THE CHOICE OF BETATRON OSCILLATION FREQUENCIES IN A SPIRAL-SECTOR FFAG ACCELERATOR (*)

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(Presented by G. Parzen)

In the design of a spiral-sector FFAG accelerator, one is required to answer the question of what is the best choice for the betatron oscillation frequencies, ν_r and ν_z. In strongly nonlinear machines the stability limits and the stability of the accelerator against field perturbations are very much affected by the choice of the tune, ν_r and ν_z.

It is difficult to establish a general, rigorous criterion for stability. The considerations that go into determining stability for a particular choice of tune depend on where this tune lies in relation to nearby resonances, and these considerations vary with the choice of tune. Determining the stability for a particular choice of tune usually requires a considerable amount of computer study of the orbit motion for the various possible field perturbations.

In the following, a general criterion for stability will be given which, while not a rigorous criterion, seems to be a reasonable criterion in the light of experience with results of a good deal of computer study of the orbit motion. With this criterion, a computer study was made on the dependence of the stability on the tune, ν_r and ν_z. The results of this stability survey indicate a fairly unique choice of ν_r and ν_z to give optimum stability.

Horizontal Stability Limit

What shall one use as a measure of the horizontal stability limit? The most obvious answer is to study orbit motions which initially have small z motions for various r amplitudes and thus determine the largest r amplitude for which the motion is stable. This largest r amplitude we will call x_{SL}. One finds that there usually exists a smaller r amplitude of motion at which considerable coupling between the r and z motions appears and the amplitude of the z motion grows[1,2]. The r amplitude of motion at which this coupling sets in we will call x_{th} (the subscript th stands for threshold).

Computer studies indicate that, in the absence of perturbing fields, orbit motions with r amplitudes larger than x_{th} but smaller than x_{SL} are stable, although there is considerable growth in the amplitude of the z motion. Nevertheless, orbits for which the r amplitude exceeds x_{th} may be considered undesirable. In some accelerators, the vertical gap requires that the z amplitude remain considerably smaller than

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the r amplitude; not much coupling between r and z motions can be allowed. In some cases it has been found that orbit motions for which the r amplitude exceeds x_{th} are particularly unstable against perturbing fields. This has been interpreted as due to the more rapid change of tune with the amplitude of the motion associated with these orbits, which may drive the motion more quickly on to a resonance introduced by the perturbing field.

Finally, it would seem clear that if one is able to find values of ν_r and ν_z for which there is little coupling and for which x_{th} and x_{SL} are essentially equal, then such an operating point would be regarded as desirable. In the absence of further computer study, one would assume that the tune ν_r and ν_z with the larger value of x_{th} is the more desirable choice. In the following, we will use x_{th} rather than x_{SL} as a measure of the r stability limit. One finds that there are values of ν_r and ν_z for which the coupling is small and x_{th} is not much smaller than x_{SL}.

Vertical Stability Limit

As in the case of the r stability limit, one can study orbit motions which initially have small r motions for various z amplitudes, and thus determine the largest z amplitude for which the motion is stable. This largest z amplitude we will call y_{SL}. In this case also there is a smaller z amplitude which may sometimes be a better measure of the stability limit. One finds that for FFAG machines the z tune, ν_z, varies rapidly with the amplitude of the z motion, much more so than the radial tune, ν_r, for the range of operating points ν_r , ν_z to be considered here. Let us define y_ν as the z amplitude required to cause the z tune, ν_z, to increase by 0.25. Vertical motions having amplitudes larger than or equal to y_ν have reached a half-integral or integral resonance in ν_z and may become unstable in the presence of perturbing fields. Computer studies indicate that in some cases the vertical motion can safely pass through the half-integral or integral resonance. However, in the absence of further detailed computer study, one should perhaps regard y_ν rather than y_{SL} as a measure of the stability limit.

Results of the Stability Survey

With the criteria for radial and vertical stability limits given in the previous paragraphs, a computer study was made of the dependence of the stability limits x_{th} and y_ν on the choice of tune ν_r and ν_z.

In Fig. 1 the threshold for y growth, x_{th}, in units of the average radius of the orbit, is plotted against the z tune ν_z for various values of k. Since the r tune depends only on k to a good approximation, each curve may be considered to be for a constant ν_r also.

