## J. Reich, S. Martin, D. Protić, and G. Riepe

Institute for Nuclear Physics of Kernforschungsanlage Julich, 517 Jiilich, W. Germany

## Abstract

The matching of particle beams to a solid state spectrometer as well as to a magnet spectrometer is investigated and the appropriate design procedures are described. In the case of a solid state spectrometer experimental and ray tracing results show, that after matching the width in the reaction angle distribution depends almost only on the acceptance angle of the detector. The matching of the Juilich cyclotron beam to the spectrograph "Big Karl" is studied for full transmission between cyclotron and target, for 5 msr solid angle of the spectrograph and for $K$-values of up to 0.3. Under these conditions a momentum resolving power of 6300 still should be available.

## 1. Introduction

Cyclotron accelerator installations often provide both, beams and detection systems of high resolving power. However, in nuclear reaction experiments the resolving power is deteriorated and the transmission is affected by the kinematics. This effect is especially large in the case of horizontal cyclotron emittances of typically several $10 \mathrm{~mm} \cdot \mathrm{mrad}$ and beam energy spreads of often several parts in 1000. This situation can be remarkably improved by a careful matching of the beam to the particle spectrometer.

## 2. General Theory

The transfer optics between the accelerator and the detection plane of the spectrometer is a function of the particle transfer through the beam line to the target, described by the transfer matrix $\bar{B}$, as well as a function of the target transformation TT and the particle transport from the target to the detection plane, described by the transfer matrix $\bar{S}$. The formulation of the transfer matrices $\bar{B}$ and $\bar{S}$, including second order effects, is available for instance in the ion optics code TRANSPORT1). The target transformation TT

$$
\begin{aligned}
& \text { target beam cyclo- } \\
& \text { plane } \\
& \left|\begin{array}{l}
x \\
\theta \\
y \\
\phi \\
\ell \\
\delta
\end{array}\right|=\bar{S} \\
& \text { target }
\end{aligned}
$$

beam cyclo-
line tron
$\left(\left.\begin{array}{ll}x_{0} \\ \theta & \\ y_{0} \\ \phi_{0} \\ e_{0} \\ \delta_{0}\end{array} \right\rvert\,(1)\right.$
describes the change of the spatial coordinates $x, \theta, Y, \phi$ as well as the change of the momentum deviation $\hat{0}$ due to the kinematics at the target. $\ell$ represents the longitudinal beam extent.

First order expressions for the target transformation are given in the following. The nominal reaction angle a (see fig. 1), which is the angle of rotation between the


Figure 1: Convention of coordinates in the horizontal plane at the target
optical axis of the beam line $z_{1}$ and that of the spectrometer $z_{2}$, as well as the target rotation angle $\gamma$ are measured clockwise, whereas the particle angles $\theta_{1}$ and $\theta_{2}$ are measured counter clockwise with respect to their optical axes $\mathrm{z}_{1}$ and $\mathrm{z}_{2}$. The horizontal position $x_{2}$ of a particle just leaving the target is to first order only dependent on the position $x_{1}$ of a particle just hitting the target

$$
\begin{equation*}
x_{2}=T \cdot x_{1}, T=\cos (\alpha-\gamma) / \cos \gamma \tag{2}
\end{equation*}
$$

The angle $\theta_{2}$ of a reaction particle is a new variable at the target, its range being defined by the horizontal acceptance of the spectrometer. To first order in the vertical plane

$$
\begin{equation*}
Y_{2}=Y_{1} \tag{3}
\end{equation*}
$$

and $\phi_{2}$ is also a new variable, its range being defined by the vertical acceptance of the spectrometer. The momentum deviation $\delta_{2}$ after the target due to kinematics is to first order given by

$$
\begin{equation*}
\delta_{2}=K\left(\theta_{2}-\theta_{1}\right)+C \delta_{0} \tag{4}
\end{equation*}
$$

where $K=\left(1 / p_{2}\right) \partial p_{2} / \partial \alpha$ and $C=\left(p_{0} / p_{2}\right) \partial p_{2} / \partial p_{0}$. Complete expressions for the target transformation in the form
$\mathrm{x}_{2}=\mathrm{x}_{2}\left(\alpha, \gamma, \mathrm{x}_{1}, \theta_{1}, \theta_{2}\right)$
$Y_{2}=Y_{2}\left(\alpha, \gamma, x_{1}, \theta_{1}, \theta_{2}, Y_{1}, \phi_{1}, \phi_{2}\right)$ and $\delta_{2}=\delta_{2}\left(\alpha, \gamma_{1}, r, \theta_{1}, \theta_{2}, \phi_{1}, \phi_{2}\right),{ }_{2}$ have been derived and inserted into the ray tracing program TURTLE 2 ). In this case the parameter $r$ describes the kinematics of the reaction. This modified program TURTLE calcu-
lates particle trajectories to second order with respect to the local effects of the elements in the beam line and the spectrometer and to higher order with respect to the global effects of lumped elements and the target transformation.

