A METHOD FOR MINIMIZING TRIM COIL POWER REQUIREMENTS IN SUPERCONDUCTING CYCLOTRONS *

G. Bellomo and F. Resmini
Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48823

Introduction

A key issue in the design of superconducting cyclotrons is to obtain a properly isochronous field over the wide range of particles and energies usually demanded. In this respect it is useful to recall that in superconducting cyclotrons the main coils produce a large fraction of the total average field, and that the field produced by the saturated iron is very nearly constant over the 20-50 kG range usually employed.

The main problem is therefore to optimize the main coil design, and to optimize the field generated by the iron, in order to minimize the total power requirements for trim coils. Detailed studies for the proposed K=800 machine at MSU, which requires isochronous fields for ion energies spanning from 5 to 200 MeV/n, led us to develop a method which effectively helps in solving the problem. In the process, we found some results which seem to have an overall validity and therefore have some implications for superconducting cyclotron design. It is the purpose of this paper to review in some detail the procedure used and the main consequences pertaining to the K=800 design study.

Outline of the Method

We assume the main coils to be split into two vertical sections, subscript α referring to the lower section (closer to the median plane) and subscript β to the upper one. The following notation will be used throughout:

- $F_\alpha(r)$, $F_\beta(r)$, $F_k(r)$ = form factors of the main coil sections and of the kth trim coil (with resistance R_k, N coils total).
- I_α, I_β, I_k = corresponding currents
- $B_{iron}(r)$, $B_{is}(r)$ = magnetic field generated by the iron configuration and isochronous field required by a given ion
- $\varepsilon(r)$ = field error, with respect to isochronism.

We shall then write, for any ion:

$$B_{is}(r) = B_{iron}(r) + F_\alpha(r)I_\alpha + F_\beta(r)I_\beta + \sum_{K=1}^{N} F_k(r)I_K + \varepsilon(r) \quad (1)$$

It is obvious that given $B_{iron}(r)$, $F_\alpha(r)$, $F_\beta(r)$, and $F_k(r)$, ordinary least squares fitting will yield the I_α, I_β, and I_k values which minimize $\sum_r \varepsilon^2(r)$.

It is also obvious that for any particular ion, and whatever the functions $F_\alpha(r)$ and $F_\beta(r)$ are, there will always be a function $B_{iron}(r)$ for which the currents I_k and the errors $\varepsilon(r)$ in that particular case equal zero.

The problem to which we address ourselves can instead be schematized as follows:

- Given a main coil, i.e. $F_\alpha(r)$ and $F_\beta(r)$, and trim coil form factors $F_k(r)$, determine, if it exists, a $B_{iron}(r)$ which minimizes: $\sum \varepsilon^2(r)$ and $P = \sum_{K=1}^{N} R_k I_k^2$ (total trim coil power) over the desired range of ions and energies.
- As a consequence, optimize the main coil design, with the aim to minimize the total trim coil power.

Our method is based upon the selection of two appropriate particles which, for the present purpose, can be thought of as representative of the extreme isochronous fields required, like the least and most relativistic particles in the operating range of the cyclotron.

If we write eq. (1) for these two ions, represented by subscripts 1 and 2 on the relevant quantities, and then subtract the two equations, we get:

$$[B_{1is}(r)-B_{2is}(r)] = [B_{1iron}(r)-B_{2iron}(r)] + F_\alpha(r)[I_{1\alpha}-I_{2\alpha}] + F_\beta(r)[I_{1\beta}-I_{2\beta}] + \sum_{K=1}^{N} F_k(r)[I_{1k}-I_{2k}] + [\varepsilon_1(r)-\varepsilon_2(r)] \quad (2)$$

with self-explanatory notation.

Let us now make the simplifying hypothesis:

$$B_{1iron}(r) - B_{2iron}(r) \equiv 0 \quad (3)$$

for every radius.

