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INJECTION STUDIES FOR THE K=800 SUPERCONDUCTING CYCLOTRON AT M.S.U.

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Introduction

Beam injection from the K=500 superconducting cyclotron, now under construction at MSU, into the proposed K=800 cyclotron obviously plays a major role in the successful coupling of the two cyclotrons. Let us recall that anticipated maximum beam energies[1] are between 200 MeV/n and 50 MeV/n for fully stripped light ions and heavy ions (uranium) respectively. Minimum energies for the same extreme cases are 50 MeV/n and 5 MeV/n. Operating center field values are between 30 kG and 47 kG, and Z/A values of accelerated particles range from 0.1 to 0.5. Focusing and bending requirements lead to a 41" pole radius machine, and a three sector geometry, having 46° wide hills, with an unusually tight spiral, i.e. with a constant of 1/13 rad/inch.

Although schemes of beam injection into superconducting cyclotrons have been studied before,[2,3] they were not addressed to machines with such a wide range of beam energies and extreme sector spiral. Furthermore, they concerned injection from electrostatic accelerators, while, as will be seen in the following, coupling between two cyclotrons imposes additional constraints on the injection scheme.

Therefore, an extensive study of the injection process was carried out. This led us to consider a rather new pole tip geometry as an alternative to a conventional spiral, in order to meet all injection requirements. It is the purpose of this paper to discuss in some detail the relative merits of the two solutions presently available.

Outline of Injection Requirements and Constraints

The main requirements are:

a. If at all possible, stripping should take place in a hill, to avoid having the stripping foil positioning and replacement mechanism inside a dee.

b. Injection trajectories should have an almost constant azimuthal entry position into the cyclotron, i.e. spanning 20-40 degrees at most.

c. As a consequence of (b), all trajectories could then originate from a fixed point outside the cyclotron. If the distance of this point is conveniently chosen, typically two to three times the cyclotron radius, then steering of just a few degrees at this point will assure for every beam the proper azimuthal entry position into the cyclotron.

d. Proper phase space matching to the cyclotron acceptance, both in the radial and axial spaces, should be possible.

Let us briefly review those aspects of the coupling of the two cyclotrons which pose intrinsic constraints on beam injection. We define: h, Z, Bo, Rex, and T/A as the harmonic number, charge state, center field value, extraction radius, and extraction energy of the first cyclotron (K=500) or second cyclotron (K=800), according to the subscript.

The basic requirement in the coupling of the two cyclotrons is that the R.F. acceleration frequencies in the two machines be the same. This hypothesis leads to the following consequences:

The second cyclotron acts just as an energy multiplier, so that in a non-relativistic approximation it must be:

$$\frac{T_2}{A} / \frac{T_1}{A} \simeq \left(\frac{Rex_2 \cdot h_1}{Rex_1 \cdot h_2}\right)^2 \qquad (1)$$

Thus, for any given final energy per nucleon, the injection energy depends solely upon the harmonic coupling ratio (HCR).

- The stripping radius R_S in the K=800, corresponding to the injection energy T_1/A, can be expressed as

$$R_S = Rex_1 \frac{h_2}{h_1} \qquad (2)$$

and therefore, for a given Rex_1 (in our case 26.4"), it is constant for any given HCR and does not depend upon charge state or energy of the ion.

- The stripping ratio Z_2/Z_1 must obey the relationship

$$Z_2/Z_1 = \frac{h_1\ Bo_1}{h_2\ Bo_2} \qquad (3)$$

On the other hand, from the hard edge approximation, a limit can be derived for the lower limit of the admissible stripping ratio for injection, i.e.:

$$Z_2/Z_1 \geq \frac{1}{2}\left(\frac{Rex_2\ h_1}{Rex_1\ h_2} + 1\right) \qquad (4)$$

In our case, since $\frac{Rex_2}{Rex_1} \simeq 1.5$, one obtains, for typical HCR of 3:1, 4:1, and 5:1, lower limits of Z_2/Z_1 = 2.8, 3.6, 4.4, respectively. Although the hard edge approximation is not really verified in a realistic cyclotron, the limits given by the above formula represent very reasonable guidelines.

