Orbit Dynamics for a Four-Spiral-Sector Cyclotron

K. M. Terwilliger

I wish to report on some calculations I made last year on stability limits and orbits for a cyclotron design for the University of Michigan. This machine was designed with a somewhat different philosophy than those that have been presented here. We wanted to make a 40-Mev deuteron machine giving a β of only about 0.2 which is quite small; it could be quite a conservative machine. We decided to use the Stahelin-type flat spiral shim design* the spiral sector variety with very small flutter, the point being to keep away from worries about satuartion effects. This was to be the key point. So we designed for fairly low fields, about 15 kilogauss, and a maximum flutter of about 7-1/2%. To do this, of course, we had to have quite a lot of spiral. We chose 4-sectors. Figure 59 indicates the design of this machine. It is quite large. The extraction radius is about 34 inches. The spiral is quite tight, as you can see, going out to a maximum tan γ (defined previously) of about 4.

The determinations of the frequencies of oscillation and stability limits were performed on the MURA IBM 704 computer using the well-tempered-V program with scaling fields, a fair approximation here. I would like to discuss the results briefly now. First, the betatron oscillation frequencies. The smooth approximation results were remarkably borne out, much more so than indicated before in the discussions.

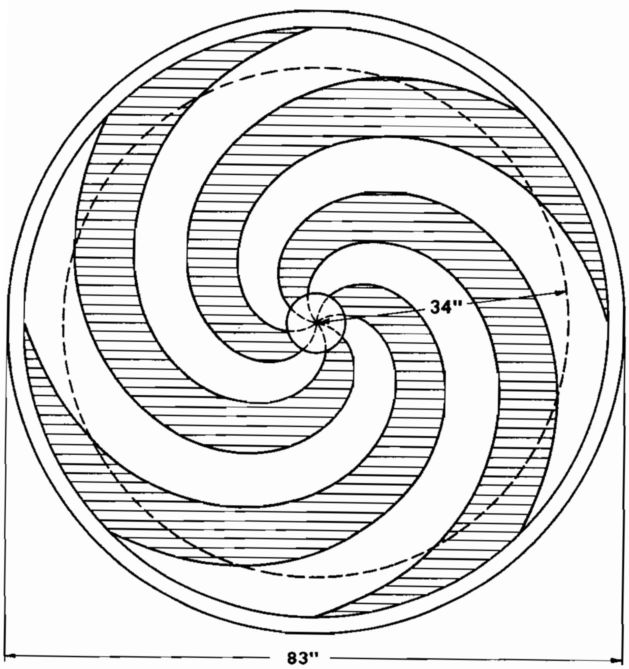

Fig. 59. Schematic diagram of a 4-spiral-sector design for 40-Mev deuterons, with flat shims cut in an Archimedean spiral.

For example, the smooth approximation for ν_x and ν_y gives

$$\nu_y^2 = -k + f^2 \left(\frac{1}{2} + \tan^2 \gamma\right)$$

$$\nu_x^2 = 1 + k,$$

where the parameters are defined from the median plane scaling field,

$$H = H_o \left(\frac{r}{r_o}\right)^k \left[1 + f \cos \left(N\theta - N \tan \gamma \ln \frac{r}{r_o}\right)\right] ;$$

At the maximum radius the parameters are

$$k = 0.043, \quad f = 0.075, \quad \tan \gamma = 4.$$

A comparison of the smooth approximation and the computer results for these parameters is:

*Bluemel, Carroll, and Stahelin, Technical Report No. 2, Navy Contract 1834 (05). University of Illinois.