This physically corresponds to an invariable field produced by the saturated iron configuration. Although not exactly fulfilled in reality, it nevertheless is an accurate enough hypothesis for the present purposes. We shall anyhow examine later the consequences of it not being strictly verified.

Equation (2) can then be written as

$$\Delta B_{is}(r) = F_\alpha(r)\Delta I_\alpha + F_\beta(r)\Delta I_\beta + \sum_{K=1}^{N} F_k(r)\Delta I_k + \Delta\varepsilon(r) \quad (4)$$

where the symbol Δ represents the differences in eq.(2).

Having eliminated $B_{iron}(r)$, equation (4) lends itself to a minimization, with an ordinary least squares fitting procedure, of the quantity: $\sum_r[\Delta\varepsilon(r)]^2$, thus obtaining values of:

$$\Delta I_\alpha = I_{1\alpha} - I_{2\alpha} \quad (4a)$$
$$\Delta I_\beta = I_{1\beta} - I_{2\beta}$$
$$\Delta I_k = I_{1k} - I_{2k}$$

One can prove[1] that if $\sum_r[\Delta\varepsilon(r)]^2$ is minimized with a least squares fitting, then also

$$\sum_r \varepsilon_1(r)^2 + \sum_r \varepsilon_2(r)^2 \quad \text{and} \quad P_1 + P_2$$

will be minimized, if

- $\varepsilon_1(r) + \varepsilon_2(r) = 0$ for every radius (5)
- $I_{1k} + I_{2k} = 0$ for every trim coil

This finding has the following consequences:

- Given two ions, and for given main coils and trim coils, there exists an absolute minimum of the trim coil power needed for providing an isochronous field in the two cases. This minimum value is obtained by least squares fitting of eq. (4), imposing thereafter condition (5). This power is thus equal for both ions, and corresponds to equal and opposite currents for each trim coil.
- To reach the condition of absolute minimum power the pole tips must produce a $B_{iron}(r)$ which satisfies for every radius the following equation:

*This material is based upon work supported by the National Science Foundation under Grant No. Phy 78-01684.

$$B_{1is}(r) + B_{2is}(r) = 2\,B_{iron}(r) + F_\alpha(r)[I_{1\alpha} + I_{2\alpha}] + F_\beta(r)[I_{1\beta} + I_{2\beta}] \qquad (6)$$

where:

- the terms of eq. (1) have been summed, rather than subtracted, for the two ions
- $B_{iron}(r) = B_{1iron}(r) = B_{2iron}(r)$ according to (3)
- the errors $\varepsilon(r)$ and the trim coil currents I_k cancel out according to (5)

In eq. (6) there are four unknowns, namely $I_{1\alpha}$, $I_{2\alpha}$, $I_{1\beta}$, and $I_{2\beta}$, while, according to (4a), ΔI_α and ΔI_β are known via least squares fitting. As a consequence, there are ∞^2 $B_{iron}(r)$ functions which yield an absolute minimum for trim coil powers. However, if one specifies in eq. (6) the $B_{iron}(r)$ values at two conveniently chosen radii R_1 and R_2, one has a set of four linear equations, which will yield $I_{1\alpha}$, $I_{1\beta}$, $I_{2\alpha}$, and $I_{2\beta}$. Henceforth the values at all radii of the ideal $B_{iron}(r)$, which assumes at R_1 and R_2 the specified values, can be calculated through eq. (6).

It will also be recognized that whatever values of $B_{iron}(r)$ are chosen at R_1 and R_2, and whatever the radii R_1 and R_2, this will only affect the partial currents I_α and I_β and not the trim coil power required for either case.

Given the "ideal" $B_{iron}(r)$ function thus calculated, and which shall henceforth be denoted by $B_{best}(r)$, one can readily calculate the trim coil powers needed for any other ion, by least squares fitting through eq. (1). One should further note that the resulting trim coil power, the trim coil contributions, and the field errors will in any case be independent[1] of the particular $B_{best}(r)$ function chosen.

