CENTRAL-REGION STUDIES FOR THE MSU CYCLOTRON(*)

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(Presented by H.G. Blosser and M. Reiser)

The initial orbits in a cyclotron are crucial in determining the properties of the final beam. At the same time, due to a complicated tripartite force (from the main magnetic field, the electric field of the dees, and space charge forces), the behaviour of these orbits poses the most difficult analysis problem in the cyclotron. Studies of the central-region problem typically involve rather crude approximation, particularly as regards axial motion, necessitated by the complexity of the problem. At East Lansing, considerable effort is now being directed to this central-orbit problem; the studies are largely of a numerical character, employing three previously described[1] computer programs.

For an initial survey of median-plane motion in various central geometries, a program known as "Cop" is employed; the accelerating gaps are represented as square-wave fields (constant E within the gap, zero E out of the gap, sinusoidal time variation) and the magnetic field is uniform.

For a detailed survey of a particular geometry the "Cartwheel" program is employed. This program integrates complete median-plane equations in arbitrary electric and magnetic fields. The fields are carried as a mesh of data points; the electric field is intended to be derived from electrolytic-tank or other analog studies.

The third program, "Silax", also has complete median-plane equations and in addition has axial equations through linear terms; the electric field in this case is restricted to a model based on the known conformal-mapping solution of a standard accelerating gap. The field is a correct representation of 180° dees without puller, slits, etc.; for other dee angles it is qualitatively correct.

I. Studies with Square-Wave Accelerating Gaps (By M. Reiser).

The RF system for the MSU cyclotron incorporates two dees and is being designed to operate in both the push-pull mode, for first- and third-harmonic acceleration, and the push-push mode, for second-harmonic acceleration. Such a double-mode system requires dummy dees to prevent neutralization of the electric fields in the push-push mode. Several specific problems with respect to ion injection and central-region layout arise, as is illustrated in Fig. 1, namely : a) The favorable starting phase (1'), which is determined by the maximum energy gain in the first gap between ion source and extractor electrode, does not coincide with the phase of maximum-energy-gain per turn (1, 2, 3, etc.). To accelerate the useful part of the beam a phase shift

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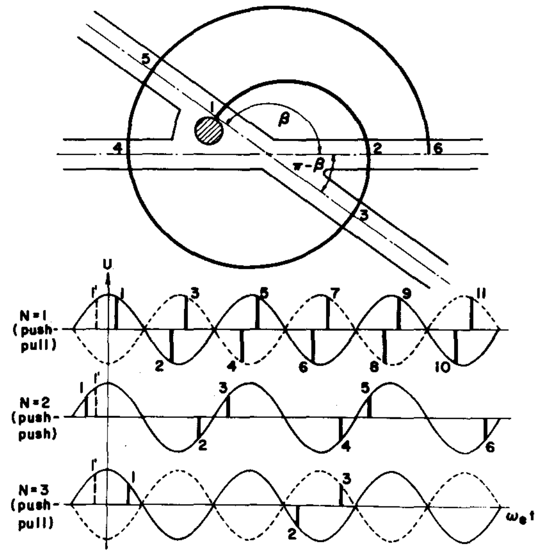


Fig. 1 Schematic drawing of acceleration sequence in a multi-mode dee system. Heavy vertical bars (1, 2, 3, etc.) mark gap crossing phase for maximum energy gain; dashed bar (1') marks the favorable starting phase.

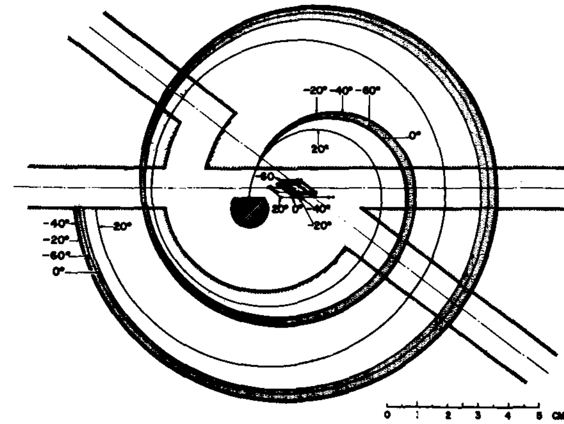


Fig. 2 Initial trajectories and instantaneous center points for particles of varying phase in the N = 1 mode, the labeling on the curves being the phase of the RF at the time the particles leave the source (0° is peak voltage). The puller-to-dee angle is 36°.

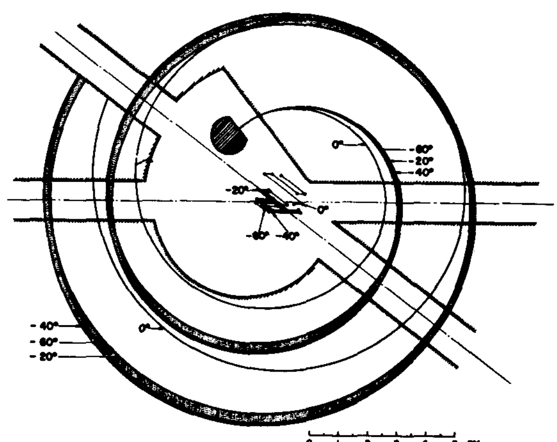


Fig. 3 Same as Fig. 2 except N = 2 and puller-to-dee angle of -16°.

