Orbit Calculations for MSU Cyclotron

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The computations which I want to describe bear on the design of a 50-Mev variable-energy multiple-particle cyclotron on which we are working at Michigan State. In this machine we would like to use three sectors because of the enhanced axial focusing thus obtained, particularly near the center. To use the three sectors you must first of all carefully check the extent to which the beam will be disturbed by the strong $\nu_r = 3/3$ resonance. The computations which I will describe are concerned mainly with ascertaining the precise effects of this resonance.

The geometry of our magnet is shown in Figure 42. The pole base is a 64-in. right circular cylinder. The pole tips which produce the hills in the field are sections of right cylinders with one straight edge, one 32-in. convex edge, and one 22-in. concave edge. I won't go into the factors which led us to this particular shape. The field produced by the magnet is determined experimentally in a model magnet study and the date are processed into a form suitable for use with the Oak Ridge orbit codes. The codes compute the properties of orbits in the measured field.

Figure 43 shows the actual shape of the field as measured in the model magnet. The curves show the variation of the field with azimuth at various radii. As you can see, the field is relatively flat on the hills; the valley field gets weaker and the hills get wider as the radius increases so that the proper increase of average field with radius is maintained.

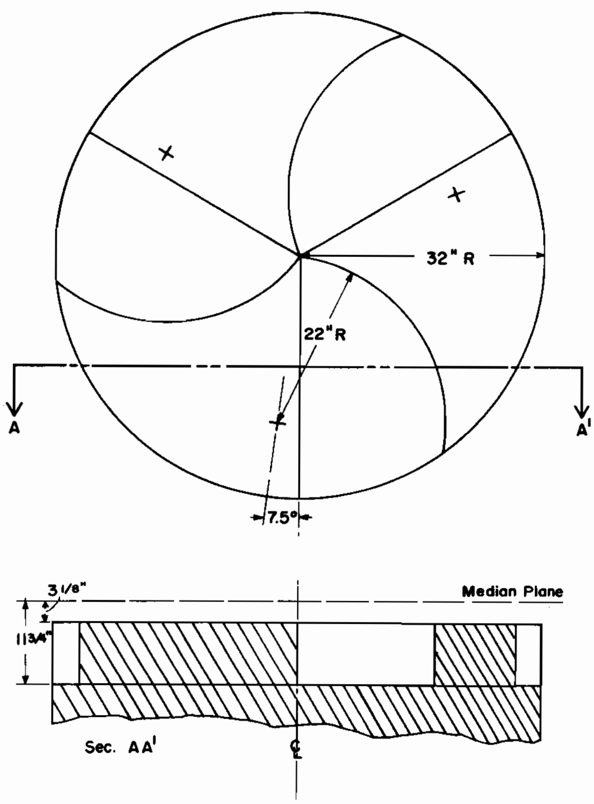

Fig. 42. Pole tip geometry of proposed MSU cyclotron.

Figure 44 shows the linear or small amplitude properties of the axial motion obtained with this field. The axial focusing frequency ν_z in units of the orbital frequency is plotted versus the number of turns for our two principal particles, protons and C^{4+} ions. The rapid rise in the ν_z near the center shows very clearly the reason why we want to use three sectors in our cyclotron. These results are with a field which is strictly isochronous from the center all the way out to the deflection radius. We feel that with this rapid rise in ν_z it won't be necessary to use an average field falloff or anything of that nature in the center. We will run it strictly isochronous all the way and avoid the possible difficulties of passing through the integral resonance encountered with falloff. Also, we plan to use focusing grids to enhance the ν_z on the first four or five turns.