In summary, the method entails the following steps:

1. Choose two ions which represent extremes in field isochronism requirements.
2. Carry out the least squares fitting over all radii, according to (4), and determine the minimum power required using condition (5).
3. Determine a convenient $B_{best}(r)$ function. In this respect, it is obvious that radii R_1 and R_2 shall typically be chosen close to the innermost and outermost radii of the cyclotron.
4. Map the trim coil power requirements for all other ions.

At this point we note that if a generic $B_{iron}(r)$ function is chosen, instead of the $B_{best}(r)$ determined by the above procedure, the following consequences will arise for the two representative ions:

- The sum of the powers, P_1 and P_2 (which will not be equal anymore) will increase by the amount $\Delta P = 2\sum_k R_k(\Delta I'_k)^2$
- The sum of the errors, $\sum_r \varepsilon_1^2(r)$ and $\sum_r \varepsilon_2^2(r)$, will increase by $\Delta\varepsilon = 2\sum_r(\Delta\varepsilon'(r))^2$

  where $\Delta I'_k$ and $\Delta\varepsilon'(r)$ are given by a least squares fitting at all radii of:

$$\Delta B_{iron}(r) = F_\alpha \Delta I'_\alpha + F_\beta \Delta I'_\beta + \Sigma_k F_k \Delta I'_k + \Delta\varepsilon'(r) \qquad (7)$$

$\Delta B_{iron}(r)$ representing the error: $B_{best}(r) - B_{iron}(r)$.

Results for the K=800 Design

The following two ions have been chosen as extremes:

- A fully stripped light ion, Z/A=.5, with center field of 34.6 kG, corresponding to a final energy of 200 MeV/n.
- A uranium ion, charge state 38^+, Z/A=.16, center field of 40 kG, corresponding to 20 MeV/n. This is the least relativistic particle, at the maximum field, which can be accelerated in 1st harmonic, the latter mode covering about 80% of the machine operating range.

Although final results suggest that different choices may be more appropriate, nevertheless this is a very realistic starting point. Isochronous fields are computed on the basis of the field modulation produced by the sectors. Fits, as required by eqs. (1) and (4), are made in .5" radial steps at all radii between 4" and 39.5". The latter value, which should be viewed against a pole radius of 41",[2] is then the last radius at which perfect isochronism is required, and has been chosen after a careful analysis. Comparison of fits with different final radii, from 38" to 40.5", indicated that 39.5" would be the upper limit compatible with: i) realistic phase behaviour in the fringing region, ii) fringing field generated by iron and coils for extraction at approximately 40.4". All the trends reported here retain however their validity over the range of ultimate fitting radii quoted above.

Twenty-two trim coils are used for the K=800, as described in Ref. 2. Main coils are spaced by 3" around the median plane and a maximum current density of 3500 A/cm^2 is allowed. Coil inner radius is 45.5" with radial width of 6". As stated above, each coil is split into two sections, α and β, and we shall use the following notation:

$h_\alpha + h_\beta = H$ coil height

$h_\alpha/H = F$ partition fraction, referred to the section closer to the median plane. The two sections are separated by 0.5" axially.

The minimum trim coil power required for either representative ion, and derived from the fits according to eq. 4, is plotted in Fig. 1 as a function of the coil height and for four different partitions F. Variation of the coil radial width by 1" around 6" produces the same results. This is physically intuitive since the coils' form factors, F_α and F_β, do not change appreciably, the height to width ratio being rather large for all cases. The curves of Fig. 1 point out that:

- For the same coil height, i.e. constant ampereturns, the larger the partition F, the lower is the power.
- For the same partition the power decreases by increasing the coil height.

