Orbit Studies Related to the Berkeley 88-Inch Cyclotron

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I would like to begin by talling you a little bit of the history of our efforts in this field at Berkeley. Our cyclotron project started rather later than most other groups who are now actively designing and constructing machines. It was perhaps in late spring of last year that we began to consider the problem of orbit calculations seriously. At that time we made use of relatively simple formulas such as Richardson and Powell have described this morning, primarily for the purpose of providing our model magnet group with a starting point. Since the accuracy of the formulas available was questionable and particularly since no spiral-ridge cyclotrons in our desired energy range have yet been put in operation, we thought it important to have better methods for predicting particle motion in the actual fields measured in the model magnet.

Our efforts went in two directions--looking for computer programs of sufficient generality and developing analytic formulas which could be applied more or less directly to model data. We have met with good fortune in both endeavors. MURA, mostly in the person of Frank Cole, supplied us with its Ill-Tempered-Five IBM 704 code, designed just for this sort of application. More recently Welton gave us the Oak Ridge code No. 1482, a very fast and efficient 704 code for obtaining equilibrium orbits and betatron frequencies from a set of measured field values.

On the analytic aspects we developed a number of formulas for equilibrium orbits, betatron frequencies, and the 3/3 radial nonlinear resonances effects in 3-sector geometry. This work is described in UCRL Report 8598, and the results are very similar to, if not identical with, those obtained by Parzen. The field is represented by a Fourier analysis in azimuth and Taylor series in radius. Axial betatron frequencies are obtained by the smooth approximation, with the difference from previous work that we retained all terms which might be relevant in the case of a medium energy cyclotron. For the radial frequencies, the smooth approximation is not as readily applicable, so we resorted to a technique involving canonical transformations on a Hamiltonian.

That is about the end of the history except to add that Al Garren and I spent the week before the New York Physical Society meeting at Oak Ridge getting the last bugs out of our input to the 1482 code and investigating the 3/3 resonance with the help of their Oracle computer.

I would like to show you a still somewhat incomplete table of axial and radial betatron frequencies computed by the various methods from some of our model data (Table 3). It would appear that for all practical purposes the results are the same. Since the 1482 program uses the measured field values directly with a minimum of intermediate processing and can produce a complete table of orbit times and betatron frequencies in about 15 minutes of computer time, we shall probably use it most often to obtain precise values, and rely on the analytic formulas to tell us which features of the field shape are most important at the various radii. For those of you who would like to have this sort of information but do not have access to a 704, I should say that the analytic results can be obtained in a reasonably short time with a desk computer; the accuracy indicated in Table 3 requires about three harmonics in the Fourier analysis of the field.

With respect to the isochronous condition, the non-circularity of the equilibrium orbits causes a deviation which ranges in the 3-fold case up to about 5 parts in a thousand. Correcting the average field according to formula (20) of UCRL Report 8598 reduces the error to a maximum of 5 parts in ten thousand.

On the matter of the non-linear resonance effects arising because the radial frequency is close to unity, there has been some confusion in the recent past. Stahelin's analysis of the problem led him to the conclusion that 3-fold symmetry was very dangerous; this conclusion was being contradicted by detailed orbit calculations on the Oracle. Encouraged by the fact that some time ago at Harwell, Marshall King and Mrs. Bell had been able to reconcile computational and analytic results on a very similar problem, we proceeded to obtain analytic formulas for the effect in the 3-sector case, again being careful to retain all relevant terms to the end. We came to Oak Ridge armed with predictions of how the particles should move in our model fields, and were much heartened to find that the Oracle agreed with us, to greater precision than we perhaps had a right to expect. Bender and Bassel found that our formulas gave similar agreement with the computational results they had been getting, and Blosser tells me that there is reasonable agreement for the Michigan State machine as well. It therefore seems safe to say that formulas now exist from which the 3/3 resonance effects may be predicted with sufficient accuracy. The 4/4 case would require somewhat more effort to carry through, but the effort would probably be rewarded with success.

This degree of understanding applies as yet only in the approximation that the energy gain of the particles is neglected. The last few days of our stay at Oak Ridge were spent accelerating particles in the Oracle for various starting conditions at the center of our model field and following them for as much as 75 turns. We do not have enough cases to say definitely how reliably the constant energy predictions may be used in the presence of acceleration, but we did find a group of particles corresponding to a radial spread of about 1/4 in. and angular spread of a few degrees at a radius of 3 in. (our final radius is 37 in.) which looked certain to go all the way with amplitudes of 1/4 in. or less. On the other hand, we found another group which appeared to increase in amplitude by 50% or more from 25 turns to 75 turns, although they seemed to be well within the limits of constant energy stability at 25 turns.*

That about ends the description of our state of knowledge of how particles should behave in a spiral-ridge cyclotron. We have no idea as yet about how we would try to extract a beam or of what peculiarities might be present in the center-most region. In the immediate future we shall try to extend the analysis of the non-linear problem by including acceleration.

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*Since the meeting we have discovered that the odd behavior of this group is due to an interaction between the fluctuations in energy gain from a smooth increase and the odd-fold flutter structure of the field, resulting in a displacement of orbits much as would arise from a first harmonic field error. The effect is not very large; the main result of understanding the anomaly has been to lead us to the conclusion that our 3-fold magnet design, with 70 kv on a single dee, is all right as far as the resonance effect is concerned.

Table 3. Betatron Frequencies Computed for Fields Measured in the Berkeley Model Magnet.*

	Approx. radius	ν_z (N = 3)			$(\nu_r - 1)$ (N = 3)			ν_z (N = 4)		$(\nu_r - 1)$ (N = 4)	
P/m_oc	(inches)	Analytic	MURA IT-V	OR 1482	Analytic	MURA IT-V	OR 1482	Analytic	MURA IT-V	Analytic	MURA IT-V
0.025	4	0.028	--	0.026	0.00227	--	0.0023	--	defocus	--	0.00057
0.05	7.5	0.096	0.093	0.093	0.00901	0.0086	0.0092	0.0481	0.0046	--	0.00335
0.10	15	0.142	0.146	0.146	0.0147	0.0144	0.0147	--	0.0633	--	0.0193
0.15	22	--	0.151	0.147	0.0201	0.0155	0.0207	0.0893	0.0996	--	0.0197
0.20	30	0.177	0.158	0.178	0.0248	0.0204	0.0249	0.129	0.130	0.0201	0.0209
0.25	37	--	0.178	0.182	--	0.0379	0.0382	0.122	0.130	0.0372	0.0379

*The flutter amplitude is about 0.2, and the spiral angle increases from 0° at 10 in. to about 60° at 37 in. The values are at high field, corresponding to 60-Mev deuterons at 37 in. radius (maximum momentum = 0.25 m_oc).