Careful analysis of the operating range of the K=800 cyclotron has determined[1] that only the following harmonic coupling ratios are possible:

- 3:1, 4:1, 5:1 for beam energies between 200 MeV/n and 18 MeV/n, the lower limit being imposed by the R.F. frequency. Corresponding average stripping radii are 8.8", 6.6", and 5.2".
- 5:2 and 7:2 for energies between 5 MeV/n and 60 MeV/n, in this case the upper limit being determined again by the R.F. frequency. Corresponding stripping radii are 10.5" and 7.5".

In view of the above constraints, the self-consistent analysis of beam injection involves the following steps:

- Determine the ranges of stripping ratios needed for each harmonic coupling, over the whole $(Bo_2, Z_2/A)$ operating range. These must satisfy the relationships (2), (3), and (4) and the Z_2 values must also, for each energy, be at, or around, the peak for maximum intensity at stripping.[1]
- Determine an injection scheme which fulfills requirements (a), (b), and (c) set forth above.
- Determine the real lower and upper limits of the stripping ratio Z_2/Z_1 which are compatible with the selected injection scheme. This must be done for each of the five possible harmonic couplings over the entire range of Bo_2, Z_2/A.
- Verify that the Z_2/Z_1 limits thus derived allow the desired coupling of the two cyclotrons.

Examine the phase space behaviour of the injected beams and establish the proper matching conditions.

Possible Injection Schemes

Injection both against and along the spiral has been considered. As expected, the former restricts severely the range of allowed stripping ratios and therefore injection along the spiral, and more precisely in a valley, was selected.

The resulting scheme is shown in Fig. 1, where most cases of light ions for a final energy of 200 MeV/n are presented, encompassing HCR of 3:1, 4:1, and 5:1. The latter two modes are needed for boron and carbon injection respectively, with corresponding stripping ratios of 5 and 6, which cannot be injected in the 3:1 mode. All trajectories, tracked up to the radius of 52", converge to a common point, not shown in the figure, which lies 150" from the cyclotron center[1] at an azimuth of 60°. Decreasing stripping ratios correspond to trajectories of progressively lower average curvature radius, as would be expected also in a hard edge approximation, being:

$$\rho_{inj} = R_S \frac{Z_2}{Z_1} \tag{5}$$

The main difficulty with this scheme is that stripping occurs naturally in a valley. This is mostly a geometrical effect, arising from the tight spiral of the sectors, and it can be predicted also on the basis of the hard-edge approximation. Displacing the trajectories to a hill, although possible in principle, is not practical because the increased average magnetic field along the trajectory restricts the allowable stripping ratios range.

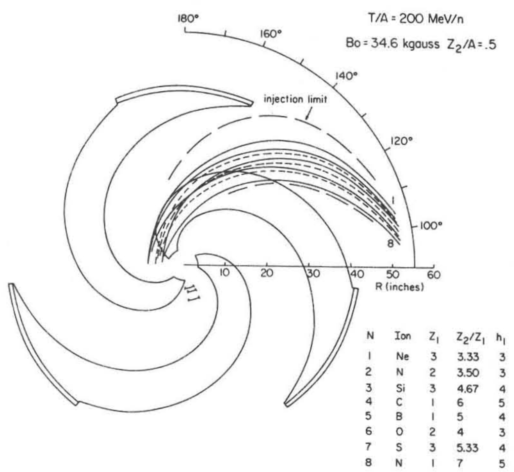

Fig. 1. Injection trajectories for 200 MeV/n light ions (conventional spiral, 1/13 rad/inch constant).

We designed therefore a different sector geometry, in which the spiral constant is positive, with the same 1/13" constant, up to a radius of 15" and it reverts to a negative constant thereafter. This scheme, shown in Fig. 2, produces the effect of substituting hills for valleys in the useful range of stripping radii (up to 12") with a narrow, 2" in radius, transition region between the two spirals.

Injection trajectories are also shown in Fig. 2, again for 200 MeV/n final energies. Now stripping for all particles effectively occurs in a hill, all other characteristics of injection remaining equal.

The only drawback of such a double spiral geometry is the decrease of vertical focusing, which can be expected in a radial region of 3"-4" around the transition radius of 15", the sectors being practically radial there.

