## Beam Deflection Systems

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Before I start on a discussion of beam extraction, I think I should take a moment and indicate how we obtain results of the sort I want to talk about. First of all, we do our model magnet work at Oak Ridge; we have a contract with the AEC which lets us use the laboratory facilities and then, with the cooperation of the group down there, we get the work done. In analyzing the data we also depend rather heavily on Oak Ridge codes. In addition, we do part of the work on our own computer, the Mistic, and for certain things we've used the MURA computer and codes. We're greatly indebted to all of these people for their assistance and cooperation.

The calculations I want to discuss first are simply the initial steps in the design of a resonant deflection system for a medium energy cyclotron. The system would be based on the ideas which Dr. Gordon discussed in the previous talk. In tackling this problem we looked first of all at the nonlinear effects in the radial or median plane motion. What we wanted to do was to get a good rate of growth, good turn separation, and to do this the field must be adjusted in someway so that $v_{r}$ goes to the resonant value. This will cause the stable region to shrink so that the beam can be pushed out of it with a small $\cos \theta$ field bump. With the beam outside the stable region, the nonlinear effects should then cause the amplitude to grow and hopefully, you will eventually get a turn separation large enough to allow insertion of a magnetic channel to pick off the beam.

Figure 239 shows the results which we got from our first series of runs of this type. For these runs we used the natural field from the model magnet. The


Fig. 239. Radial phase plots in the deflection region.
average field rises with radius out to a certain point, then it flattens off and begins to fall rapidly across the edge of the magnet. We figured the natural flattening off near the edge would take us to the resonance and, we hoped, would give as good deflection.

Instead, a phenomena which we had not at all expected popped up; the orbits which we had been calling unstable turned around and closed on themselves giving "pockets" or "eyes" in the stability region. Around the whole pattern a superstable region appeared, as you see in the $38-\mathrm{Mev}$ and $40-\mathrm{Mev}$ patterns of the figure. As the normal stable region shrinks (it doesn't actually show in any but the 30-Mev plot, but for each of the others it would be a successively smaller triangle nested in the center), the eyes also become smaller with smaller separation between points, which is equivalent to smaller turn separation. By the time the normal stable region gets small enough that you can push the beam out of it, the turn separation in the "unstable" eyes is essentially all gone so that the net result makes for very poor deflection.

It turns out the eyes were already well known in the theory on nonlinear resonances but back in September when we were making these runs we weren't aware of this, and so we simply followed our intuition in trying to get rid of the trouble. The thing which we noticed at once about these plots was a very large shift of $v_{r}$ as the amplitude built up. In the normal stable region we had a frequency which was slightly above unity and went to unity as the amplitude reached the critical value. In the superstable region the frequency was below unity and as you can see from the $40-\mathrm{Mev}$ run it shifted further below unity quite rapidly as the amplitude went up. The difference between $v_{r}$ and unity is given by the reciprocal of the number of points on a complete closed phase-space curve. For the inner $40-\mathrm{Mev}$ curve there are about five times as many points as on the outer.

We explained the eyes by saying that as the amplitude grew the frequency moved away from unity causing the particle to precess and move out of phase with the driving term and finally to precess into anti-synchronism with the driving term and get driven back into the machine. The frequency shift is then clearly the cause of the difficulty and after a series of discussions with Ted Welton we came up with a field shape gauged to eliminate the frequency shift. At $\nu_{r}=1$, its actually fairly simple to figure out how to do this. You want $v_{r}$ to be one and you want it to remain one as the amplitude grows. This is equivalent to saying that orbits shall close smoothly on themselves, even if their centers are displaced from the cyclotron center, and closing smoothly simply means that the integral of the curvature around the orbit must be $2 \pi$.

Consider a closed orbit centered in the cyclotron with energy equivalent to the maximum energy and a second similar orbit displaced from it. For half of its trajectory the displaced orbit lies inside of the centered orbit; in this region you have to leave the field alone because other centered orbits of lower energy also use this part of the field. You can, however, easily compute the accumulated curvature of the displaced orbit in this region and then when it moves into the half of its trajectory outside of the centered orbit you are free to adjust the field such that the accumulated curvature is just equal to $2 \pi$. The adjustment can be accomplished most simply by merely adding a circularly symmetric correction to the average field in the region beyond the radius of the final energy equilibrium orbit, and so this is what we did.


Fig. 240. Radial phase plots for protons for revised average field.

The phase plots (Fig. 240) which resulted from this revised field show at once that the situation is greatly improved. The stable region shrinks smoothly as the energy increases, and at the $38.6-\mathrm{Mev}$ resonance there is a relatively pure radial growth with increasing rate of growth per turn. The behavior for deflection is now reasonably favorable. The field used for these runs was the same as for those of Figure 239 out to the deflection radius. The average field rises in the center, then flattens off, and has zero slope at the deflection radius. Beyond the deflection radius the average field was set to be simply the symmetric image of the field inside that radius. Although we weren't thinking of it in these terms at the time, this reflection means that all odd derivatives of the average field are zero at the deflection radius, or more importantly, if you expand the average field in a power series about the deflection radius the $\mathrm{X}^{3}$ term in the expansion would be zero. Since we made these runs Dr. Bassel at Oak Ridge has checked through the theory and I think in the next paper he'll explain that these are really the crucial requirements.