The computer runs were done for N = 48. One can obtain a set of curves which can be applied to accelerators with other values of N by plotting $N^2 x_{th}/f$ against ν_z/N with k/N^2 held constant, where f is the flutter. For the computer runs f = 1. This

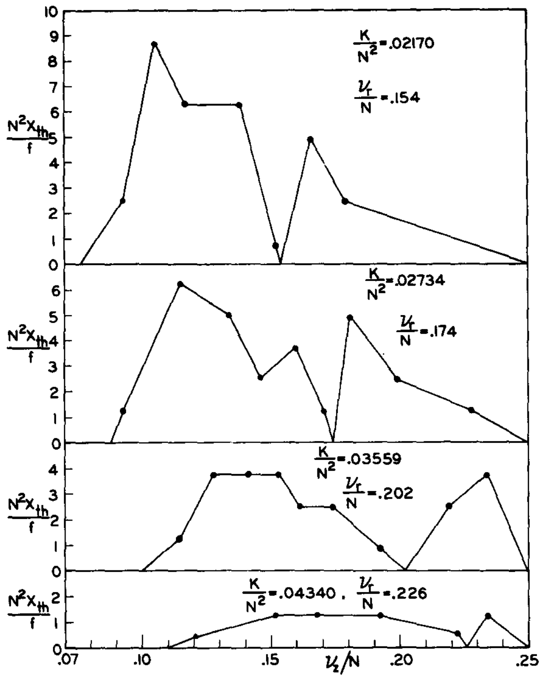

Fig. 1 Plots of the r threshold, x_{th}, for the onset of appreciable coupling with the z motion against the z tune, ν_z. For each curve, the r tune is held constant as the value indicated. The data was obtained for an N = 48, f = 1 machine, but the results are believed to be applicable to other spiral sector machines. The r threshold is in units of the average radius of the orbit at the azimuthal position where the magnetic field is largest and has an accuracy of about 25%.

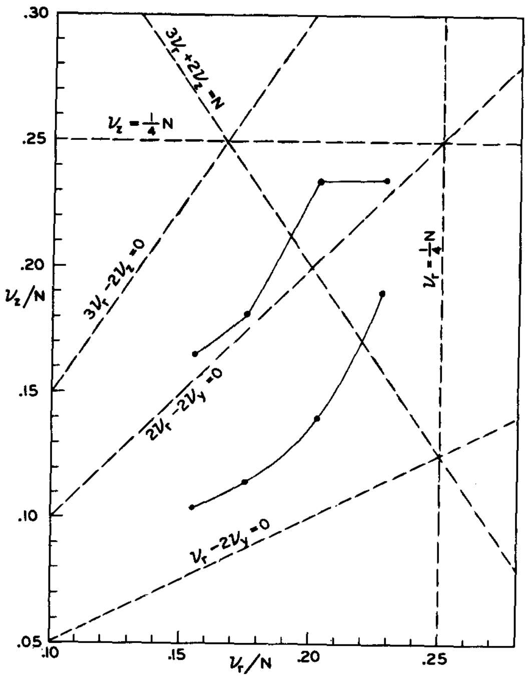

Fig. 2 A plot of ν_z against ν_r for the operating points corresponding to the peak value in x_{th} shown in Fig. 1. Two curves exist; one below the resonance line, $\nu_r = \nu_z$, and one above the resonance line. The important nearby resonance lines are indicated by dashed lines. Both curves cross the $3\nu_r + 2\nu_z = N$ resonance; some distortion of the curves may be expected in the neighborhood of resonance line which the computer study was insufficient to show.

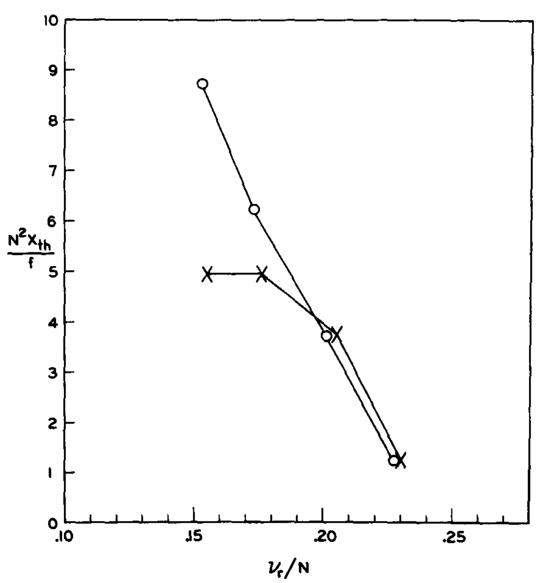

Fig. 3 A plot of the r threshold, x_{th}, which corresponds to the peak found in Fig. 1, against the r tune. The peaks threshold points with tunes ν_r, ν_z below the resonance line $\nu_r = \nu_z$ are indicated by circles, and the peak points above the resonance line $\nu_r = \nu_z$ are indicated by crosses. x_{th} is measured in units of the average radius of the orbit at the azimuthal position where the magnetic field is largest.