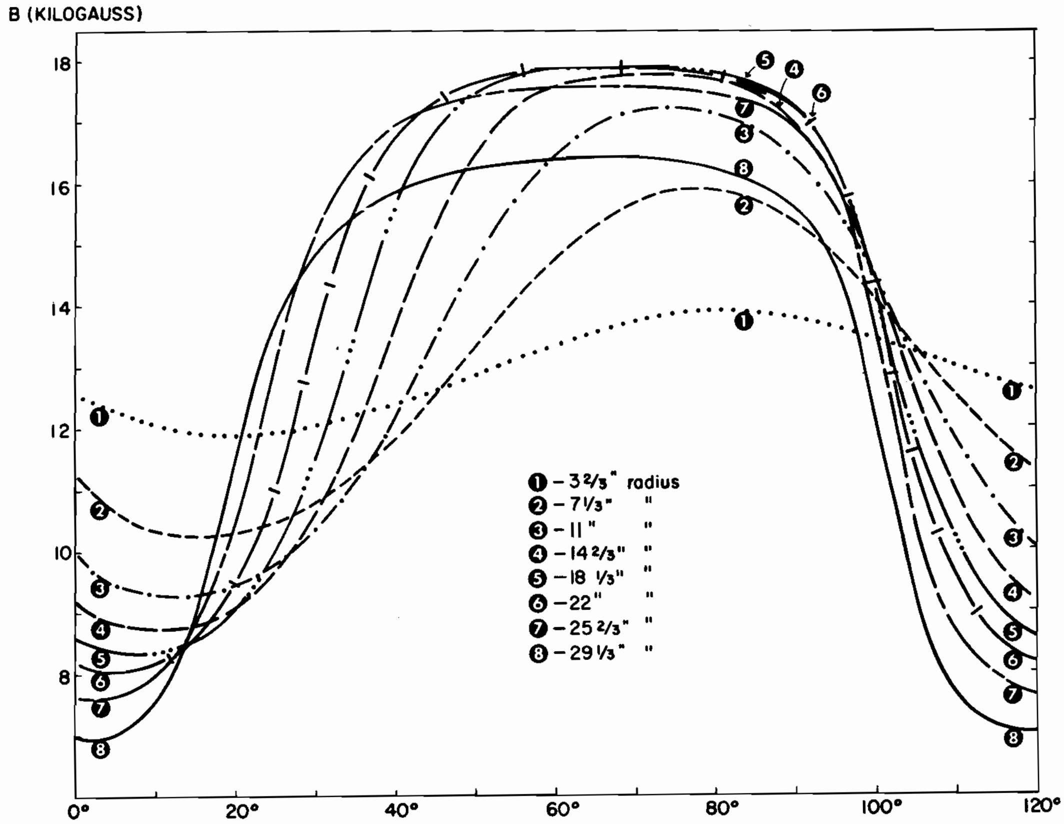

Fig. 43. Azimuthal plot of magnetic field obtained from model measurements.