What this means in terms of actual coil operation can be readily seen if, according to eq. 5-6 and the procedure outlined above, actual $B_{iron}(r)$ values are introduced, thus obtaining the currents I_α, I_β for either ion. Values of 19.27 kG and 16.12 kG were chosen for the radii of 10" and 38", as computed for the iron configuration. This allows us to construct a diagram like the one shown in Fig. 2 for the 200 MeV/n ion. The current density needed in the upper coil, J_β, is plotted as a function of the coil height, and for

0018-9499/79/0400-2096$00.75 © 1979 IEEE

the same four partitions F. Corresponding J_α values can be read off the scale on the right. Also shown are lines of constant trim coil powers.

Several features emerge from Fig. 2, namely:

- For the same coil height, the decrease of the trim coil power for increasing partition fractions F is obtained by running the upper coil to progressively lower currents, and ultimately reversing the sign. Clearly, F values larger than 0.6 are not practical because one would rapidly approach the J_β=-3500 A/cm^2 limit.
- Reduction of the power along an F=constant line, accomplished by increasing the coil height, means also progressively larger negative values of J_β, and lower J_α.
- Optimal coil design implies therefore a compromise between: i) minimum trim coil power desired, ii) practical limit on how large a negative current can be tolerated in the upper coil because of stresses, etc., and iii) cost of progressively larger main coils.

For the K=800, we choose a total height of 26.5" and a partition fraction of .6, thus leading to minimum negative $J_\beta \simeq -1000$ A/cm^2 and power requirements of about 36 kW for the two extreme ions.

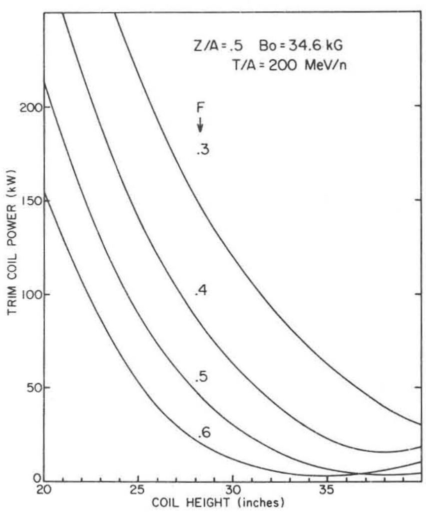

Fig. 1. Minimum trim coil power for 200 MeV/n.

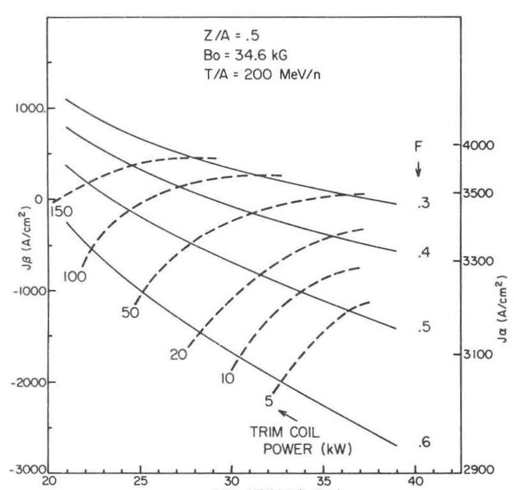

Fig. 2. Current density needed in the main coils, as a function of coil height, for 200 MeV/n.

Mapping of the powers throughout the operating range is done, as explained above, by ordinary least squares fitting once the function $B_{best}(r)$ is calculated. For these coils, and on the basis of the two ions chosen, this field is presented in Fig. 3 and compared with the actual field produced by the iron configuration. The most interesting feature is the increase of about 180 Gauss required for the ideal field around the radii 39"-40". Calculations with local corrections of the hill profile show that there is no problem in matching the desired field. We also recall that, according to eq. (7), failure to provide the required field shape would enhance the power from 36 kW to about 80 kW for the 200 MeV/n ion.