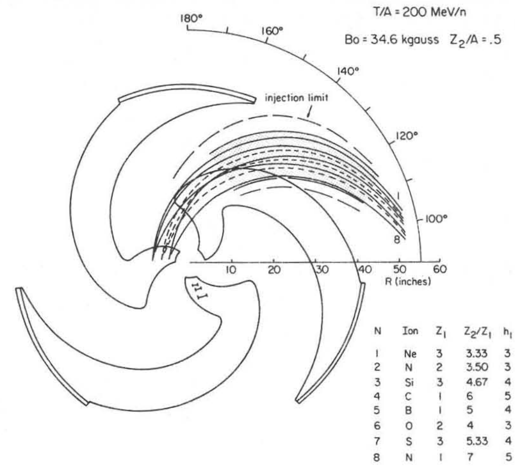

Fig. 2. Injection trajectories for 200 MeV/n light ions (double spiral scheme).

Equilibrium orbit data show, however, that the transition region presents no danger of beam losses. Vertical focusing frequencies for 200 MeV/n and 20 MeV/n particles are shown in Fig. 3, for both the conventional and double spiral geometries. Since the minimum ν_z values are about .1 and .14 respectively, we concluded that the double spiral scheme is a realistic alternative to injection in the K=800 machine.

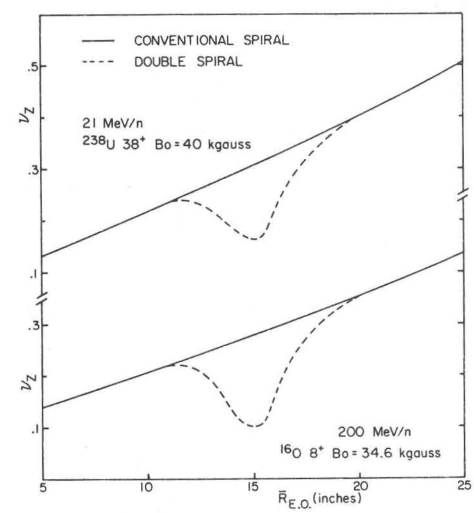

Fig. 3. Axial focusing frequencies, ν_z for the conventional spiral and the double spiral scheme.

Detailed Analysis of the Injection Solution

The ranges of stripping ratios compatible with the proposed scheme have been studied in great detail. For each h_1/h_2 ratio, minimum and maximum values of the allowed stripping ratio will exist, as shown by the trajectories indicated as injection limits in Fig. 2. In particular the maximum Z_2/Z_1 value usually corresponds to the hill azimuthal limit at the proper stripping radius. The minimum is more often associated with the intrinsic limit on the radius of curvature of the injection trajectory, like the one given by eq. (5).

The actual limits of the stripping ratios are of course a function of the center field level, Bo_2, and Z_2/A, as can be expected because of the very different isochronous field shapes and hence the different average field experienced by the injected particles. An example is shown for the case of h_2:h_1=3:1 in Fig. 4, where maximum (solid lines) and minimum (dashed lines) values of Z_2/Z_1 are plotted as a function of Z_2/A for different field levels. Marked variations are indeed observed, especially for the minimum Z_2/Z_1 values.

Such an analysis has been carried out for all other harmonic coupling modes, thus giving a very comprehensive picture of the injection solution.

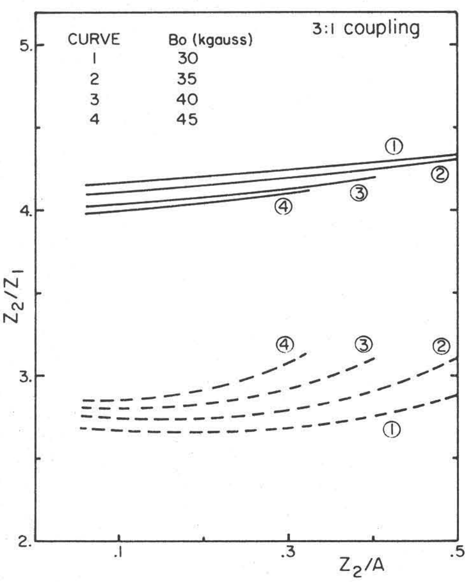

Fig. 4. Maximum and minimum stripping ratios allowed for injection, as a function of Z_2/A, for different field levels.

