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## ABSTRACT

The results of numerical calculations on the passage of the integral resonance $\mathrm{Q}_{\mathrm{r}}=2$ in a cyclotron are presented. It is shown that the resonance is passed at a moderate gain of energy per turn and at a sufficiently high value of the second harmonic of the magnetic field.

The idea of producing a high-intensity generator of above-threshold kaons on the basis of twostage cyclotron, with the first stage of $\mathrm{SIN}^{1}$ ) or TRIUMF ${ }^{2}$ ) type for proton energies between 500 and 900 MeV playing the part of injector and pion faci1ity, while the second accelerates protons up to $\approx 5 \mathrm{GeV}$, which was advanced by the author in 1970 ${ }^{3-7 \text { ), }}$ is gaining an increasingly wider acceptance ${ }^{8}$ ).

In accelerating protons to an energy of 4.4 GeV , the central problem is the passage of a number of integral resonances in radial oscillations in the second stage $\left(\mathrm{H}_{0}=2 \mathrm{kG} ; \mathrm{N}=24 ; \mathrm{Q}_{\mathrm{z}}=1.1 ; \sim 2 \leq \mathrm{Q}_{\mathrm{r}} \leq 6 ;\right.$ $0.2 \leq \varepsilon \leq 1 ; \quad r_{\infty}=1563.72 \mathrm{~cm}$ ). The existing theory of integral resonance passage has been developed in the linear approximation ${ }^{9}$. In this approach, it has been assumed that the force due to the lowest harmonic of the vertical magnetic field component of index $\mathrm{S}=\mathrm{Qr}$ is always in phase with betatron oscillations, i.e., continuously acts upon the radial amplitude of a particle in the course of passing a resonance zone. The behaviour of an amplitude after passing a resonance was estimated on the basis of an asymptotic formula according to which an increase in amplitude was proportional to the number of turns in the resonance zone (the particle energy changes adiabatically) which led to infinitely large deviations of the amplitude of radial particle oscillations from an ideal orbit. This treatment is incorrect because the character of gaining energy per turn and an increase in energy and particle radius, associated with it, are not considered. Furthermore, the equations of motion and the magnetic field of the isochronous cyclotron are essentially nonlinear, which must bound the growth of an amplitude in a resonance ${ }^{10}$ ). Therefore, the integral resonance passage in a cyclotron was investigated by numerical computer simulation involving the complete equations of motion according to the procedure described in ref. ll. The dynamical regime was investigated in passing the most dangerous resonance, $\mathrm{Q}_{\mathrm{r}}=2$, occurring at initial radii of the second stage. These conditions were investigated with cyclotron parameters ( $\mathrm{H}_{0}=2 \mathrm{kG}$ $N=20 ; Q_{z}=1.1 ; \varepsilon=1 ; r_{\infty}=1563.72 \mathrm{~cm}$ ) for particles with various initial coordinates

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\left(\xi=\frac{r}{r_{\infty}}, \quad \xi^{\prime}=\frac{r^{\prime}}{r_{\infty}}, \quad \text { and } \quad r=\frac{z}{r_{\infty}}, \quad \eta^{\prime}=\frac{z^{\prime}}{r_{\infty}}\right)
$$

and while varying the injection energy, energy gain per turn, amplitude of the second harmonic of the vertical magnetic field component, and phase shift $\delta$
between the fundamental and second harmonics of the magnetic field ( $0^{\circ} \leq \delta \leq \pi / \mathrm{N}=9^{\circ}$ ). The initial magnetic field was given as a table containing 44 radial points with a radial step of 4 cm . The energy per turn was achieved by using two slits (the slit azimuthal extension cavering 8 points) installed $180^{\circ}$ apart at a fixed azimuth. The particles were injected symmetrically relative to the accelerating slits. With 800 steps per turn, the computing time of a БэСМ-6 computer was 70 min for 104 turns.

Figure 1 shows phase trajectories of the beam as functions of the number of turns at an energy gain of 0.8 MeV per cycle for a reduced radial distribution of the second harmonic of the mangetic field (given at seven radially-equidistant points). The beam trajectories have been constructed on the basis of four-particle calculations at injection energies of 853 MeV . It is seen that in the resonance zone, the initial amplitude of radial oscillations of beam of 0.8 cm does not increase (the beam emittance remains essentially constant) which confirms the coherent character of the integral resonance mechanism. The distortion of the mean radii of the orbits is finite, being $\because 3 \mathrm{~cm}$.


Fig. 1 Radial phase trajectories in passing the $\mathrm{Q}_{\mathrm{r}}=2$ integral resonance in the presence of the second harmonic of the vertical magnetic field component. Turns are labelled by numbers. Energy gain per turn $=0.8 \mathrm{MeV}$.

For an energy gain of 1.4 MeV per turn, phase trajectories of the beam are plotted in Fig. 2. It is seen that in passing an integral resonance, the initial amplitude of radial oscillations of the beam of 0.8 cm is practically unchanged. The distortions of the mean radii of equilibrium orbits are of the same order as for the energy gain of 0.8 MeV per turn. The situation did not change in the case of coupled $r$ and $z$ - oscillations at the initial amplitude of the vertical oscillations of the beam of 0.5 cm .


Fig. 2 Radial phase trajectories in passing the $\mathrm{Q}_{\mathrm{r}}=2$ integral resonance. Energy gain 1.4 MeV .

From the results of the numerical simulation described above, one may draw the following conclusions:

1. The consideration of the passage of integral resonances in a cyclotron on the basis of the asymptotic formula is incorrect; it should be based on the complete equations of motion and should take into account the character of energy gain per turn.
2. In the second, high-energy stage of a cyclotron, it is possible to use an energy gain of $\sim 0.8-2 \mathrm{MeV}$ per turn at a permissible value for the second harmonic of $\approx 10$ to 15 G . For the lower field harmonics, beginning with the third, these values in the zones of the corresponding integral resonances will be higher in accordance with the ratio $S / 2$, where $S$ is the number of the lower field harmonic.
3. In order to decrease the effect of slits on the orbit displacement and exclude the high-frequency resonance, the number of spiral-shaped resonators in the second stage should be taken greater than six, for example, eight. This implies that the voltage of a single resonator should be 250 kV at an energy gain of $2 \mathrm{MeV} /$ turn.
4. For a successful passage of the integral resonance $\mathrm{Q}_{\mathrm{r}}=2$ with finite radii of the first stage at $\mathrm{eV}=0.8-1.5 \mathrm{MeV} /$ turn and $\mathrm{H}_{2}=10-15 \mathrm{G}$, the periodicity of the field structure $N$ should be taken equal to 10. In this case, the integral resonance $\mathrm{Q}_{\mathrm{r}}=2$ is separated in radius from the nonlinear structural resonance of the forth order

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p=\frac{N}{Q_{r}}=\frac{10}{2.5}=4
$$

and occurs before it.
We note that studies carried out in Zurich in the course of adjustment of the SIN cyclotron ${ }^{1}$ ) and in Dubna involving the electron connection with the assuring of complete separation of an orbit at an
edge, enables one to say that a mean intensity of quasi-continuous beam in a cyclotron kaon facility is at a level of $0.5-1.0 \mathrm{~mA}$, which is unattainable for a linac-synchrotron cascade ${ }^{12}$ ).

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