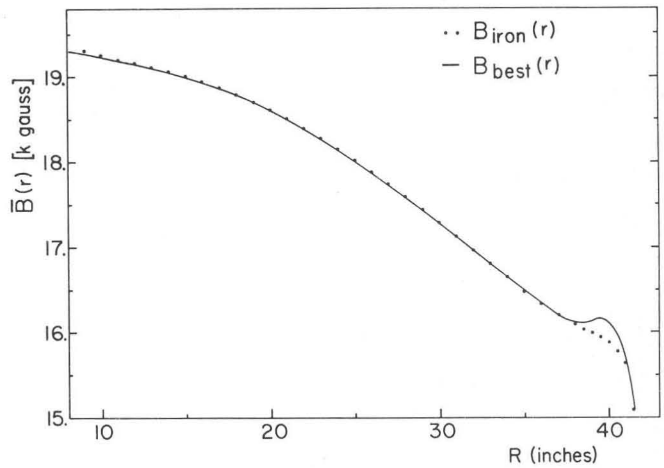

Fig. 3. $B_{best}(r)$ and $B_{iron}(r)$ (see text for details).

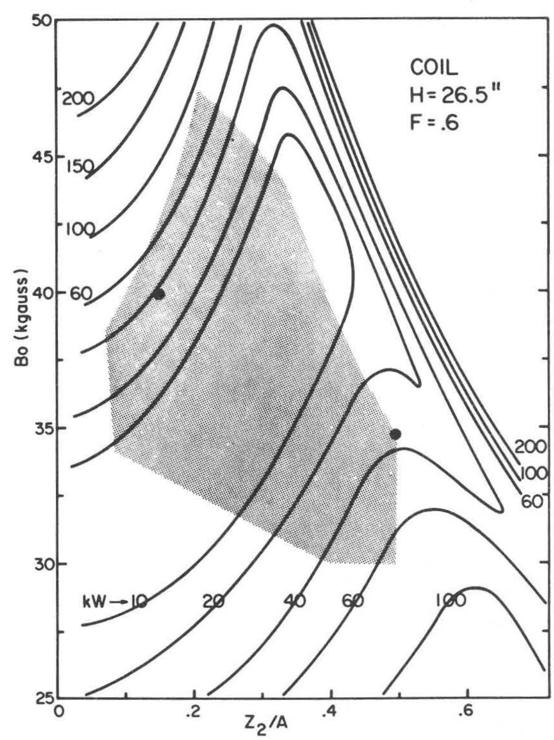

Fig. 4. Trim coil power contours for the K=800 operating range. Coil height=26.5", partition F= .6

Contours of constant trim coil power obtained by using the calculated $B_{best}(r)$ are presented in Fig. 4, in the (Bo, Z/A) plane, with reference to the operating diagram of the machine.[2] This is also done for comparison for two other coils, namely a coil of the same height but partition fraction equal to 0.4, Fig. 5,

0018-9499/79/0400-2097$00.75

and a coil of larger height, 30", and the same partition of .6, in Fig. 6. The positions of these coils on the diagrams of Figs. 1 and 2 can be easily recognized. Needless to say, the $B_{best}(r)$ function for each coil is slightly different, although it still assumes the values of 19.27 and 16.12 kG at 10" and 38" radius respectively, and is calculated on the basis of the same two representative ions. All the fits reported here turned out to be of remarkable quality, with departures from the ideal isochronous field confined to a few Gauss at most. This is obviously to be ascribed to the use of a theoretical $B_{best}(r)$, and therefore shows the importance of approximating the above function as closely as possible in the real field.

Comparison of the three figures shows that the power requirements for the different coils follow throughout the operating region the same pattern apparent from Figs. 1 and 2, and therefore that the procedure established on the basis of just two representative ions allows a meaningful optimization of the main coil design.