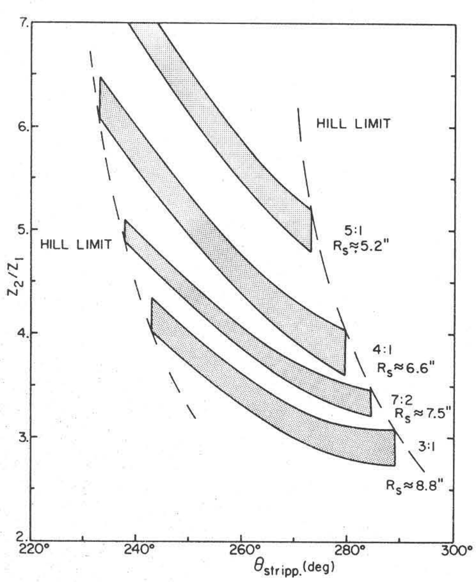

Fig. 5. Allowed stripping ratios, as a function of the stripping angle, for different harmonic couplings.

The overall results are shown in Figs. 5 and 6, where the stripping ratios compatible with the injection scheme are plotted as a function of $\theta_{stripping}$ and θ_{entry} at 40", respectively. The latter radius has been chosen because, as apparent from Fig. 2, the bundle of injection trajectories reaches there the maximum width. Fig. 5 shows that, for any given HCR, the allowed stripping ratios lie within a band. The latter is defined by:

a. The hill width limits, which are functions of the stripping radius.

b. Two curves which are roughly defined, at each stripping angle θ_S, by the maximum and minimum ion energy compatible with the given harmonic coupling. Therefore, upper limits of the 3:1, 4:1, and 5:1 bands are defined by the 200 MeV/n energy limit at B_o=34.6 kG and Z_2/A=.5. Likewise, the lower limits belong to 20 MeV/n cases. All other intermediate cases lie essentially within the band.

The same considerations apply to Fig. 6.

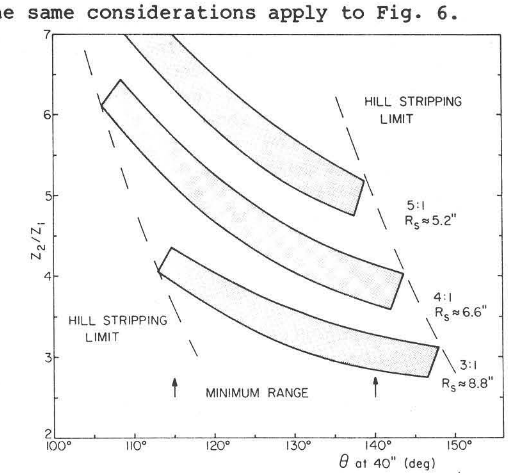

Fig. 6. Stripping ratios as a function of the entry azimuth at 40" radius, for different harmonic couplings.

As a result of this analysis we have been able to define a minimum range of entry azimuths at the radius of 40", as indicated in Fig. 6, which still allows coupling of the two cyclotrons for all required particles and energies. This minimum range corresponds to the dashed region of Fig. 2 and implies in turn a restricted range of stripping azimuths.

The injection solution thus emerging is characterized by a maximum angular width at 40" of about 25°. The corresponding azimuthal range of stripping azimuths is about 26°, i.e. considerably less than the hill width of 46°. All injection trajectories converge to a point 150" away from the cyclotron center, where a steering of ±1.5° will be sufficient to properly match all injection paths.

Phase Space Behaviour

Examples of radial and axial phase space behaviour of typical beams along the injection trajectory are shown in a self-explanatory way in Figs. 7 and 8. The calculations were made on the assumption of matching the accelerated beam eigenellipses of 6 mm·mrad emittance at the stripping radius. As expected, radial defocusing of the beam is produced by the cyclotron fringing field, thus requiring a radially focused beam at the entry point at 52" radius, while practically an axial waist is needed at the same point.

The same pattern of Figs. 7 and 8 is maintained throughout the operating range of fields and stripping ratios. A marked correlation showed up, however, between the extent of radial defocusing and the position of the injection trajectory with respect to the sectors. This is illustrated in Fig. 9, where the maximum x and p_x values at 52" radius, corresponding to the extreme points of the radial phase space figure, are plotted as a function of the stripping ratio. Since the case presented here refers to the trajectories of Fig. 2, it is easy to appreciate that the maximum defocusing occurs for those trajectories

