mathbf{C}$$
$$\mathbf{R}_D=\begin{pmatrix}\mathsf{R}_x & 0\\ 0 & \mathsf{R}_y\end{pmatrix};\ \mathsf{R}_{x,y}=\begin{pmatrix}\cos\mu_{x,y} & \beta_0\sin\mu_{x,y}\\ -\sin\mu_{x,y}/\beta_0 & \cos\mu_{x,y}\end{pmatrix}$$

where $\mathbf{D}^{-1/2}$ is an inverse drift matrix for a half of the cooler, $\mathbf{R}_D$ is the revolution matrix for no-field and no-current solenoid, and $\mathbf{C}$ is the on-field and on-current solenoid matrix. For specific parameters of ACR (RIKEN) ion ring, the eigenvalues of that sort of matrix were numerically calculated in Ref. [11]. Growth rate as a function of the tune separation is presented for this case in Fig. 1.

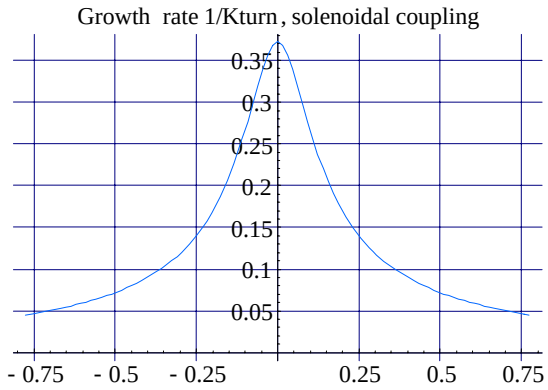

Fig. 1: Ion-electron growth rate, in inverse Kturns $(10^{-3}/T_0)$ as a function of the tune separation $\mu_x - \mu_y$, reproducing Fig. 1 of Ref. [11].

It can be shown that this growth rate is described by the following formula:

$$\Lambda = \frac{\alpha\beta_0}{4T_0 l}\frac{1}{\sqrt{1+(\mu_x-\mu_y)^2/\psi_{iL}^2}} \quad . \qquad (10)$$

Note that the width of the coupling resonance is determined by the (uncompensated) ion Larmor phase advance ψ_{iL}.

If the solenoid is compensated, the revolution matrix is written as $\mathbf{R} = \mathbf{C}_0^{-1/2}\cdot\mathbf{R}_D\cdot\mathbf{C}_0^{-1/2}\cdot\mathbf{C}$. Numerical eigen-value calculation for the same parameters as Fig. 1, but with this compensated coupling, reduces the width of the growth rate resonance ~ 300 times, see Fig. 2.

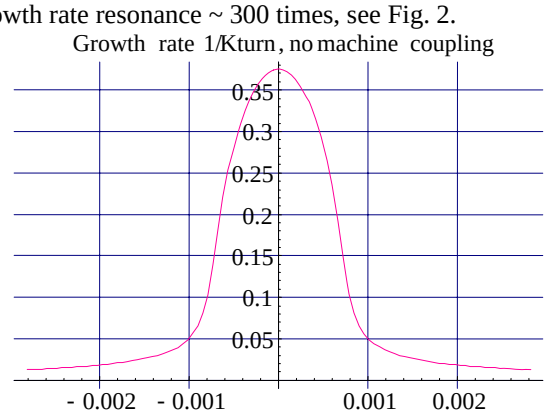

Fig. 2: Growth rate plot similar to Fig. 1, except the solenoid coupling is compensated. The peak maximum is the same, but the width is reduced ~300 times.

FAST HIGH ORDER MODES

The two-beam interaction is local. That is why the growth rates do not depend on the longitudinal wave number *n*. This makes a difference between the two-beam instability and instabilities driven by the chamber impedance. The chamber impedance destabilizes slow modes (*n<0*), and stabilizes fast modes (*n>0)*. From other side, the Landau damping is not effective at $n \cong n_* \equiv \xi/\eta$. To be stabilized by the chamber impedance, these modes have to be fast. That is why, conventionally, the sign of the chromaticity ξ is set identical to the sign of the slippage factor η. The dipole ion-electron instability would be most likely suppressed either by Landau damping or by the chamber impedance.

The situation is different for the quadrupole and higher order modes though. The rate of the mode *m* scales with the aperture *b* as $\propto mb^{-2(m-1)}$ in the units of the dipole rate [8]. The same is true for the ion-electron rate, with the electron beam radius a_e as the aperture. Usually $a_e << b$, so the quadrupole or higher order modes, most likely, are insufficiently suppressed by the chamber impedance. Since there is no Landau damping at $n \cong n_* \equiv \xi/\eta$, these fast quadrupole or higher order modes are, most likely, linearly unstable.

At some level, these oscillations would be stopped by their own non-linearity and then stay forever. Due to vanished Landau-damping, transfer of this coherent motion into incoherent one is strongly suppressed. This suppressed energy transfer still has a place, and results in slow emittance growth and lifetime degradation. This problem can be fixed by stepping away from the coupling resonance, or / and reduction of the coupling area.

Recycler Ring used to stay at the coupling resonance, having ~100% coupling. There was emittance growth, strongly correlated with the electron current and the pbar linear density. At “mining” state (max bunching), and 100mA of e-beam, the typical rate was ~ 30 pi mm mrad/hr, or ~0.001 of the calculated quadrupole rate for these parameters. The described theory pushed the author to request more tune separation. For the new working point, the coupling parameter reduced ~10 times, and the emittance growth reduced by about the same factor [12].

I am thankful to Martin Hu, Sergei Nagaitsev, Lionel Prost, and Alexander Shemyakin for numerous details of the observations in the Recycler, and to Alexander Valishev for the Recycler optical file. My special thanks are to Valeri Lebedev for his multiple remarks related to various aspects of this paper.

REFERENCES

[1] A. Burov, S. Nagaitsev, NIM-A, **441**, Nos 1-2, p.18 (2000).
[2] D. Reistad et al., CERN 94-03, p.183 (1994).
[3] V. Parkhomchuk, D. Pestrikov, ibid, p. 327 (1994).
[4] A. Burov, Part. Acc., **57**, p.131 (1997).
[5] P. Zenkevich, A. Bolshakov, NIM-A, **441**, Nos 1-2, p.36 (2000).
[6] A. Burov, ibid, p. 23.
[7] V. Parkhomchuk, V. Reva, JETP, **91**, 5, p.975 (2000).
[8] A. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", J. Wiley & Sons Inc.
[9] V. Lebedev, S. Bogacz, JLAB-ACC-99-19 (1999).
[10] L. Landau, E. Lifshits, “Quantum Mechanics”, Elsevier Science Ltd.
[11] P. Zenkevich, ITEP 26-01 (2001).
[12] L. Prost et al., Proc. HB2006, Tsukuba, Japan (2006).