# Peculiarities of charged particle motion in the monoenergetic cyclotron 

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Presented by A. A. Glazov

## 1. THE CONDITIONS FOR LOW ENERGY SPREAD

The heightened requirements for cyclotron beam energy spread ( 1 in $10^{4}$ ), as well as the necessity for varying the external beam energy, results in the need to consider a number of additional conditions which are usually neglected in ordinary cyclotrons. These conditions are as follows:
(a) equal ion energy gain per turn to an accuracy of 1 in $10^{4}$,
(b) isochronism of closed orbits,
(c) rigid restrictions for all the processes related to increasing free oscillation amplitudes,
(d) external injection with limited beam emittances,
(e) the necessity to take account of the effects of space charge.

The first condition makes it necessary to obtain a 'flat' form for the accelerating voltage with a corresponding degree of amplitude stabilisation. The accelerating system is shown schematically in Dr. A. A. Glazov's report presented to this conference. ${ }^{1}$

The second condition is the isochronism of closed orbits. It should hold for monoenergetic cyclotrons to higher accuracy since the maximum phase shift of the beam during acceleration should not exceed 2 to $3^{\circ}$. Beam phase correction in the acceleration process is to be reported at the present conference by Dr. Yu. N. Denisov et al. ${ }^{2}$

The requirement for orbit separation during acceleration leads to strict tolerances in all the processes which may cause the generation of free radial oscillations. ${ }^{3}$ Since the frequencies of radial oscillations at the injection radius are always close to unity, a requirement arises for making the acceleration symmetrical if excitation both by integral and parametric resonances is to be avoided.

The increase of the radial oscillation amplitude with single-electrode acceleration is found from the expression:

$$
\begin{equation*}
\Delta a_{\rho}=R \frac{\Delta W}{W} \quad \frac{1+\cos \varphi_{1}}{4 \sin \pi \delta_{1}} \tag{1}
\end{equation*}
$$

where $\Delta W$ is the energy gain at the accelerating gap, $\varphi_{1}$ is the electrode aximuthal width $\delta_{1}=v_{\rho}-1, R$ is the radius of the corresponding kinetic energy $W$, and $v_{\rho}$ is the radial oscillation frequency. Eqn (1) leads directly to the necessity for symmetrical arrangements both for the main and the additional electrodes. A less strict requirement is imposed on the parametric oscillation by the rf accelerating field. In order to avoid the resonance it is necessary to follow the inequality:

$$
\begin{equation*}
v_{\rho}^{2}-1>\delta_{2}\left(1+\frac{\epsilon_{4}}{2}\right) \tag{2}
\end{equation*}
$$

where $\delta_{2}$ is the value of the second harmonic of the $\Delta p / p$ function, $p$ is the momentum, and $\epsilon_{4}$ is the amplitude of the fourth harmonic of the magnetic field.

The value of the radial oscillation at the end of the acceleration cycle $a_{\rho}$ should be considerably smaller than the radial gain per turn $(\Delta R)$ :

$$
\begin{equation*}
a_{\rho}<\frac{\Delta R}{2} \tag{3}
\end{equation*}
$$

This relation shows the advantage of using in the accelerator a relatively low average magnetic field, since $\Delta R \propto 1 / B$, where $B$ is the mean field induction.

The small amplitudes of free oscillations required to ensure beam separation at each turn restricts the emittance of the injected beam into the accelerator:

$$
\delta_{\rho} \leq(5 \text { to } 10) \pi \mathrm{mm} \operatorname{mrad} \quad \delta_{z} \leq(30 \text { to } 50) \pi \mathrm{mm} \mathrm{mrad}
$$

With such emittances the particle density in a bunch ( $\kappa$ ) amounts to ( 5 to 8 ) $\times 10^{7} \mathrm{~cm}^{-3}$ which limits betatron oscillations to space-charge effects. The proper components of the electric field $E$ for a particle bunch of constant density having dimensions $b_{x}, b_{y}, b_{z}$, can be found from the expressions with $b_{y} \gg b_{x}, b_{z}$ (in m.k.s. units):

$$
\begin{align*}
\frac{E_{z}}{e_{\kappa} / \pi E_{0}} & =\frac{b_{x}}{2} \ln \frac{b_{x}^{2}+\left(b_{z}+z\right)^{2}}{b_{x}^{2}+\left(b_{z}-z\right)^{2}}+\left(b_{z}+z\right) \arctan \frac{b_{x}}{b_{z}+z} \\
& -\left(b_{z}-z\right) \arctan \frac{b_{x}}{b_{z}-z} \tag{4}
\end{align*}
$$

The components $E_{x}$ and $E_{y}$ at $\pi=0$ are obtained from Eqn (4) by changing $z$ for $x$ (for $E_{x}$ ) and $z$ for $y$ (for $E_{y}$ ).

The evaluation of the transverse effect (the component $E_{z}$ ) shows that in order to obtain average currents of $100 \mu \mathrm{~A}$, the axial focusing of the accelerator should satisfy the condition

$$
\begin{equation*}
\nu_{z}>0.5 \tag{5}
\end{equation*}
$$

The numerical evaluation of the effects related to the conductivity of the chamber walls have shown that this effect for transverse oscillation is negligibly small, if the aperture of the conducting surfaces is several times larger than the axial beam size.