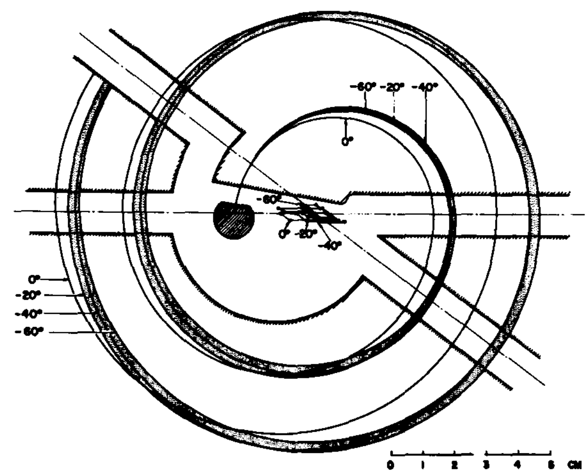


Fig. 4 Same as Fig. 2 except N = 3 and puller-to-dee angle of 27°.

between the first and second gap crossing is necessary; the ion source and puller have to be moved to different azimuthal positions when the mode of operation is changed.
b) The presence of dummy dees which must extend into the very center of the machine imposes geometric limitations to large readjustments of the injection system, especially to radial displacements, and as a consequence one must try to keep the orbit geometry practically constant in all modes of cyclotron operation.

Unfortunately, it is not possible to rigorously achieve constant orbit geometry in a double-mode dee system. The maximum energy gain per turn in such a system is a function of the dee angle β and the harmonic number N and, for delta function gaps, is given by the formula

$$\Delta E_{max}/4eU_0 \equiv U_N/U_0 = \sin(N\beta/2) \quad , \qquad (1)$$

where U_0 denotes the peak dee voltage, and U_N is defined as an effective voltage in the N-th mode of operation. Since the energy gain, and hence the radius of curvature, of the useful part of the beam in the first gap is determined by the peak voltage U_0, whereas the turn pattern as a whole is determined by the effective voltage U_N, Eq. (1) establishes that the raltionship of the first half turn to the remainder of the turn pattern will vary with N. The optimum ion source position will therefore vary with N and β even though the voltage and field are adjusted to give a constant turn pattern in the cyclotron as a whole.

A dee angle of 144° causes the ratio of voltages in Eq. (1) to be the same for both second and third harmonics; to the extent to which Eq. (1) is applicable, a convenient similarity in the turn patterns (tending to minimize required radial adjustment of the source) is implied. In view of this, a dee angle of 144° was selected for initial investigation.

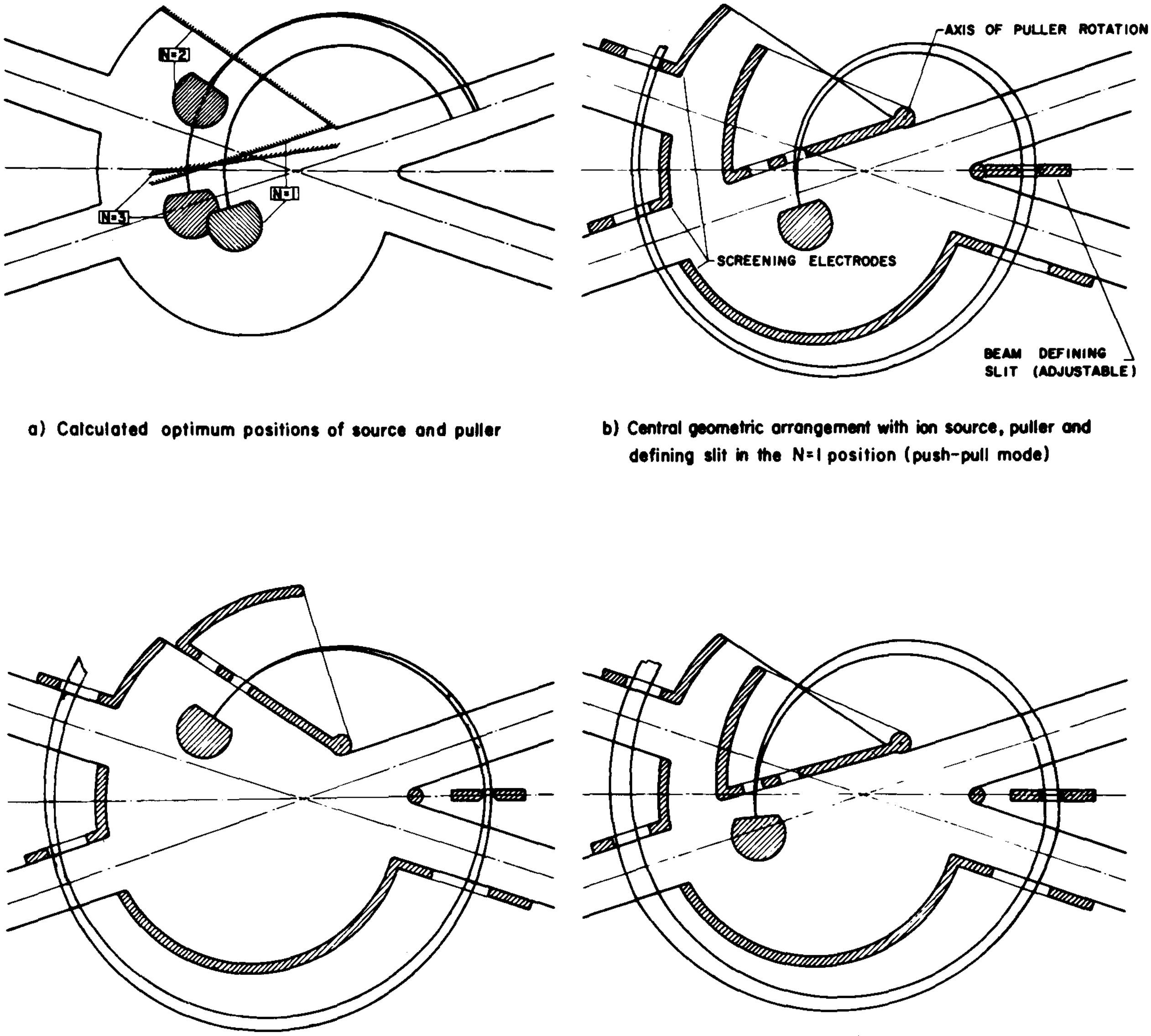
To obtain an initial determination of the optimum positions for source and puller, orbit studies[2]) were made with the Cop code for protons, deuterons, and C^{4+} ions operating on the first, second, and third modes respectively and with an RF of 21 Mc/s and a central magnetic field of 13,744 gauss. The peak dee-to-ground voltages were set at 70 kV for protons, 56.6 kV for deuterons, and 37.7 kV for C^{4+}, the voltages for the deuterons and carbon ions being selected to yield the same number of turns as for protons (for acceleration to full energy).