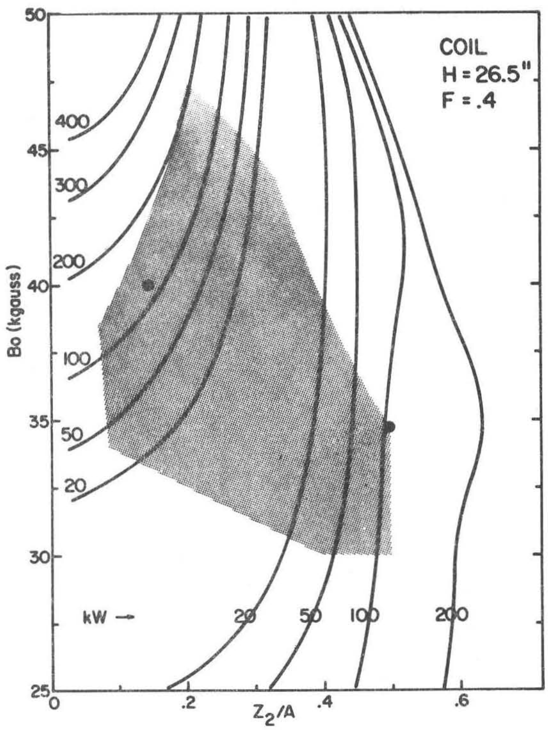

Fig. 5. Trim coil powers contours for the K=800 operating range. Coil height =26.5", partition F=.4.

For a closer analysis of the cases pertaining to our optimized coil design, the contours of constant current densities J_α and J_β are shown, again in the (Bo, Z/A) plane, in Fig. 7. The margins with respect to the 3500 A/cm^2 limit and the need to have J_β negative over a part of the operating range can be appreciated.

Typical field corrections given by the trim coils are presented in Fig. 8, both for the two representative ions and for other cases corresponding to lower and higher powers according to the coils and diagrams of Fig. 4. As anticipated, the field corrections have opposite signs and the behaviour as a function of radius is very much the same, different power levels being mostly characterized by different amplitudes of the field correction.

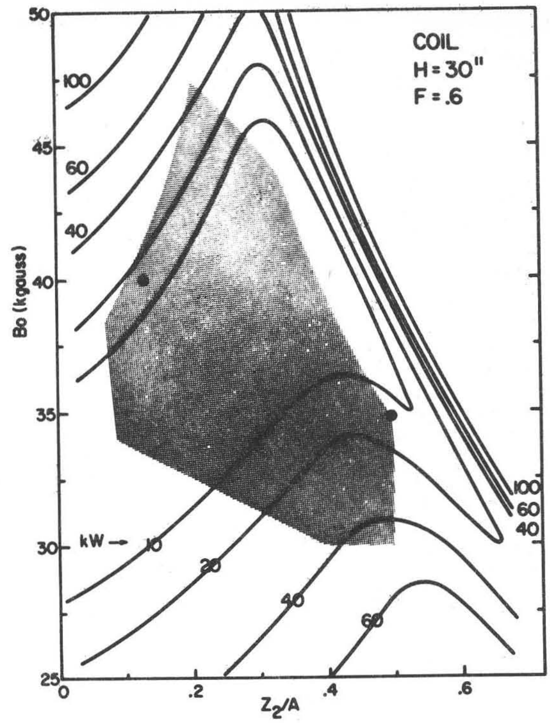

Fig. 6. Trim coil power contours for the K=800 operating range. Coil height=30", partition F=.6.

