

SOLUTION OF POTENTIAL PROBLEMS FOR VARIABLE PERMEABILITY
MATERIALS BY COMPUTER TECHNIQUE

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Advances in technology during the past years have resulted in improved grades of steel. Consequently, flux densities up to 23 kG are now commonly used in the planning and construction of large magnets. These greater flux densities increase the difficulties in the estimation of the proper magnetic potential drop inside the steel. Flux measurements made by the integration method help in this case only to locate possible saturated sections and to correct the yoke or pole profile of simple geometries. However, for detection of local saturation effects, as for example in the Rose shims, even this method becomes ineffective because it gives only the average flux in a cross-section. Until now the most helpful method has been the use of the model magnets, but the necessary shape of shims or pole-shoe profiles can be obtained only by laborous and protracted magnetic field measurements¹⁾.

The problem is becoming more pronounced for the AVF-cyclotron magnets with flux densities at the hill sector in the range of 20 kG, and over. The difference between the flux densities in the hill and in the valley develops a flux equalizing process inside the steel. The result is a broad variety of flux densities in the magnet pole, which, in turn, cause various magnetic potential drops in the steel at different radii. If, for example, a radial-sector type is required and the field correction for the isochronism should be made only by a change of the flux density at the hill, the shaping of the pole-shoe surface will change the flux and resistance at all radii. It is, therefore, impossible to predict for flux densities over 18 kG the fast gap length at a specific radius. Until now several experiments and repeated reshaping of the pole face on model magnets was necessary for final solution.

Another possible way for solving problems of this kind makes use of theoretical calculations based on Maxwells' equations. For regions with no current, the magnetic field \vec{H} may be represented as a gradient of the magnetic potential V_m . The general differential equation for V_m is, then, of the form $\Delta V_m = - \text{grad}(\ln\mu) \times \text{grad} V_m$, where μ is the absolute permeability of the medium. This potential equation degenerates to the Laplace equation for materials with constant μ .

If the permeability is a function of coordinates, the use of the relation $\mu = k/H^n$ (k is approx 23, if H is in kilooersted, n approx 0.9), which is very well satisfied for flux densities between 12 and 25 kG, leads to a quasilinear partial differential equation for V_m of the second order. The combination of the Laplace equation for air and the potential equation for steel regions, under consideration of

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boundary conditions at the transition from air to steel, will completely describe the potential distribution for a certain pole-face geometry. Generally, for profiles appearing in practice, an analytical solution for such a combination of equations does not exist; the problem may be solved either by electrical networks with variable resistors²⁾, or by the methods of numerical mathematics.

If the use of large computing machines is possible, the numerical methods show some advantages, of which accuracy and speed are the most important. The solution of certain problems can be obtained by the difference method. The region in question must be covered by a lattice and the potentials at the intersections evaluated according to the difference equations converted from the differential equations. For this purpose the potential V_m near a point V_{m0} is expanded into a Taylor series; for further calculations only the terms up to the cubic are taken into account. The process of calculating the potential distribution must be repeated until the distribution with the required accuracy is obtained.

There are, however, several difficulties in connection with the numerical calculations which must be explained in more detail. At first, the number of iterations depends very much on the required accuracy. Otherwise, the convergence of the V_m in the steel is very slow and, as an example, with a net of 20 x 30 points about 100 iterations are required for an accuracy better than 1%. The number of iterations depends also on the zeroth approximation of the potential distribution; it is important, therefore, to start with the best possible approximation. This can be obtained if the portion of the maximum potential at some point on the steel surface is evaluated under the assumption of a linear change of the potential in the air as well as in the steel. The percentage of the maximum potential follows from the boundary conditions and the corresponding value of V_m must be attached to the intersections on the surface. If a large difference in flux densities along the steel surface can be expected, as in the AVF cyclotrons, the region must be halved and the calculation made separate for hill and valley sectors.

The next difficulty occurs while determining the potential distribution near sharp corners where the gradient V_m is large. The V_m in the neighbourhood of the corners will be calculated more exactly with a small-mesh grid. However, this means a great number of intersections, which will slow the rate of convergence. A compromise consists in choosing a lattice with large mesh at the beginning of the calculation and after 20-40 iterations, depending on the size of the region, the lattice must be doubled. Thus, the total number of iterations can be decreased; the coarse net will deliver faster an approximate, although less exact, distribution. For some cases it may be necessary to again reduce the mesh of the complete net, or some its portions near the critical corners. Somewhat faster and more accurate result will be obtained by taking terms up to the seventh order in the expansion for V_m , but the gain can be observed only with coarse nets.

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To avoid large oscillations of V_m values on the boundary due to the zeroth approximation, a special calculation procedure must be introduced. In principle it consists in considering the constant in the μ law as a variable. The potential V_m follows then by a linear interpolation between the value of the constant estimated from the boundary condition and the actual constant given by the magnetic characteristics of the steel. If desired, the procedure may be repeated several times for the same iteration.

Although it is possible to show that a convergence does exist it is very difficult to predict the rate of convergence of V_m for a certain steel profile. Generally, up to 1000 iterations may be expected for nets with several thousand intersections, when no special precautions for the net refinement or zeroth approximation are made. The time required for calculation in such a case by a high-speed computer of the type IBM 7090 would exceed three hours. This is relatively uneconomical and demonstrates the necessity of introducing the tricks mentioned above.

On the basis of the procedure described above a computer program for two-dimensional potential problems was written, and the flux density distribution inside the magnet pole of an isochronous cyclotron was estimated. The influences of vertical and tapered transitions from the hill to the valley were also investigated. The results are in good agreement with the measurements except for some effects near the edge of the pole, where a three-dimensional solution is necessary.

References

- 1) Mates, J., Magnet Report, Carnegie Inst. of Tech., Pittsburgh, 1953.
- 2) Tschopp, P.A., Arch. f. Elektr., Bd. 44, 441 (1959).

DISCUSSION

SNOWDON : Have you investigated any accelerated convergence techniques? These techniques amount to replacing the Laplace operator by a diffusion type operator which, in the limit of a large number of iterations, reduces to the Laplace operator. The effect is that the rate of convergence may be increased. For example, at MURA, an iterative magnetic field calculation which was originally performed in two hours was finally run in 20 min when accelerated convergence techniques were employed.

BERKES : There are a few tricks which we introduce to accelerate the convergence, but even with these tricks we need a net with 500 points for one iteration in 0.4 s. This is a very nice time but unfortunately 500 points intersections of lattice is not enough for certain problems; if we use more intersections the time increases rapidly.

RESMINI : What agreement did you get between calculated values and measured ones in the region where two-dimensional approximation seems to be good?

BERKES : We calculated a special profile of the AVF cyclotron, and we made an azimuthal cross-section. The agreement between the field in the mid-plane obtained by the calculations and otherwise obtained by measurements was certainly better than 1%. Of course, at the moment our 2-dimensional solution is not valid for regions near this axis of the machine, and it is not valid for outer parts of the machine near the edges because of some effects due to the flux equalizing process in the steel itself and due to the fringing fields.