The results of the calculations in the proton case are shown in Fig. 2. The puller forms a large protrusion of the dee with an off-set angle of 36°, which increases the angle between first and second gap crossing to 180°. A large group of particles, in the starting phase interval $-60^\circ \leq \vartheta_0 \leq 0^\circ$, is well centered and bunched radially (shaded beam), while beyond $\vartheta_0 = 0^\circ$ the eccentricity and divergence in radial motion increases quickly, as indicated by the 20° trajectory.

In the case of second-harmonic acceleration (deuterons) the dee angle between first and second gaps must be reduced from 180° to 128°, as is illustrated in Fig. 3, whereas the situation for third-harmonic acceleration (Fig. 4) is similar to the proton case, the first-to-second angle being 171°.

II. Cartwheel Studies (by M. Reiser).

The calculations from the previous section formed the basis for an initial central-region layout which is shwon schematically in Fig. 5 and which was used to build a 3il scale electrolytic-tank model (Fig. 6) to obtain detailed electric field information for studies with the Cartwheel code. The puller is seen to have two slits and can be rotated about a fixed axis. Such an arrangement allows a close match to the calculated source-puller positions with a single motion which is highly desirable as regards simplifying the associated mechanical mechanism. The two figures also show an



a) Calculated optimum positions of source and puller

b) Central geometric arrangement with ion source, puller and defining slit in the N=1 position (push-pull mode)

c) Source, puller and defining slit in the N=2 position (push-push mode)

d) Source, puller and defining slit in the N=3 position (push-pull mode)

Fig. 5 Schematic layout of central geometry showing positions of source and puller in the three modes of operation.

adjustable beam defining slit in one of the dummy dees and various electrodes for defining the electric field in a favorable way (particularly to shield the beam from undesired radial field components). Studies with the electrolytic tank are in progress; an initial scan of the field has been completed and orbit calculations with Cartwheel are, at the time of writing (March, 1963), partially completed.

In parallel with the studies of the full central region a separate detailed study of the source-to-puller region in an enlarged scale of 10 : 1 has been conducted[3]. This study is facilitated by the fact that the geometry envisaged is approximately two-