## 3. Matching the Beam to a Solid State Spectrometer

The most common spectrometer in nuclear reaction experiments today is a setup equipped with solid state detectors. In this case the beam matching results in a minimization of the spread of the reaction angle $\triangle \alpha$ (FWHM), which in turn means a reduction of the kinematic contribution to the particle energy resolution. This, of course, only makes sense, if the beam is prepared by a magnetic beam analysing system of adequate resolving power.

### 3.1 Geometrical Consideration and First Order Optics

Assuming a point focus and a point detector, $\triangle \alpha$ (FWHM) vanishes, if the focus $F$ (see fig. 2), the detector $D$ as well as the target are located on a circle, which is well known as the Rowland circle. If $\overline{\mathrm{TF}}=\mathrm{LF}$


Figure 2: Rowland circle as matching location for point focus $F$, point detector $D$ and target, shown for forward and backward scattering angles
and $\overline{\mathrm{T}} \overline{\mathrm{D}}=\mathrm{L}_{\mathrm{D}}$, the radius $\rho$ of this circle and the rotation angle $\gamma$ of the tangent to this circle at the target position $T$ are given in the following expression

$$
\begin{align*}
& \rho=\frac{1}{2 \sin \alpha} \sqrt{L_{F}^{2}-2 L_{F} L_{D} \cos \alpha+L_{D}}{ }^{2}  \tag{5}\\
& \operatorname{ctg} \gamma=-L_{F} \cdot \sin \alpha /\left(L_{F} \cdot \cos \alpha-L_{D}\right) \tag{6}
\end{align*}
$$

If $L_{F}=L_{D}, \gamma$ has to be equal to $\alpha / 2$.
In the case of common cyclotron emittances one has to deal with some first order beam optics to fulfill the condition

$$
\begin{equation*}
\Delta \alpha=\left(\theta_{2}-\theta_{1}\right)=0 \tag{7}
\end{equation*}
$$

In case of a point detector we have

$$
\begin{equation*}
\theta_{2}=-x_{2} / L_{D}=-T x_{1} / L_{D} \tag{8}
\end{equation*}
$$

The matching condition in equation (7) can
be expressed in the following form: Using as a calculation aid for the required beam matching a matrix $\bar{M}$ with $m_{21}=T / L_{D}$,
$m_{61}=T K / L_{D}, m_{62}=\mathrm{K}$ and $m_{66}=C$, all other $m_{i j}$ corresponding to the unity matrix, and considering the matrix product $\bar{R}=\bar{S} w \bar{m} \bar{B}, \bar{R}$ determines $\Delta \alpha$ and $\delta 2$ as the second and sixth element of the particle coordinate vector in the correct form. The matching condition is then equivalent to the vanishing of the matrix coefficients $\mathrm{R}_{2} 1$, $\mathrm{R}_{22}$ and $\mathrm{R}_{26}$. This consideration is helpful to the users of the code TRANSPORT, since one can insert $\bar{M}$ as an arbitrary matrix and fit the quadrupole settings in the beam line for the constraints $\mathrm{R}_{21}=\mathrm{R}_{22}=\mathrm{R}_{26}=0$.

### 3.2 Experimental Results

Spectra of elastically scattered particles from ${ }^{1} \mathrm{H}(\mathrm{p}, \mathrm{p})-$ and ${ }^{12} \mathrm{C}(\alpha, \alpha)$-reactions at 44.1 and 155.5 MeV incident beam energy, respectively, have been taken for different distances $L_{F}$. The reaction angle $\alpha$ and the target rotation angle $\gamma$ were set to 150 and 7.50, respectively. The experiments were performed with a high purity germanium detector in an experimental set-up described in ${ }^{3}$ ). The beam was analysed by a double monochromator system 4 ) operated in dispersive mode to cut down the energy spread of the cyclotron beam to $\triangle \mathrm{E}(\mathrm{FWHM}) / \mathrm{E} \sim 10^{-4}$.