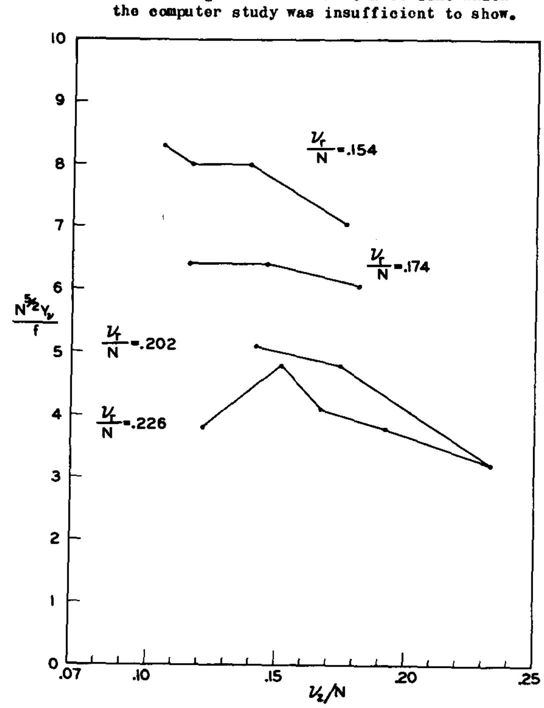

Fig. 4 A plot of y_ν, the z-amplitude required to increase the z tune, ν_z, by 0.25, against ν_z. The ν_r is held constant for each curve. The data was obtained for an N = 48, f = 1 machine, but the results are believed to be applicable to other spiral sector machines. The y_ν is measured in units of the average radius of the orbit at the azimuthal position where the magnetic field is largest.

manner of scaling the result for different values of f and N is predicted by theory.

One observes in Fig. 1 that each curve reaches a peak at some value of ν_z which lies between the two resonance lines $\nu_z = \nu_r/2$ and $\nu_z = \nu_r$. The tune (ν_r, ν_z) associated with these peaks is plotted in Fig. 2. It will be suggested later that the optimum choice of the tune will lie on the curve in the (ν_r, ν_z) diagram shown in Fig. 2, which lies below the $\nu_z = \nu_r$ resonance.

In Fig. 3, the peak value of x_{th} obtained for a particular ν_r/N is plotted against ν_r/N.

Now let us consider the vertical motion and how it is affected by the choice of tune. In Fig. 4 the stability limit y_ν, in units of the average radius of the orbit, is plotted against the z tune ν_z for various values of k.

One can obtain a set of curves which will apply to accelerators with different N by plotting $(N^2 y_\nu/f)\, N^{\frac{1}{2}}$ against ν_z/N. This manner of scaling the y_ν follows from the theoretical result that the change in tune ν_z with the amplitude z goes according to

$$\Delta\nu_z/N \simeq (N^2 z/f)^2 \;,$$

and y_ν is that amplitude for which $\Delta\nu_z = 0.25$.

The variation of y_ν with ν_z/N for a particular choice of ν_r is not as marked as the variation of x_{th}. Thus to choose the optimum tune ν_z/N, one can to first approximation choose ν_z/N to give the best x_{th} and neglect the variation of y_ν.

Possible Defects of the General Stability Criterion

In previous paragraphs, a general criterion for stability was proposed, and a choice of operating point was outlined on the basis of this criterion. In this section, we would like to point out some of the effects which are likely to arise which are not covered well by the criterion.

In considering the stability limit for the vertical motion, the variation of the z tune with amplitude was taken into account. However, the variation of the tune with the r amplitude was not considered in establishing the stability limit for the r motion. The change in tune with r amplitude is shown in Fig. 5, for a typical operation point. It is clear that for some particular operating point the tune may reach some resonance before the r amplitude has reached the value x_{th}, and this resonance, if it cannot be crossed, would determine the actual stability limit.

Nevertheless, the threshold value x_{th} seems a useful measure of the stability limit. In general, a large x_{th} indicates a slower variation of the z tune with the r amplitude; thus one is likely not to cross a resonance very soon. In the absence of more detailed calculation, x_{th} seems to be a useful measure of the stability limit.

In Fig. 6, the variation of the tune with z amplitude of the motion is shown for the same operating point as was used for Fig. 5. One may notice that y_{SL} is considerably larger than y_ν, and y_{SL} may be the actual stability limit if the resonance reached when $y = y_\nu$ can be crossed.