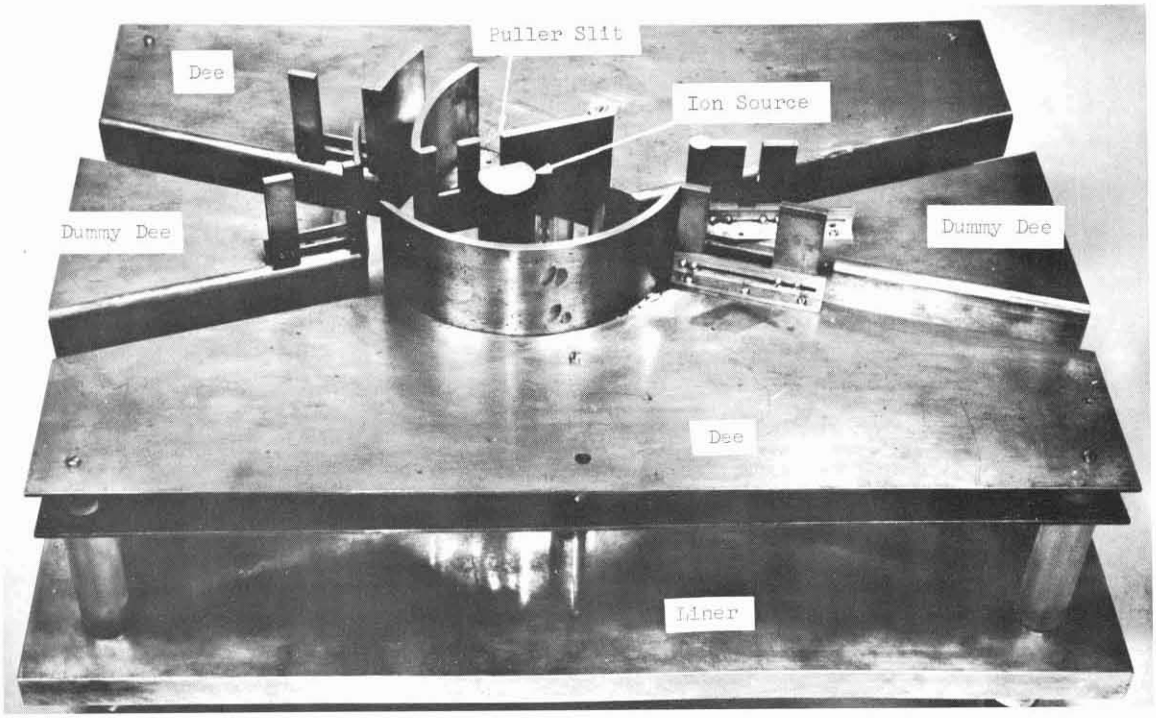


Fig. 6 Electrolytic-tank model of the central region with geometry as per Fig. 5 (source and puller in the N = 1 position). Due to the symmetry of the problem it is only necessary to fabricate the structure on one side of the median plane. The top surfaces of the carious electrodes and slits at the center of the picture mark the median plane. This is also the water surface for the measurements.

dimensional and hence accurate electric field data can be obtained by conducting-paper techniques. Fig. 7 shows the equipotentials and several trajectories of protons starting at the phase $\vartheta_0 = -20^\circ$ in the case of a cylindrical ion-source structure. The particles leave the source at different points with an initial energy of 2.3 eV and with initial directions at each point of -45°, 0°, and 45° with respect to the normal to the source surface. Differences between the trajectories emerging from the same point are minor because the chosen initial energy is small compared with the energy gain in the gap; the transverse momentum is hence also small even for divergence angles of $\pm 45^\circ$; as a result, the particles essentially follow the direction of the field lines despite the widely differing initial directions. The strong beam divergence caused by the non-uniform field of the circular source can be substantially reduced if a flat source is used (Fig. 8). Further improvement of the beam optics is possible by providing a slight recess as shown in Fig. 9, in which case a very favorable "parallel-beam" situation is achieved. A larger recess produces a strong focusing effect with an undesirable cross-over (Fig. 10). - The figures illustrate the considerable importance of the electric-field configuration close to the ion source; by properly shaping the source structure one can obtain a well focused beam. When defocusing due to space-charge forces, which were neglected in these studies, is taken into account the optimum geometry is probably between the cases of Fig. 9 and 10.

III. Axial-Motion Studies (by H.G. Blosser and M.M. Gordon).

Studies of axial motion in the central region were made with the Silax code. The electric field in this code is given by the gradient of the potential function $U(r,\vartheta,t) = U_0V(r,\vartheta)\cos(\omega_{rf}t)$, where $\omega_{rf} = N\omega_0$ ($N = 1, 2, 3,\ldots$), with ω_0 being the isochronous angular frequency. The function $V(r,\vartheta)$ is an involved analytic form depending on the dee angle, the height and width of the accelerating gaps, and the mode; the general form of the function is shown graphically in Fig. 11 for odd-N modes (push-pull) and in Fig. 12 for even-N (push-push). In each case the V function is rigorously correct in the limit of large r and qualitatively reasonable at small r.

Two magnetic fields were used in the axial-motion studies, both based on an early trhee-sector, high-flutter model-magnet study and identical except that one is accurately isochronous, while the other contains a central magnetic cone designed to give an axial frequency of about 0.2 at all energies. The equilibrium orbit properties for both these fields have been previously presented[4)].

On the fundamental ($N = 1$) mode, the radial motion in the Silax field is well behaved, as is seen in Fig. 13 and 14, which show a polar graph (solid curve Fig. 13) and phase-space displacement plots (Fig. 14) of an $N = 1$ orbit starting from approximately optimized initial conditions in the cone field. Initial and final turn results for a family of orbits distributed in phase space are also shown in Fig. 14; the distortion induced in passing through the $\nu_r = 1$ resonance is seen to be small.