To extract the angular spread $\Delta \alpha$ (FWHM) from the spectra, the assumption was made that the overall energy resolution is obtained by the quadratic sum of the different contributions. Therefore

$$
\begin{gather*}
\Delta \alpha(F W H M)=(1 / 2 E K)\left[\Delta E(C ; H)^{2}-\Delta E(A u)^{2}\right.  \tag{9}\\
\left.-\Delta E_{S T R}(C ; H)^{2}+\Delta E_{S T R}(A u)^{2}\right] 1 / 2
\end{gather*}
$$

$\Delta E(C ; H)$ and $\Delta E(A u)$ are the experimental line widths (FWHM) obtained for the reactions on $C ; H$ and $A u$, respectively. Since for the scattering on Au practically no kinematic contribution but all the others (detector, electronics, beam) are present, only the energy loss straggling in the target $\Delta E_{S T R}(C ; H)$ and $\Delta E_{S T R}(A u)$ had to be taken into account, calculated from the known target thicknesses. Fig. 3 shows the experimental results for the two reactions (points) in comparison with ray tracing results using the modified TURTLE code. The calculation was based on the actual quadrupole settings and the typical Jülich beam qualities (see table 1). In the ${ }^{12} \mathrm{C}(\alpha, \alpha)-$ experiment the beam at the target was blown up in divergence to show the effect of matching more drastically. Note, that a broad minimum in $\triangle \alpha$ (FWHM) occures at $L_{F}=L_{D}$ amounting to approximately 3 and 2 mrad, respectively. The main fraction of these values is due to the horizontal acceptance angle of the detector of 2 mrad. Fig. 4 shows particle spectra for each reaction experiment with and without beam matching.


Figure 3: Spread of reaction angle $\Delta \alpha$ (FWHM) versus the focus shift $L_{F}$


Figure 4: Particle spectra with and without beam matching
4. Matching the Beam to a

A magnet spectrometer has a feature, the dispersion, which was not available in the previous case. The dispersion can be used to match the beam for more beam intensity at the target and at the same time for a good momentum resolving power at the detecting plane. Therefore a magnetic analysing system is needed in the beam line to produce the appropriate dispersion to be matched with the dispersion of the spectro-
meter.

### 4.1 First Order Design Procedure

We display equation (1) for the horizontal displacement $x$ for trajectories at the detection plane of the spectrometer

$$
\begin{align*}
x & =x_{o}\left(s_{11} b_{11} T-s_{16} b_{21} K\right) \\
& +\theta_{0}\left(s_{11} b_{12} T-s_{16} b_{22} K\right)  \tag{10}\\
& +\theta_{2}\left(s_{12}+s_{16} K\right) \\
& +\delta_{0}\left(s_{11} b_{12} T-s_{16} b_{26} K+s_{16} C\right)
\end{align*}
$$

To optimize the combined system consisting of beam line and spectrometer, the coefficients at $\theta_{0}, \theta_{2}$ and $\delta_{o}$ should be zeroed. A way to eliminate the factors at $\theta_{0}$ and $\theta_{2}$ is the kinematic displacement of the focus at the target and at the detection plane of the spectrometer, respectively. The elimination of the coefficient at $\delta_{0}$ is done by dispersion matching. If the coefficients at $\theta_{0}, \theta_{2}$ and $\delta_{o}$ vanish, we call the coefficient at $x_{0}$ the overall magnification $M_{o v}$. The momentum resolving power of the combined system is to first order then given by

$$
\begin{equation*}
R_{1}=s_{16} / 2 M_{O V} \cdot x_{O} \tag{11}
\end{equation*}
$$

Before we again apply a calculation aid matrix $\bar{M}$, we make sure that the coefficient at $\theta_{2}$ is zeroed. If the dispersion at the detection plane shall not be changed and the vertical imaging in the spectrometer shall not be deteriorated, a pair of quadrupoles in front of the bending magnets of the spectrometer should be available. If after this the matrix $\bar{M}$ is inserted with $m_{11}=T, m_{22}=0, m_{62}=-K$ and $m_{66}=C$, all other $m_{i j}$ corresponding to the unity matrix, the vanishing of the coefficients at $\theta_{0}$ and $\delta_{0}$ is equivalent to the vanishing of the coefficient $R_{12}$ and $R_{16}$ in the product matrix R .