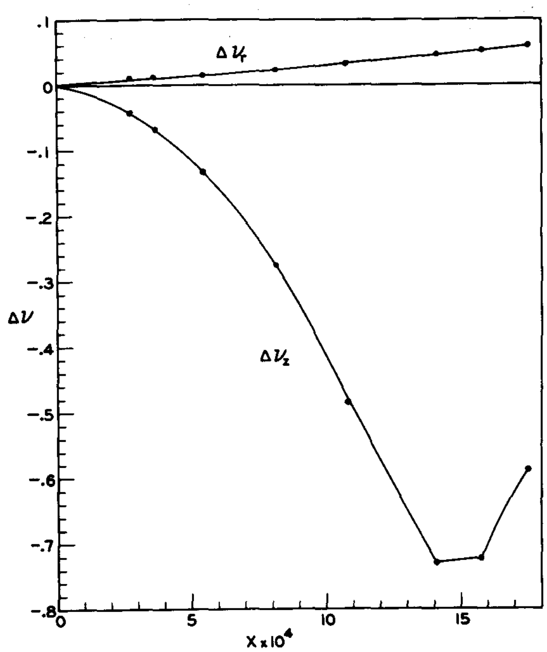

Fig. 5 A plot of the change in tune, ν_r, and ν_z, as a function of the r amplitude of motion, x. The initial z motion is held constant at the small value of 10^{-5} in units of the average radius, and x is measured in units of the average radius at the azimuthal position where the magnetic field is largest. For an N = 48, f = 1 spiral-sector machine with $\nu_r/N = 0.202$, $\nu_z/N = 0.140$.

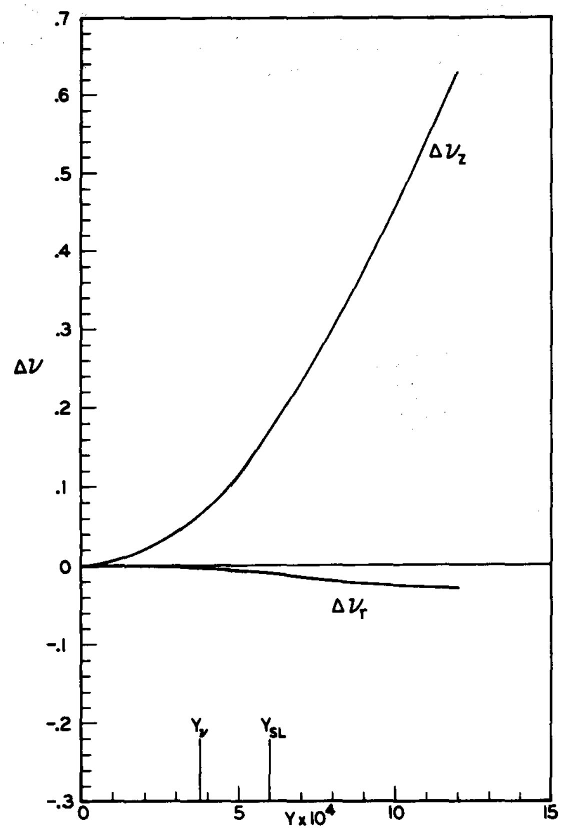

Fig. 6 A plot of the change in tune, ν_r and ν_z, as a function of the z amplitude of motion, y. The initial r motion is held constant at zero, and y is measured in units of the average radius at the azimuthal position where the magnetic field is largest; y_ν and y_{SL} are indicated on the graph.

References

1. L.J. Laslett and A.M. Sessler, Rev. Sci. Instr. 32, 1235 (1951).
2. W. Walkinshaw, A.E.R.E. report, Harwell (1956, unpublished).

DISCUSSION

TENG : Usually in planning an accelerator one injects with a definite phase space or beam quality. How can you decide how much spread in the x direction you need until you know the field gradient?

PARZEN : In principle, if one injected so as to make use of all the possible radial phase space, that is what one should do. But in the case of our accelerator, no great attempt is made to use the p_x direction. In our multi-turn way of injecting we use more the x distance rather than the p_x distance. I think it is just peculiar to the manner of the scheme we use for injection. If you can find a place where you have the largest radial stability limit you also have some assurance of having very nearly the largest phase space, too.

TENG : It's not obvious that for larger x threshold you necessarily get larger total radial phase space.

PARZEN : I think it is more or less true, but there are other factors involved.

KERST : Was the shaped pole shown in the previous paper a realistic pole, and about what was its size?

SNOWDON : The shaped pole that was shown previously was in fact a scale model for a 10 GeV machine, r = 30 m. The azimuthal width might have been somewhat smaller than would normally be used in such a machine.