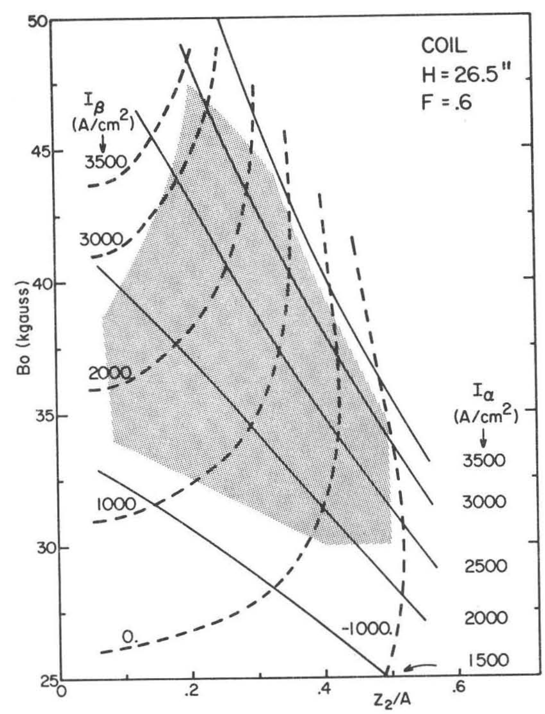

Fig. 7. Constant J_α and J_β contours for the K=800 operating range. Coil height=26.5", partition F=.6.

Trim coil currents for the three cases at the top of Fig. 8 are listed in Table I. A very smooth behaviour of the currents is observed, both in value and sign. Opposite signs hold, of course, for the cases at the bottom of Fig. 8.

Table I. Trim coil currents.

T. Coil No.	R_k (Ω)	I_k (A) $\Sigma P_k \simeq 10$ kW	$\Sigma P_k \simeq 36$ kW	$\Sigma P_k \simeq 55$ kW
1	.026	-27	-85.	-89
2	.031	-42	-127	-135
3	.035	-54	-161	-173
4	.039	-66	-191	-207
5	.043	-76	-216	-235
6	.047	-84	-234	-259
7	.051	-94	-246	-281
8	.056	-97	-250	-285
9	.060	-96	-246	-278
10	.064	-94	-234	-267
11	.068	-90	-213	-247
12	.072	-83	-185	-219
13	.076	-73	-150	-183
14	.080	-61	-108	-139
15	.085	-45	-61	-89
16	.089	-25	-12	-33
17	.093	-2	+37	+27
18	.097	+26	+82	+86
19	.101	+55	+115	+139
20	.105	+87	+133	+181
21	.109	+111	+120	+192
22	.114	+151	+93	+202

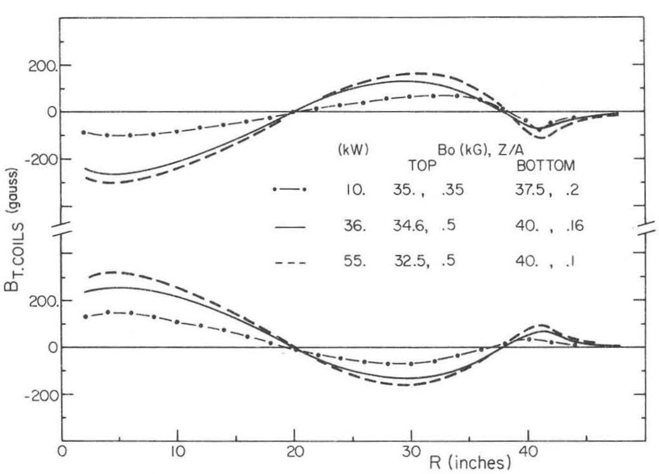

Fig. 8. Trim coil field contributions for different power levels and ions.

The peculiar behaviour of the equipower contours suggests that:

- The representative ions for defining the power limits should really be the ion with maximum Z/A at the minimum field, and that with minimum Z/A at the maximum field.
- Different choices might lead to very misleading results, or in other words to an equipower contour diagram not properly centered over the machine operating range.

As an example, Fig. 9 presents the contours which would be obtained should one choose as representative ions the same most relativistic particle (200 MeV/n, Z/A=.5, Bo=34.6 kG) and the least relativistic particle at the minimum field, i.e. 4 MeV/n, Z/A=.08, Bo=34 kG. It is quite evident that with respect to the overall operating diagram there is a definite unbalance of the required trim coil powers.