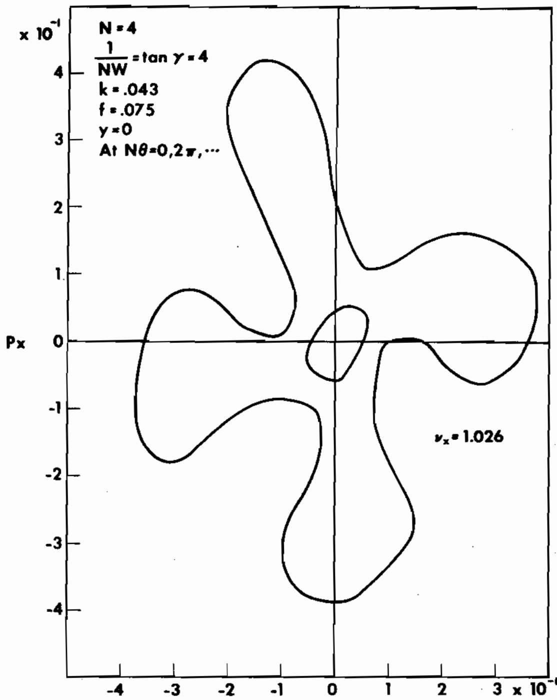

Fig. 60. x-motion phase curve; x is measured in units of the radius; the radius; $p_x = \frac{dx}{d}$ in radians.

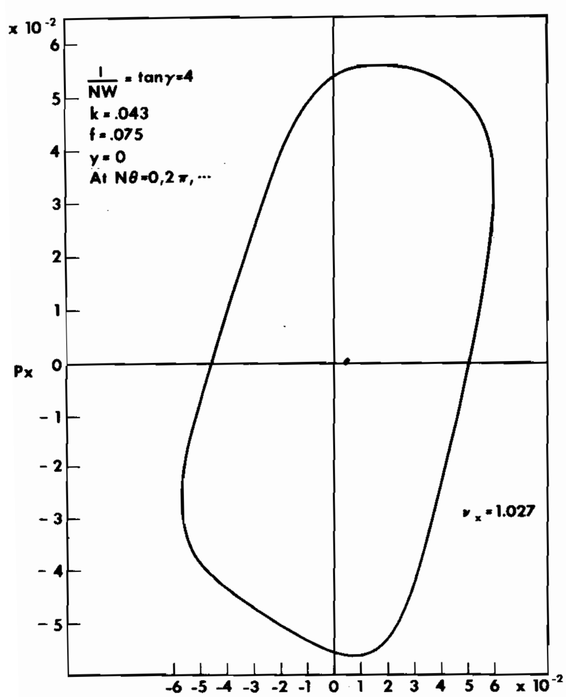

Fig. 61. A large amplitude x-motion phase curve, the central curve of Fig. 60.

	Smooth App.	Computer
ν_x	1.022	1.028
ν_y	0.223	0.231

Now for the stability limit, which was what we were really interested in, since this is a very tight spiral machine. We started by running some phase plots with zero y-motion. Here is such a phase plot (Fig. 60). The point near the center is a fixed point; there is only a small amount of displacement from zero because the flutter is so small. The curve, as we will see, is about halfway out to the stability limit. The particles, after processing around, seem to fall on this curve with very little scatter.

Now to investigate what happens with larger oscillations still without any y-motion. Figure 61 shows what happened as we doubled the initial x-amplitude. The inner curve is the previous phase plot. The character of the motion changed quite violently, as you can see. The reason has been indicated in previous talks. Inside these arms are undoubtedly four stable fixed points, corresponding to an orbit that would repeat after one revolution, or after four sectors. Around these four fixed points are stable island-like curves, and in between these islands and the central curve are four unstable fixed points. This is all in agreement with the pictures drawn in the 3-sector case Blosser talked about. This outside curve is an apparently stable envelope around the five inner islands. Although in the intermediate region the motion perhaps is considered unstable, it really isn't over-all. You have to take this into account in any extraction process.