### 4.2 Matching Design at JULIC

The beam line to the spectrometer includes a double monochromator ${ }^{4}$ ) (see fig.5). To fulfill the condition of high transmission between cyclotron and target, only its preferable single achromatic mode operation is considered. The spectrograph "Big Karl"5), a high despersion, large solid angle magnetic spectrometer of the type QQDDQ is in progress. The specifications of the cyclotron and the spectrograph, important for the matching design, are summarized in table 1.

Table 1: Important specifications of cyclotron and spectrograph

| CYCLOTRON | $\varepsilon_{x}=\varepsilon_{y}=20 \mathrm{~mm}$ mrad |
| :--- | :--- |
|  | $\delta_{\mathrm{FWHM}}=1.5 / 1000$ |
| SPECTRO- | $\mathrm{S}_{1} 6=17 \mathrm{~cm} / \frac{\mathrm{y}}{6} \Omega=5 \mathrm{msr}$ |
| GRAPH | $-20 \leq \theta_{2} \leq 20 \mathrm{mrad},-60 \leq \phi_{2} \leq 60 \mathrm{mrad}$ |



Figure 5: Lay out of beam line and spectrograph at JULIC

Limits in the design, especially due to the present cyclotron beam qualities, arose from the fact, that the horizontal beam size at the end of the beam line and at the target becomes larger for larger K-values. A large horizontal beam size requires costly large quadrupole apertures, restricts the scattering at forward and backward angles and makes the higher order correction more difficult. Fig. 6 shows a chart of beam envelopes for $K$-values up to 0.3 ,


Figure 6: Chart of beam envelopes for $K-v a l-$ ues up to 0.3
the target being positioned either for transmission ( $\mathrm{T}=1$ ) or reflection ( $\mathrm{T}=-1$ ) of the outgoing particles. $K=0.3$ corresponds for instance to reactions like ${ }^{12} \mathrm{C}(\alpha, \alpha){ }^{12} \mathrm{C}$ at $90^{\circ}$ or ${ }^{7} \mathrm{Li}(\alpha, \alpha)^{4} \mathrm{He}$ at 450. In the Jülich case the overall magnification stays at $M_{O V}=1.57$, independent of the K-value. The first order resolving power is therefore given by $\mathrm{R}_{1}=17000 / 3.14 \mathrm{x}_{\mathrm{O}}$ for all K -values. The present horizontal cyclotron emittance only allows a waist image at the slit AS11 of $x_{0}=1 \mathrm{~mm}$, which in turn determines the momentum resolving power of the combined system for $100 \%$ transmission between cyclotron and target.

Ray tracing calculations have been performed with the modified program TURTLE, and appropriate second and third order correction has been investigated for several K-values. The results of the optimized line widths (s. fig. 7) stayed the same for


Figure 7: Line forms at the detection plane applying no, second and un to third order correction for $\mathrm{K}=0.1$, respectively
different $K$-values. Therefore the curve in figure 8 , calculated for $K=0.1$, represents the resolving power versus the transmission for all considered K-values.


Figure 8: Momentum resolving power versus transmission calculated for $K=0.1$, the dotted line representing $\mathrm{R}_{1}$ of equation (11)
5. Conclusion

Since from the previous chapter it is clear, that an improvement of the cyclotron beam quality directly will pay off with respect to the performance of the combined system beam line/spectrograph, an appropriate program is in preparation. A remarkable change of the beam quality is especially expected from the modification of the center region at JULIC, which is described in another paper of this conference ${ }^{6}$ ).

## Acknowledgement

The authors acknowledge with pleasure the very valuable cooperation with K.L. Brown. They wish to thank D. Ingham and J. Meissburger for fruitful discussions, the members of the detector laboratory for their excellent technical support and the operators at the cyclotron for their careful beam handling during experiments.

References

1) K.L. Brown, S.K. Howry, SLAC 91, (1970)
2) D.C. Carey, NAL-64, (1971)
3) G. Riepe, D. Protić, J. Reich, Nucl. Instr. and Meth. 124 (1975) 527
4) J. Reich, C.U. Mayer-Böricke, K.L. Brown, F.E. Johnson, AIP Conf. Proc. no. 9 (1972) p. 716
5) A. Abdel-Gawad, A. Hardt, S. Martin, J. Reich, K.L. Brown, K. Halbach, Proc. of the 5 th Intern. Conf. on Magnet Technology, Rome (1975)
6) J. Linz, P. Wucherer, J. Reich, Contribution to this conference