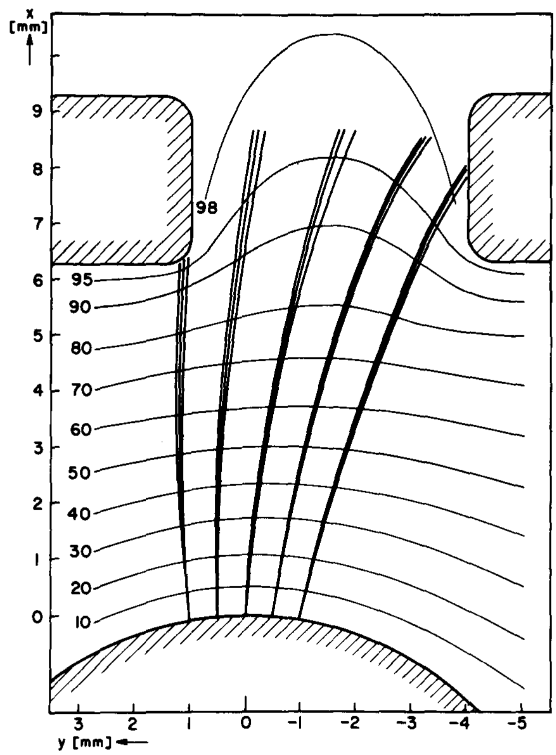


Fig. 7 Equipotential lines and proton trajectories in the case of a cylindrical ion-source structure. Peak voltage U_0 = 70 kV, magnetic field B = 13.774 kG, starting phase ϑ_0 = -20°.

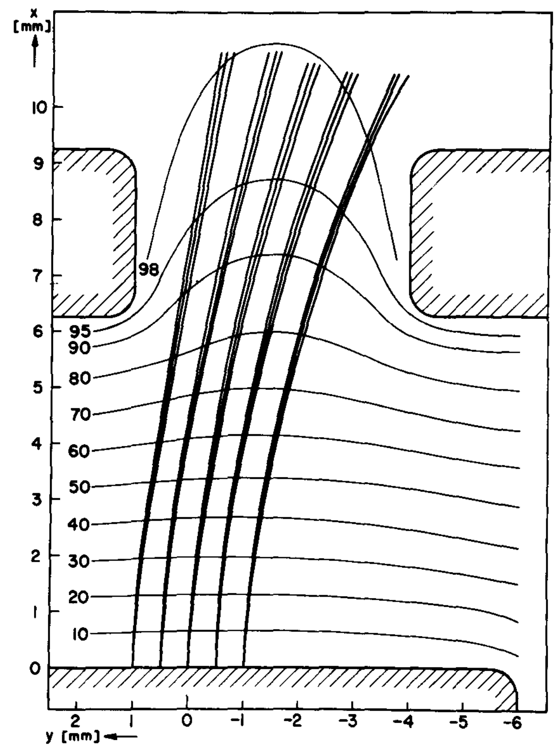


Fig. 8 Same as Fig. 7 except with flat source.

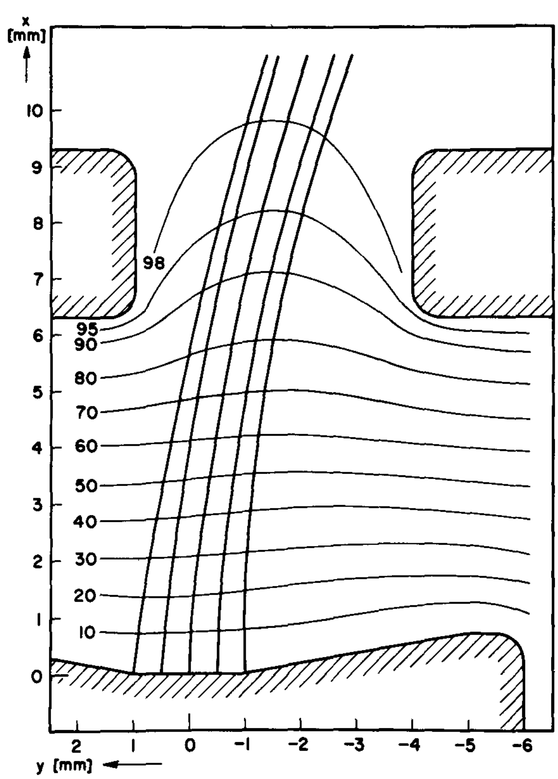


Fig. 9 Same as Fig. 7 except with face of source slightly recessed (angle 10°).

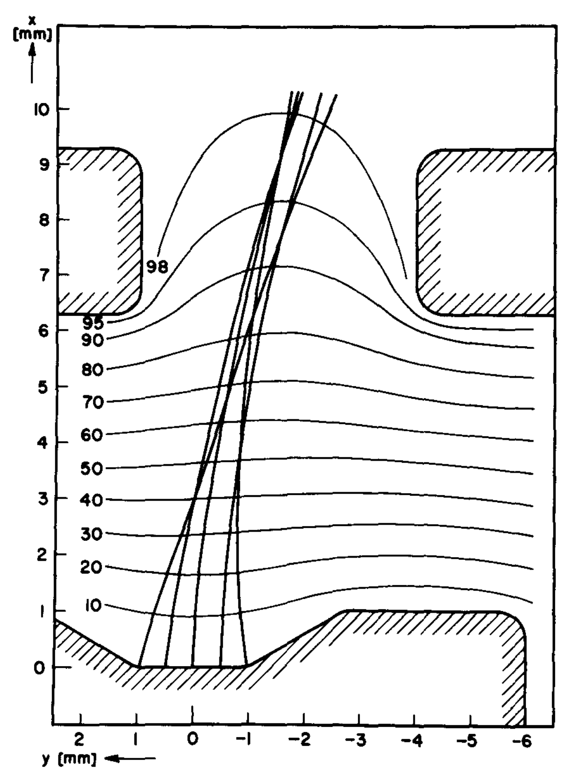


Fig. 10 Same as Fig. 7 except with face of source deeply recessed (angle 30°).