The effect of adjacent bunches ${ }^{4}$ is also small, since in the region of maximum particle density ( $R_{\text {min }}$ )

$$
\begin{equation*}
\kappa=\frac{i}{8 e a_{\rho} a_{z} R_{\min } \alpha f} \tag{6}
\end{equation*}
$$

the distance between the orbits is commensurate with the vertical aperture of the conducting surfaces. [In expression (6) $a_{\rho}$ and $a_{z}$ are free oscillation amplitudes, $\alpha$ is the beam angular width, and $f$ is the rotation frequency.] The effect of conducting surfaces on the longitudinal component of electric field due to the space charge is more noticeable. ${ }^{3}$ However, with the above densities ( $\kappa$ ) the value of this component does not exceed $10^{3} b / \mu$, which corresponds to a potential distortion along the beam length of 400 to 600 V .

## 2. PARAMETERS OF THE MONOENERGETIC CYCLOTRON

The above considerations were the basis for the development of the Dubna monoenergetic cyclotron. Its parameters are given in Table 1. The coherent


Fig. 1. A general plan of the accelerator

Table 1. MAIN PARAMETERS OF THE 80 MeV PROTON MONOENERGETIC CYCLOTRON

|  | Units |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. Accelerated particles |  | $p$ | $d$ | $\alpha$ | ${ }^{3} \mathrm{He}$ | ${ }^{6} \mathrm{Li}$ |
| 2. Maximum energy | (MeV) | 80 | 60 | 120 | 120 | 180 |
| 3. Injector potential | (MeV) | 1.290 | 1.045 | 1.045 | 1.030 | 1.043 |
| 4. Intensity of the proton external beam | ( $\mu \mathrm{A}$ ) | 100 |  |  |  |  |
| 5. Energy spread of the external beam |  | $1 \times 10$ |  |  |  |  |
| 6. Mean value of the magnetic field at the full radius | (G) | 3296 | 3985 | 3960 | 3460 | 3967 |
| 7. Mean value of the magnetic field in the central region | (G) | 3037 | 3861 | 3837 | 3317 | 3844 |
| 8. Mean radius of the full orbit | (cm) |  | 400 |  |  |  |
| 9. Mean radius of the full orbit at injection | (cm) |  | 54 |  |  |  |
| 10. Free oscillation frequencies <br> (a) axial <br> (b) radial |  |  | $\begin{gathered} 0.8-0.9 \\ -07-1.1 \end{gathered}$ |  |  |  |
| 11. Rotation rate of accelerated particles | (MHz) | $4 \cdot 638$ | $2 \cdot 948$ | $2 \cdot 948$ | $3 \cdot 377$ | 2.948 |
| 12. Acceleration multiplicity |  |  | 2 |  |  |  |
| 13. Number of main dees |  |  | 2 |  |  |  |
| 14. Amplitudes of accelerating voltage on main dees | (kV) |  | 50 |  |  |  |
| 15. Distance between adjacent orbits at full radius | (cm) | 0.44 | 0.6 | 0.6 | 0.6 | 0.6 |
| 16. Range of smooth energy variation of the external beam |  |  |  | 1:4 |  |  |
| 17. Electromagnet power supply | (kW) |  |  | 1500 |  |  |
| 18. Power loss in the rf system | (kW) |  |  | $2 \times 70$ |  |  |
| 19. Weight of the magnetic system | (tons) |  |  | 1200 |  |  |

effects of space charge do not appear as the beam energy is not high ( $\gamma=E / E_{O} \approx 1$ ). Fig. 1 shows a general view of the accelerator.

A number of calculations have been performed to investigate the effects of free oscillations induced by the accelerating electric field at energies close to the injection energy $v_{\rho} \approx 1$ ). With these numerical calculations it turned out to be necessary to take into account the shape of the electric field, except when changing the particle momentum.

Table 2 shows the necessity for making the accelerating system symmetrical.

Table 2

| Version No. | $\nu_{\rho}$ | $r_{\infty}$ <br> cm | Number of main dees | $\begin{gathered} V_{g} \\ \mathrm{kV} \end{gathered}$ |  | Number of additional dees | $\begin{gathered} a_{\rho} \\ \mathrm{cm} \end{gathered}$ | Resonance type |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.067 | 1029.5 | 2 | 50 | 90 | 2 | None | None |
| 2 | 1.067 | 1029.5 | 2 | 50 | 90 | 1 | $\begin{aligned} & 0.8-0.9 \\ & \text { per } 10 \text { turns } \end{aligned}$ | Integral |
| 3 | 1.067 | 670 | 2 | 100 | 45 | - | $\begin{aligned} & 1.2 a_{o} \text { per } \\ & 35 \text { turns } \end{aligned}$ | Parametric |
| 4 | 1.067 | 670 | 2 | 100 | 45 | 1 | $\begin{aligned} & 3 \text { per } 30 \\ & \text { turns } \end{aligned}$ | Integral plus parametric |

## CONCLUSION

The investigations carried out at the Laboratory of Nuclear Problems (Dubna) as well as at other laboratories ${ }^{5,6,7,8}$ show that the design and construction of a monoenergetic cyclotron having 1 in $10^{4}$ energy resolution are quite possible from the technical point of view. Important studies performed at the Michigan State University ${ }^{9}$ are an experimental proof of this conclusion.

## DISCUSSION

## Speaker addressed: A. A. Glazov (JINR, Dubna)

Question by T. Khoe (Argonne): For a given harmonic number, how stable can you keep the phase between the main dee and the third harmonic dee?

Answer: $0.1^{\circ}$.
Question by T. Khoe: How do you measure this?
Answer: It is not necessary to measure the phase shift with such accuracy. It is necessary only to discriminate such a phase shift.

## REFERENCES

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