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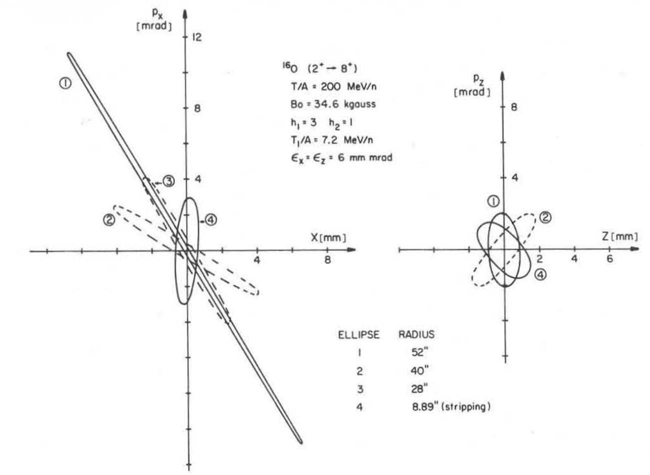

Fig. 7. Radial and axial phase space of an injected oxygen beam for a final energy of 200 MeV/n.

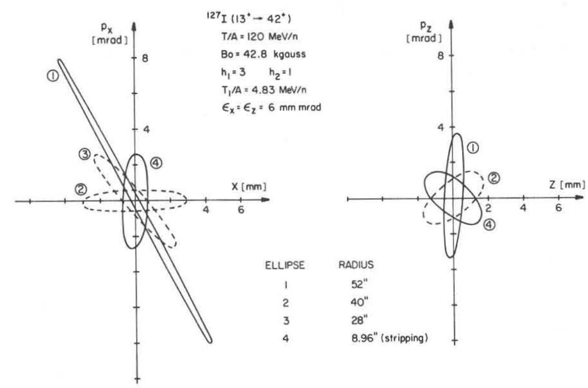

Fig. 8. Radial and axial phase space of an injected iodine beam for a final energy of 120 MeV/n.

which coast along the hill-valley boundary, thus experiencing a stronger, radially defocusing field gradient. The effect is even stronger in the case of a conventional spiral (dashed lines of Fig. 9) as examination of the trajectories of Fig. 1 would also lead to expect.

The corresponding analysis of the (z,p_z) phase space is presented for the same cases in Fig. 10. The relative behaviour of (z,p_z) for the double spiral reflects a constant axial waist situation at 52", as anticipated above.

In the conventional spiral, the counterpart of a large radial defocusing is instead a sharp axial focusing. Comparison of the radial and axial phase space pattern suggests therefore that the double spiral scheme should be preferred.

Since the effect on phase space is clearly dependent only upon the geometry of the injection trajectory relative to the sectors, and since the latter span the same space region for all harmonic coupling modes, one should expect two consequences:

- The same behaviour should be present for all HCR.
- The radial defocusing effect should be stronger the lower the field, since the local defocusing field index, as seen by the particles along the sector, is only dependent upon the field level.

Both these conclusions are supported by a number of calculations. This can be seen in a self-explanatory way in Fig. 11 for the first point and Fig. 12 for the second.

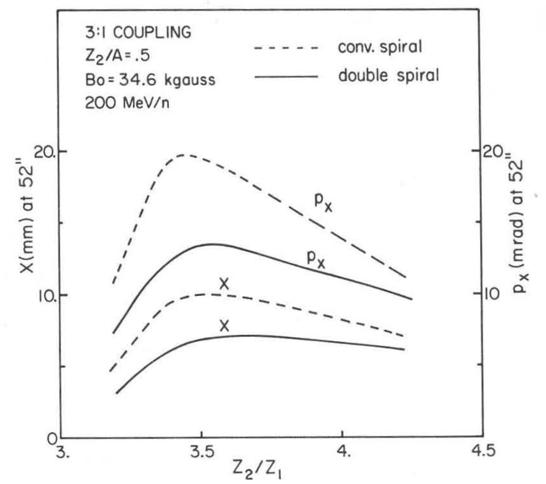

Fig. 9. Maximum (x,p_x) values at 52" radius for injected beams of 6 mm·mrad emittance (see text for details).

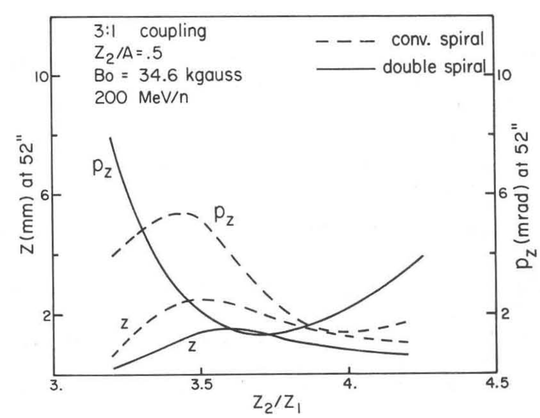

Fig. 10. Maximum (z,p_z) values at 52" radius for injected beams of 6 mm·mrad emittance (see text for details).