We consider now what happens as we add a bit of y-motion to the problem. The previous work here was for zero y-amplitude. We started with that previous x-curve we had before (in Fig. 60), the one with $x_0 = 5 \times 10^{-2}$, and increased the initial y-amplitude to see

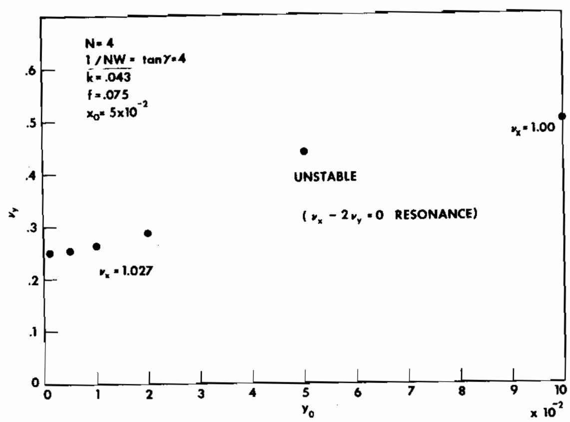

Fig. 62. y-betatron oscillation frequency as a function of initial y-amplitude.

where we would run into trouble. As indicated in Figure 62, the original y-frequency started out about 0.25 and increased as the y-oscillation amplitude increased. This is as you would expect, on a qualitative basis, because you are seeing a larger flutter as you get off the median plane, and the y-frequency goes up with the flutter. ν_x apparently has very little dependence on the y-amplitude, at least for a while. It stays essentially constant in this region at about 1.027. This is also reasonable, since in the smooth approximation ν_x does not depend on the flutter.

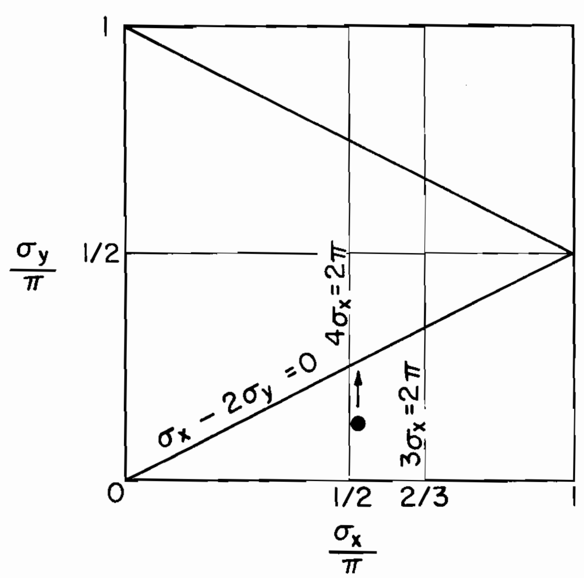

Fig. 63. Sector resonance plot.

After we got past the point of initial y-amplitude of 2% of the radius, 2×10^{-2}, the next point we ran was about 5×10^{-2} and the motion was clearly unstable. We then doubled the amplitude and got a sort of locked-in motion which was clearly a coupling resonance. The unstable motion, $y_0 = 5 \times 10^{-2}$, took off in about two or three revolutions, so clearly you cannot work with y-oscillations of this magnitude. Anything below this unstable region is perfectly satisfactory. There both the x-motion and the y-motion fell pretty well on invariant curves.

The type of instability involved can be understood most simply with the aid of a sector resonance plot (Fig. 63). The quantity σ is the betatron phase change per sector and is related to ν by $\sigma/\pi = 2\eta/N$. (So here $\sigma/\pi = \nu/2$, with N = 4.) The lines correspond to the major sector resonances. For small y-amplitude, the σ's for the given parameters are indicated by the dot, just above the $4\sigma_x = 2\pi$ (or $\nu_x = 1$) resonance. The resonance which causes the trouble in the coupling is the $\sigma_x - 2\sigma_y = 0$ resonance, called the Walkinshaw resonance. On this resonance the x-frequency is twice the y-frequency. As we increase the y-amplitude, the tune point moves up toward this coupling resonance, parallel to the $4\sigma_x = 2\pi$ resonance line, until the motion becomes unstable or locked-in.

One more thing we did was investigate the motion at different radii, adding a field perturbation. We applied a first harmonic ripple to the field, to see if this changed the stability limits. With a perturbation, which we estimated would drive the equilbrium orbits to an amplitude of 1 cm, I found that the motion at different radii, (full radius, half, one-quarter, and one-eighth) was perfectly stable, even with coupled betatron oscillation amplitudes of 2 cm. Since we hope to keep our oscillation amplitudes less than 2 cm during acceleration, I would think this would be a fairly reasonable design.