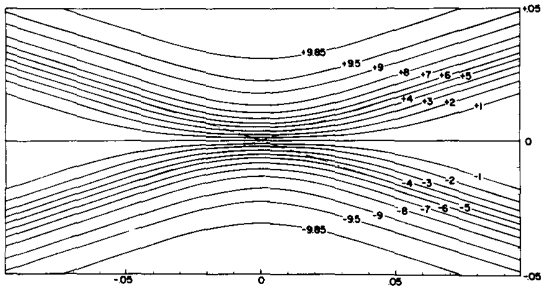


Fig. 11 Median plane equipotentials of the push-pull Silax potential function V. Dee angle 144°. The horizontal and vertical scales are in cyclotron units (for all runs herein the cyclotron unit is equal to 90.4").

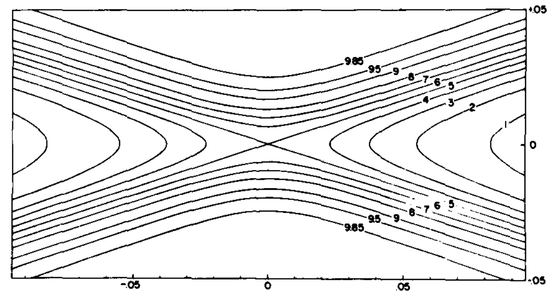


Fig. 12 Same as Fig. 11 except push-push mode.

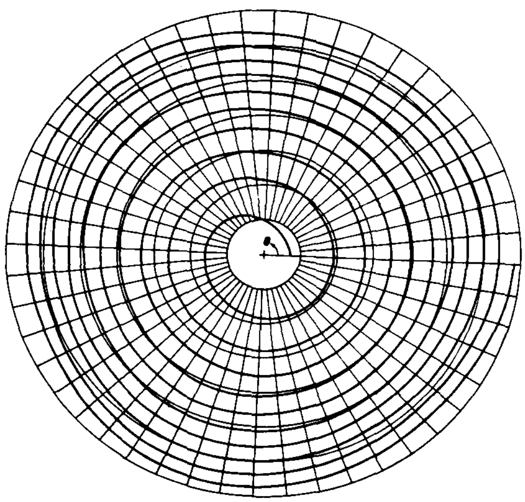


Fig. 13 Polar graph showing the first ten turns of a well-centered Silax orbit accelerated with $U_0 = 70$ kV and N = 1, in the cone magnetic field. Radial divisions in the grid are 1" steps.

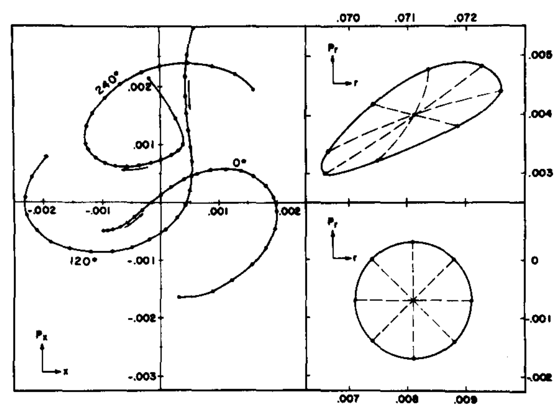


Fig. 14 On the left - a phase space plot showing the instantaneous displacement of the first 25 turns of the Fig. 13 orbit from its equilibrium orbit, the three curves giving the displacement at azimuths 0°, 120°, and 240°, and the points marking successive revolutions. The "residual" amplitude of about 0.15" is due to the gap crossing resonance. At the right - r, p_r plots, initially, and after 9 turns, of a group of orbits centered on the Fig. 13 orbit and forming an initial circle 0.2" in diameter.

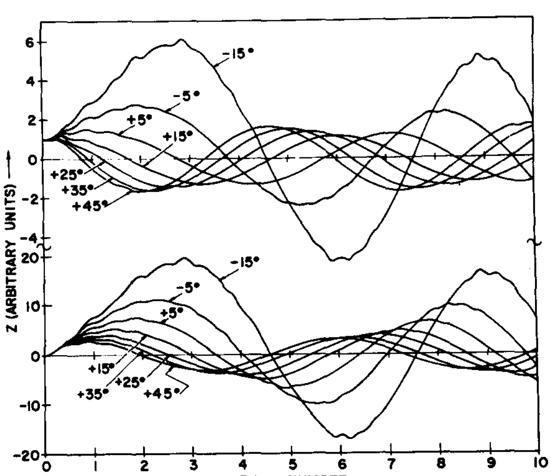


Fig. 15 Plots of axial motion versus azimuth for the first 10 turns of a family of orbits with various initial phases. The +15° curve is the orbit from Fig. 13. All orbits have identical initial conditions except for phase. The upper and lower curves give linearly independent z solutions for each phase.