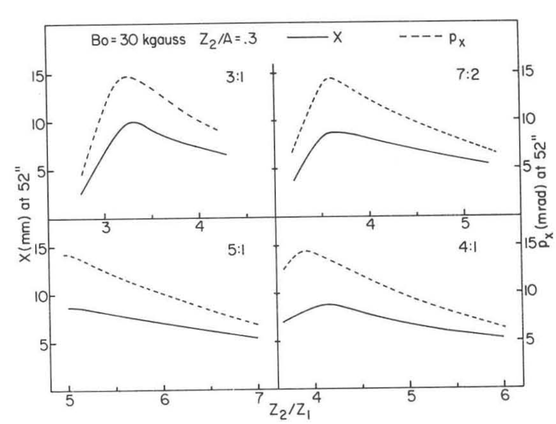

Fig. 11. Maximum (x,p_x) values at 52" radius for different HCR as a function of the stripping ratio (6 mm·mrad emittance).

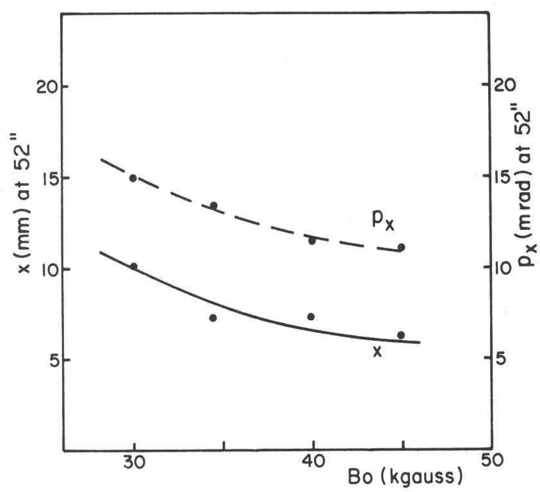

Fig. 12. Absolute maximum values of x and p_x at 52" radius as a function of the field level for a 6 mm·mrad emittance.

Conclusions

The analysis carried out so far indicates that the double spiral scheme has two main advantages over a conventional spiral:

- Hill stripping is possible for all harmonic coupling modes and all stripping ratios necessary for the proper coupling of the two cyclotrons.
- It presents a better phase space behaviour.

Admittedly, this scheme also has the disadvantage of a limited decrease in vertical focussing, albeit restricted to a narrow radial region, and of a somewhat more complex mechanical design for the pole tip. In our view, however, these should be considered as minor problems when compared to the advantages outlined above.

Phase space studies show that no extra focusing channel is needed inside either the yoke or the cryostat. In fact, if the matching quadrupoles can be positioned close enough to the cyclotron yoke, say 20" to 40", the results of phase space tracking indicate that quadrupole apertures of 4" are adequate for matching the required radial and axial beam shapes. This would of course involve a displacement of the quadrupoles, together with the central ray injection trajectory. This displacement being, however, limited to a few degrees, it should certainly be preferred to a movable focusing element inside the cyclotron.

No second order calculations have been carried out so far. This refers mostly to matching in the (E,t) subspace, where other effects may be present. Transit time spreads from the entry point at 52" to the stripping radius have, however, been computed for all beams investigated in this analysis. Maximum spreads between 2^o and 3^o were found, which should constitute no real problem for the cyclotron coupling.

Although more detailed calculations will undoubtedly be needed on these aspects of injection, the presently envisaged scheme seems to constitute a very realistic and practical one.

References

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[2] J.H. Ormrod et al., "Status of the Chalk River superconducting heavy ion cyclotron," Proceedings of the 1977 Particle Accelerator Conference, IEEE NS-24, (1977) 1093.

[3] E. Acerbi, G. Bellomo, C. de Martinis, and F. Resmini, "Injection studies for the proposed superconducting cyclotron at the University of Milan," Proceedings of the 1977 Particle Accelerator Conference, IEEE NS-24, (1977) 1112.