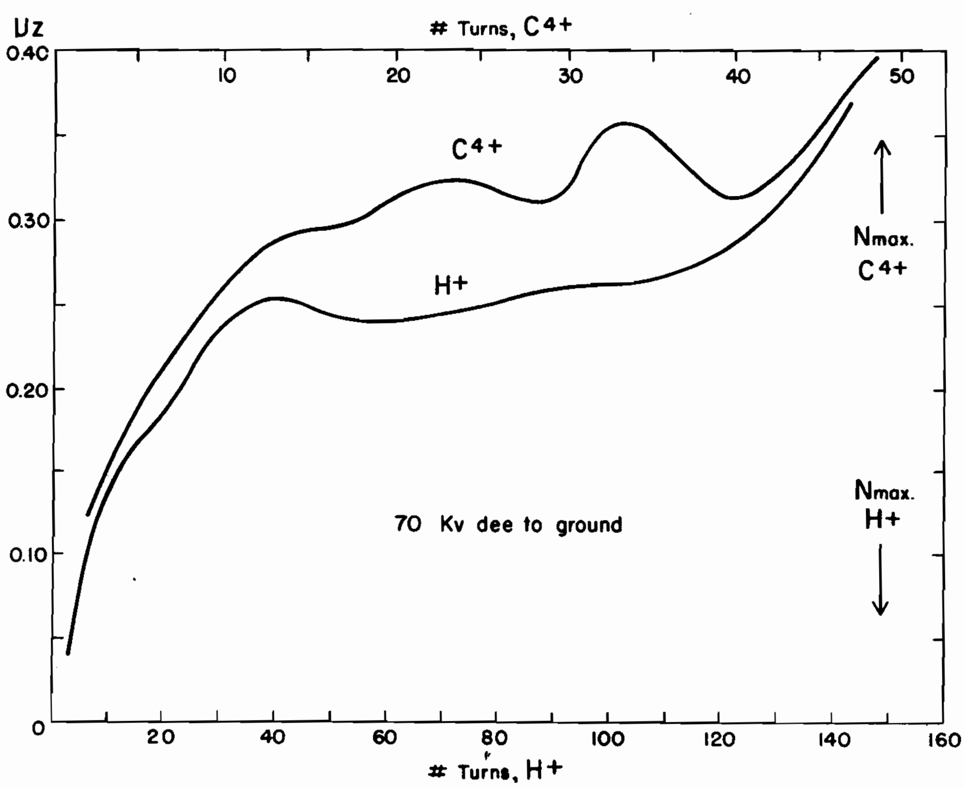

Fig. 44. Small amplitude axial focusing frequency obtained from orbit integrations.

Figure 45 shows the linear properties of the radial motion. We have plotted both the computer results and results obtained from an analytic formula sent to us by Dr. Parzen, and discussed by him in an earlier paper. We have plotted $(\nu_r)^2$ in this figure and on a scale with the zero way off in the basement so that errors are highly magnified. The results from Parzen's formula track along with the computer results quite nicely; they even do a fair job of reproducing the small bumps which the computer showed. I have also checked the Smith and Garren formula and find that for our machine the coefficients of the various items come out to be the same as Parzen's so that in this particular case their formula gives the same agreement with the numerical results.

I also have plotted on the figure the ν_r which is obtained if one neglects the flutter field, as is often done, and computes the ν_r which would be produced by the circularly symmetric part of the field. This curve is labeled "H_o terms only". You see that the deviation of ν_r from one is determined by the flutter field. The deviation is quite substantial, and it turns out that it is really this large deviation from the resonant value which in the end results in comfortable stability limits.

Figure 46 gives phast plots of the sort which Lloyd Smith explained in his talk this morning. Plots are given for protons of four different energies. For a given orbit, Δr and Δpr are plotted as a heavy point once per sector; the points resulting from a given set of initial conditions are connected by a light line. Plots are given for protons of four different energies; in each case the scales have been adjusted so that the equilibrium orbit for the particular energy lies at the origin of the system of coordinates.

For each of these runs we've made comparisons with the predictions of Smith and Garren on stability limits, and in each we've drawn on the graph their predicted

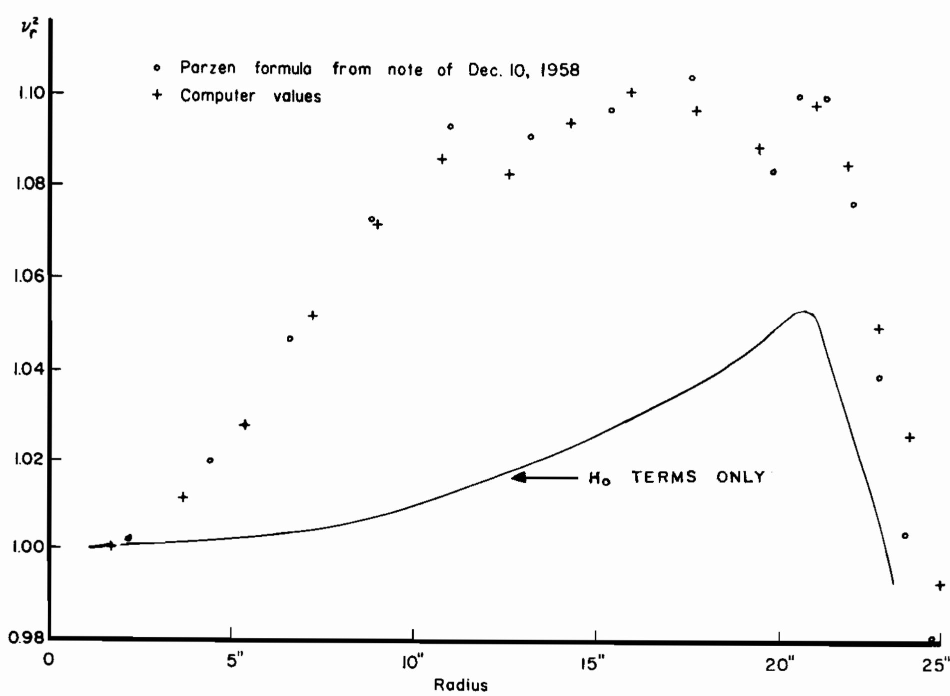

Fig. 45. Small amplitude radial focusing frequency obtained with Parzen's formula.