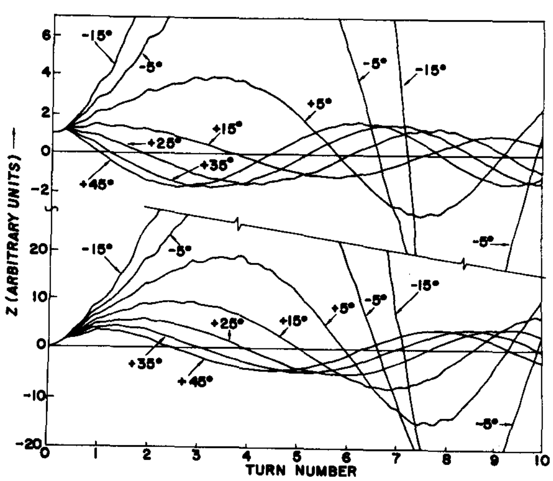


Fig. 16 Family of axial motion plots similar to Fig. 15 except with the isochronous magnetic field rather than the cone field. Radial initial conditions are chosen to produce well-centered orbits after 10 turns.

The axial motion for the Fig. 13 Silax orbit is shown in Fig. 15, along with results for other orbits starting at different RF phases but with initial conditions otherwise identical. For each phase two linearly independent z solutions are shown. Since the z motion is linear, the scale is arbitrary (for the lower orbits the initial z momentum is numerically equal to the initial z for the upper orbits).

The very strong phase dependance of the axial motion is nicely illustrated in Fig. 15. This strong dependance offers a convenient mechanism for accomplishing phase selection, when desired, as has been pointed out by Hagedoorn[5]; the source would be designed to give a spread in initial z much larger than the spread in initial p_z, so that particles with given initial phase would periodically pass through a well-defined axial node. Since the nodes for other phases would occur at widely differing points, as is clear from Fig. 15, a small axial slit would transmit particles with phase such that the axial-motion node occured at the slit and would discriminate against other particles with rather high efficiency. When it is desired to operate the cyclotron in a mode where phase selection is undesirable, the selection could easily be removed by opening or removing the slit.

Fig. 16 shows z motion for a set of N = 1 orbits starting from optimized initial conditions in the isochronous magnetic field. Comparison with Fig. 15 indicates claerly the substantial value of the magnetic cone.

In contrast with results for N = 1, considerable difficulty is encountered when third-harmonic (N = 3) acceleration is employed with the Silax field. The difficulty is largely caused by deccelerating forces deep in the dee coming from the substantial field penetration into the dee. Fig. 17 is a graph of energy versus azimuth for the first several turns of an approximately optimized N = 3 orbit, showing the large energy oscillations. Due to these large oscillations the orbit is, on the average, quite far in phase space from the instantaneous equilibrium orbit and hence subject to large non-linear forces due to the $\nu_r = 1$ resonance. As a result the radial motion is severely distorted as is shown in Fig. 18 which shows two typical groups of particles after 9 turns.

The penetration of the electric field into the dee also gives large axial forces deep within the dee. Fig. 19 shows the axial motion for a family of orbits with different initial phases. At positive

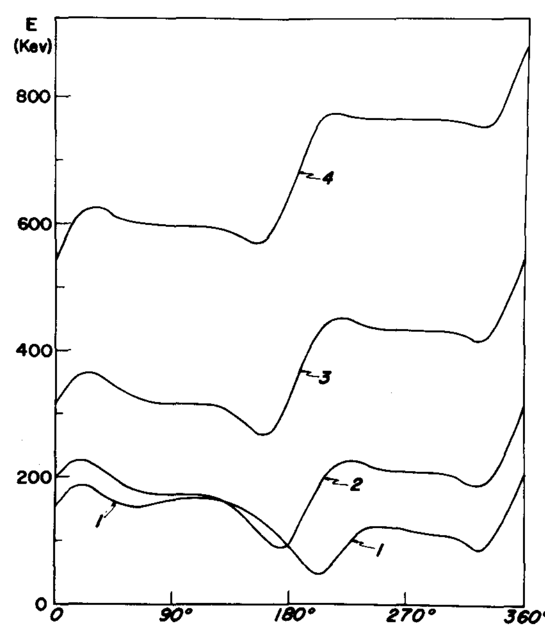


Fig. 17 Plot of energy versus azimuth for the first four turns of a well-centered (after 10 turns) N = 3 orbit in the Silax field with U_0 = 70 kV and q/m = 1/3 (proton units).