From these results then we know how to get the basic radial phase patterns into the shape needed for deflection. I should emphasize though that this is only the first step in the deflector design. We need to quantitatively check the effect of the $\cos \theta$ bump in starting the oscillations; all the calculations which I've described have included only $\cos 3 \theta, \cos 6 \theta, \cos 9 \theta, \ldots \ldots$. . . components. We need to include acceleration; most importantly, we need to check the axial motion. It's quite possible that instabilities may crop up and will have to be dealt with. So we still have lots to do.

I have a few comments on a separate subject. We have done a few preliminary calculations on an electrostatic deflector and have been checking on a slightly new wrinkle which, on the basis of our preliminary results, looks quite promising. I
should make clear that we are not at all pessimistic about the resonant magnetic deflection. We hope to use that systern, but just in case it does not work out we have been thinking a bit about electric systems.

The main difficulty with electrostatic deflection in our cyclotron is that we don't have turn separation. The spread in radial amplitudes at the outside is greater than the radius gain per turn due to the acceleration so that you have no place to put in a septum without knocking out a great deal of beam; of course, if you knock out a lot of beam you need a very tough septum, which is no small problem by itself. What is needed is a method of enhancing the radius gain/turn so that the beam can slip by the septum without large losses.

In the conventional d-c electrostatic deflection system, the septum is at ground and the voltage is applied to a plate lying outside the septum so that you have no electric field inside the septum and a strong field just outside, between the septum and the outer electrode. We have been exploring what happens if you reverse this usual arrangement, put the outer electrode at ground, and put the voltage on the septum. You then have the same strength field but of opposite sign between the septum and the outer electrode and, in addition, inside the septum you now have an electric field which falls off in normal fringing fashion as you move in radius away from the septum. The fall off is, in fact, approximately linear.

To do a preliminary evaluation of the performance of such an assembly we made a number of simplifying assumptions. First of all, we assumed a field with positive $K$ such that $v_{r}$ was equal to 1.04 throughout the region of the calculation, we used a voltage gain of 280 kv per turn, we assumed 100 kv on the septum, and that the septum was $1 / 15$ of a turn in length. We assumed that the electric field inside the septum fell off linearly from the septum radius of 66 cm to zero at a radius of 62 cm , with the strength of the field adjusted to give the proper voltage if you calculate the line integral of $E$ up to the septum. The magnetic field strength was 14 kilogauss, corresponding to $40-\mathrm{Mev}$ protons, and finally, we used an impulse approximation to calculate the effect of both the deflegtor and the acceleration. I think that covers the assumptions.

The performance of the system is shown somewhat schematically in Figure 241. In this figure we have plotted the motion of the orbit center; for simplicity we've shown only four turns whereas actually it makes about 15 turns in going through this process. We assume the


Fig. 241. Motion of orbit center for revised electrostatic deflector arrangement.
particle enters the region with a small residual amplitude equal to about $1 / 4$ of the radius gain per turn (which is as small as the amplitude could be) and displaced in the positive $y$-direction away from the deflector as indicated by point 1 on the figure. Particles rotate counterclockwise, and the deflector is connected so that the fringing electric field gives the particles an outward kick. On the "first" passage through this field the particle gets a small kick, causing the center to move in the $x$-direction to point 2. The particle makes a revolution and the center
precesses through about $15^{\circ}$ to point 3. The next time into the deflector the particle feels a stronger field because its radius of curvature is larger due to the acceleration and because the precession has moved its center closer to the deflector. It, therefore, gets a stronger kick from the fringing electric field, which moves the center from point 3 to point 4 , and the process repeats. On each turn the precession is about the machine center so that the $\Delta y$ due to the precession is determined by the accumulated $x$-amplitude and, because of this, the final gain in the $y$-direction, the $\Delta y$ of the figure, can easily be as large as the final $x$-kick, the jump from 7 to 8 in the figure. The fact that we've assumed only a few turns in drawing the figure makes the $x$-gain appear larger.

On the final turn the $\Delta y$ is sufficient to swing the particles around the septum and into the strong $100 \mathrm{kv} / \mathrm{cm}$ field between the septum and the outer electrode. This gives the particle center a strong kick back in the negative $x$-direction as shown by the jump from 9 to 10 in the figure. It appears that the separation between orbits centered at points 1 and 10 is smaller than between orbits centered at 9 and 10. Actually this is not the case, for considerable acceleration has taken place between 1 and 9 , so that 9 and 10 have a considerably larger radius than 1 , and the actual separation at the outside of the machine is just the distance from 9 to 10 plus one turn's acceleration.

Using the parameters I gave above, we got a final separation at the channel of $0.3 \pm 0.1$ in. due to the energy gain so that we got an amplification of about 4 in the turn separation which gives plenty of room to swing past the septum. Then with $100 \mathrm{kv} / \mathrm{cm}$ outside the septum we got a final separation of 1.2 in . on the jump from 9 to 10 , which is in the range of feasibility for peeling it off with a magnetic channel.

We have not looked into the radial or axial focusing of this system so it is really in a very preliminary state. It does appear, however, to be a promising way for swinging particles around a septum.