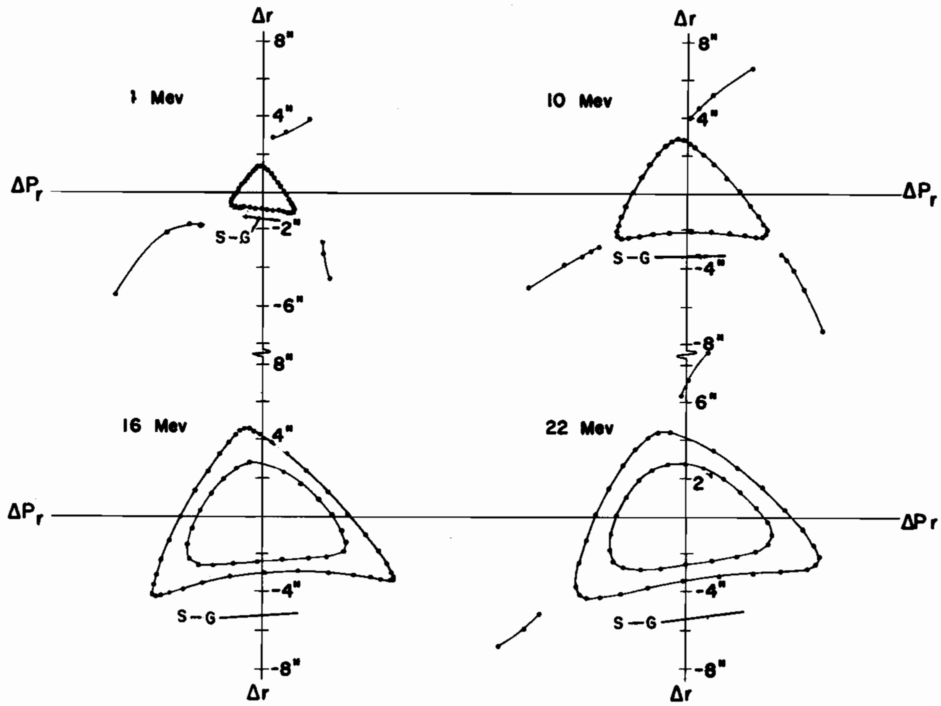

Fig. 46. Radial phase-space plots for protons.

value for the minor axis of the triangle, labelled S-G. We don't have sufficient numerical data to accurately specify the actual stability limit, so the comparison is not a really stringent test of their theory. On the basis of the results which we do have, the theory certainly looks very good, though.

In Figure 47 we have the same sort of information for C^{4+} ions. Possibly somewhat surprising is the fact that the stable region is essentially the same size as it was for the protons. One might think that because of the reduced relativistic effect the heavy ions would be much closer to the 3/3 resonance. Actually the pertinent factor is the deviation of ν_r from 1; which, as you recall from a previous figure, is determined by the flutter field. When we flatten the azimuthally symmetric part of the field to provide isochronism for the heavy ions it gives essentially no change in ν_r; hence, it is not at all surprising that you end up with comparable stability limits.

I should note also that, here as in all of these computations, we are still using a field which is strictly isochronous out to the deflection region. A circularly symmetric correction, which in the actual magnet will be produced by circular trimming coils, has been added to the measured field to give isochronism. The strength of this correction is appropriately adjusted when we shift from protons to heavy ions.

In addition to the results shown a run was made for C^{4+} ions of 4-Mev energy and with an initial amplitude of 1.2 in. In 45 sectors no detectable growth or precession took place; the points simply plotted on top of themselves. It would be impossible to say, absolutely, whether the motion is stable or unstable without an exhorbitant amount of computation. The motion is effectively stable however, since after only four turns the energy would have increased to 8 Mev where the comfortable limit from the first plot of Figure 47 is applicable.