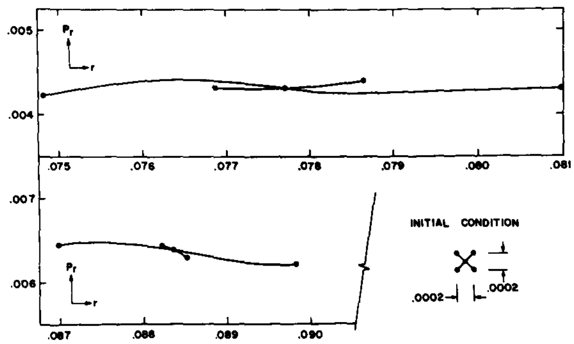


Fig. 18 Phase space plots after nine revolutions showing two typical groups of N = 3 orbits. The initial conditions for each set formed an X of size as shown at the lower right. The upper group, the most severely distorted, is at the same time better centered after nineturns than the lower group. Scales are in cyclotron units.

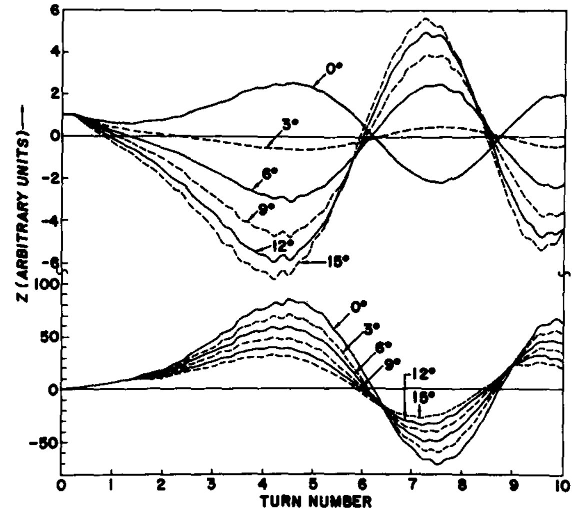


Fig. 19 Family of axial motion plots similar to Fig. 15 except with N = 3. Radial initial conditions are chosen to produce well-centered orbits after nine turns.

phases the amplitude grows due to over-focusing from the electric field within the dees; at negative phases the usual defocusing occurs. The range of usable phase between these two extremes is quite narrow.

The results shwon in Fig. 17-19 are for the cone field. Similar studies with the isochronous field yield results which are qualitatively the same and are not presented here. Extensive studies have also been made with N = 2 acceleration; the results are intermediate in behaviour between N = 1 and N = 3 but much closer to N = 1. These results are also not presented here.

IV. Conclusions.

The studies with square-wave accelerating gaps give an initial indication of highly satisfactory performances in a double-mode acceleration system; a broad range of initial pahses can be started in well-centered orbits and positioned in phase to give good axial focusing and acceleration. The Cartwheel studies, which are just beginning, will allow detailed design of the central geometry without recourse to approximations other than single-particle dynamics. The Cartwheel studies of the source-to-puller region indicate the importance of properly shaping the electric field in this region. The axial motion studies clearly indicate the dominant role of the electric field in determining axial motion in the initial turns. The strong phase dependence of the electric forces offers a promising mechanism for accomplishing phase selection. The N = 3 axial motion results show that it is essential to localize the electric field, by means of slits, etc., if acceleration on high harmonics is to be of significant quality.

References

1. T.I. Arnette et al., Nuc. Inst. and Meth. 18-19, 343 (1962).
2. M. Reiser, MSU Cyclotron Report 15, March, 1963.
3. M. Reiser, MSU Cyclotron Report 16, March, 1963.
4. J.E. Stover, MSU Cyclotron Report 3, August, 1960.
   M.M. Gordon and W.S. Hudec, Nuc. Inst. and Meth. 18-19, 243 (1962).
5. H.L. Hagedoorn (private communication).

DISCUSSION

VERSTER : In your automatic measurements in an electrolytic tank, how do you keep your probes from hitting the electrodes where they intersect the water surface.

REISER : The probe is touching the electrodes; we have done intensitve studies for some months to get rid of the voltage drop at the edges. We have succeeded with an accuracy of better than 1%.

LAPOSTOLLE : How far outward does the cone in the center of your machine extend?

BLOSSER : It extends to about 5 orbits; the $\nu_r = 1$ resonance occurs here in our machine.

POWELL : Regarding this field bump, did you say that it gave you advantage where vertical focusing was concerned, and also advantages for the horizontal motion?

BLOSSER : No. Actually, there is some debate whether we should build a cyclotron with a cone in it. I favor the cone.

POWELL : To my surprise when we did our first studies on the external beam we did actually need to increase the height of our bump about 1% to optimize the beam. Also we found that sometimes the beam defining slits seem to disturb later turns; perhaps they introduce distortions in the horizontal field which continue several turns.

REISER : In our case the turn separation is much larger 280 KeV/turn, and we put the defining slit in the dummy dee where it does not introduce serious distortions.

GORDON : In response to Powell's question about the effects of the bump in the center of the field, in addition to the axial focusing there is the shift in the RF phase. If precaution is taken in moving the source and puller to appropriate angles to optimized the phase difference, as Reiser discussed, then the cone may provide an additional advantage by shifting the phase of the ions coming from the puller in such a way that they get good axial focusing where they really need it. This effect is perhaps an important factor in the superior operation of certain machines with the cone, as opposed to the isochronous operation.