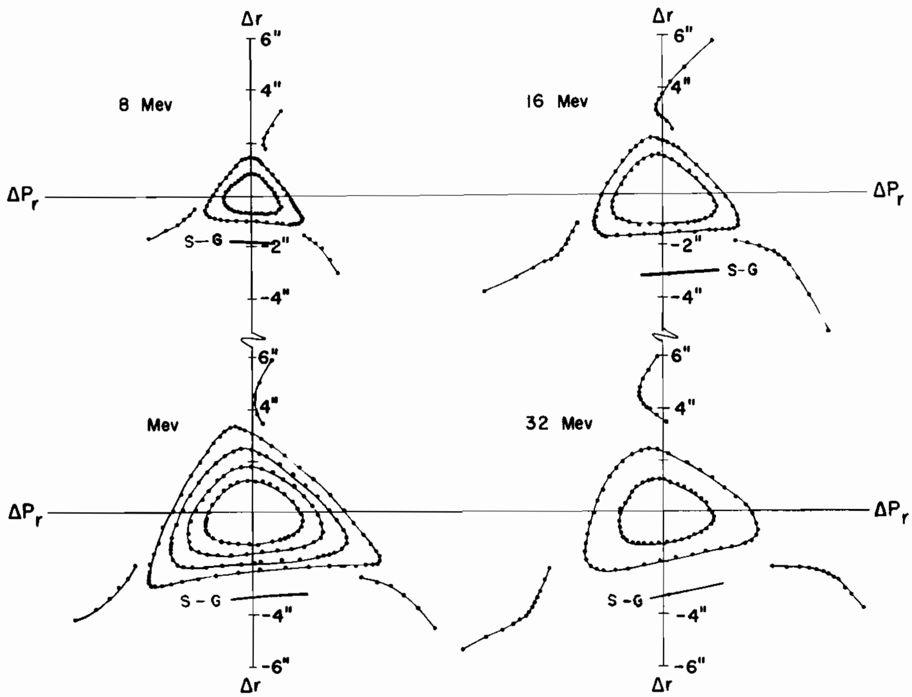

Fig. 47. Radial phase-space plots for C^{4+} ions.

On the basis of these runs we've concluded that there will be no difficulty from the 3/3 resonance in our cyclotron. We expect a distribution radial amplitudes of $\pm 1/8$ in. in our cyclotron, and the stability limit is in every case far larger. I should point out that all of these runs are with coasting particles, no acceleration is included. With the large stable region which we've found one would not expect any difficulty to be introduced by the acceleration, but we do plan some runs to check this quantitatively.

Stability limits for the axial motion would normally not be expected to present any difficulties in a cyclotron such as we are designing. Iron and currents must be fairly far from the median plane to provide for dee insulation; fields should therefore be essentially linear in the beam space. We made several runs with the MURA IT-V code to verify this. We tracked a series of particles with 3/4 in. initial displacement from the median plane, corresponding to our maximum beam space, and in every case the axial motion was completely stable. Several of the runs included coupled r and z oscillations and there was no indication of any substantial reduction in the radial stable region due to the coupled motion.

Finally I wanted to present the results of some comparisons which we made between the Mura IT-V code and the Oak Ridge 589 code. We went into this comparison because in one of our radial stability runs with the Oak Ridge code we got a queer behavior in a phase plot; Bender and Bassel at Oak Ridge have also found a number of similar anamolous plots. We suspected some difficulty in the 589 code and made the comparisons as a check.

First of all, in these comparisons you run into the difficulty that the Oak Ridge code works from a grid of measured field values, whereas the MURA code specifies

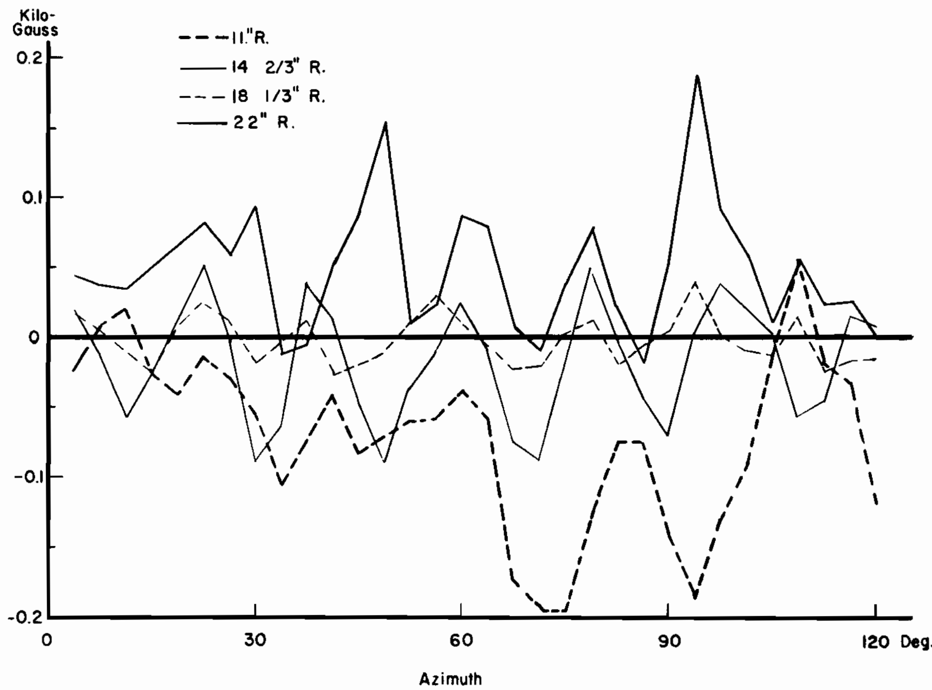

Fig. 48. Residual error for fitting of model magnet fields with IT-V field formula. $\bar{B}$ is 14 kilogauss.

the field by means of an analytical formula. We adjusted the parameters in the formula to give a least-squares fit to the measured fields over the region of interest. In Figure 48 the residual difference between the two fields is plotted at four radii. The difference in fields goes up as high as 1% at the worst places. Actually, we are fitting, of course, over quite a broad range, but one has to because the stability diagram extends over quite a broad range. Thus, we are really somewhat limited with the IT-V in how well we can reproduce actual fields. I think this relates to Dr. Teng's question this morning about the differences in results from the two codes, you just can't get the fields closely enough alike with them.

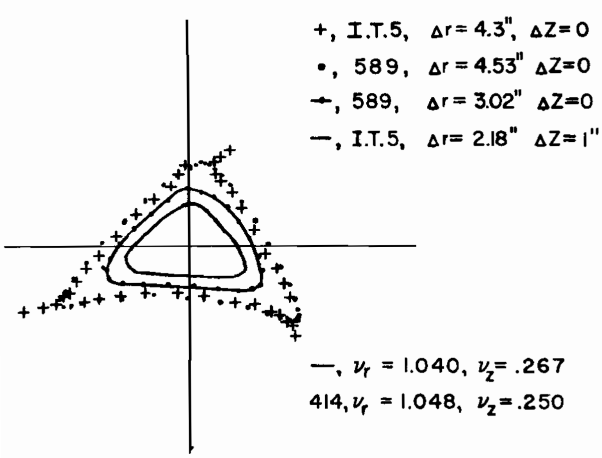

Fig. 49. Phase diagram obtained.

Figure 49 shows the phase diagrams which were obtained with the two codes. Unfortunately, we made a mistake in parameters and did not start with the same Δr for the two codes so that the results form an interweaving net. In general however, the agreement is fully as good as one would expect considering the difference in the input fields. The small amplitude ν values checked out quite closely, as is indicated on the figure.

Since these runs were made, a coding error has been uncovered in the 589 code which fully accounts for the particular anamolous phase plot which we observed. This error was of a nature such that when it occurred a grossly incorrect field value was fed to the code. From this and from the favorable results of the code comparisons we've concluded that the 589 code is in normal circumstances completely trustworthy, and we therefore have full confidence in the reliability of the results we have presented.