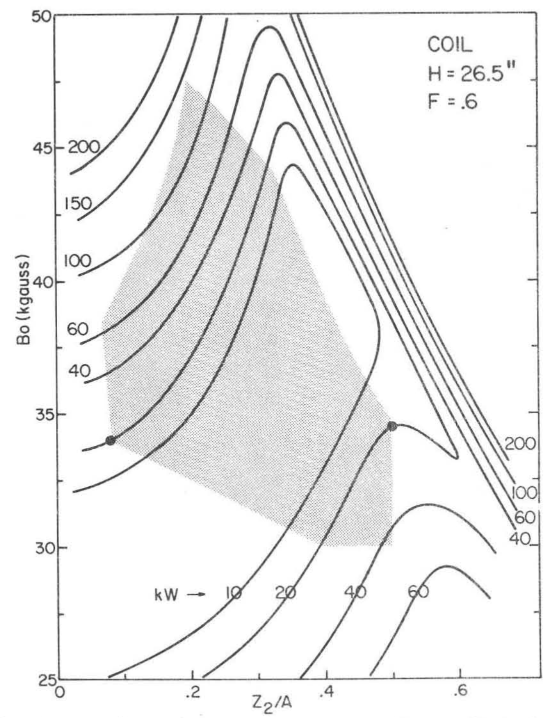

Fig. 9. Trim coil power contours for coil height = 26.5", F=.6. The two new representative ions chosen are marked on the figure.

Further Considerations

All calculations reported here are consistent with the hypothesis of a function $B_{best}(r)=B_{iron}(r)$ independent of the field level. It is therefore appropriate to ask which variations can be expected in reality, when the $B_{iron}(r)$ will vary both in level and shape depending upon coil excitation.

So far we have carried out only a limited number of calculations. In Fig. 10 we present the iron fields which are expected at various coil excitations. Curve (1) corresponds to the B_{best} used so far (see Fig. 3) and therefore to the coil excitation needed for Z/A=.5, Bo=34.6 kG. Curve (2) shows the expected B_{iron} field for the coil excitation needed for the other representative ion, i.e. U^{38+} at Bo=40 kG.

The field differences between curves (2) and (1) are then plotted in Fig. 11, curve (a). Fit for the uranium ion in the sense of Eq. (1), using the B_{iron} of curve (2), yields a lower trim coil power i.e 22 kW instead of the original 36 kW. This is reflected in a smaller field contribution of the trim coils, curve (d) in Fig. 11, with respect to the original one, curve (c). Analysis of this unexpected result shows that, due to the level variation between the two iron fields, there is also a difference in the field produced by the main coils as shown in curve (b) Fig. 11. Since the difference between curves (a) and (b) has the same behaviour as the field correction produced by the trim coils, the consequence is the observed reduction in power.

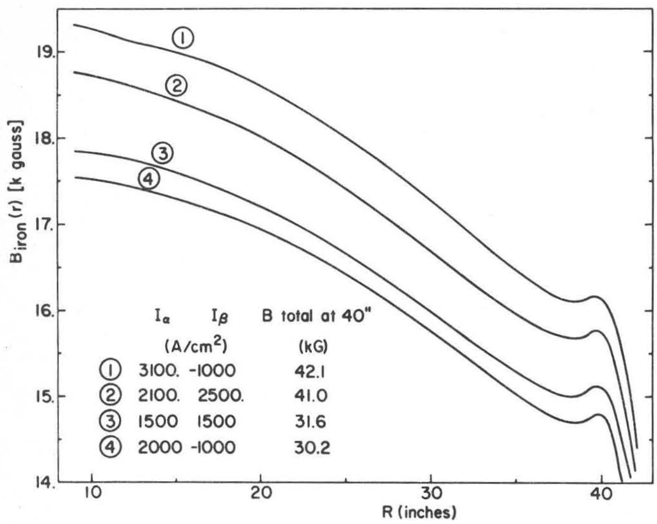

Fig. 10. Calculated B_{best}(r) variations for the indicated coil excitations.

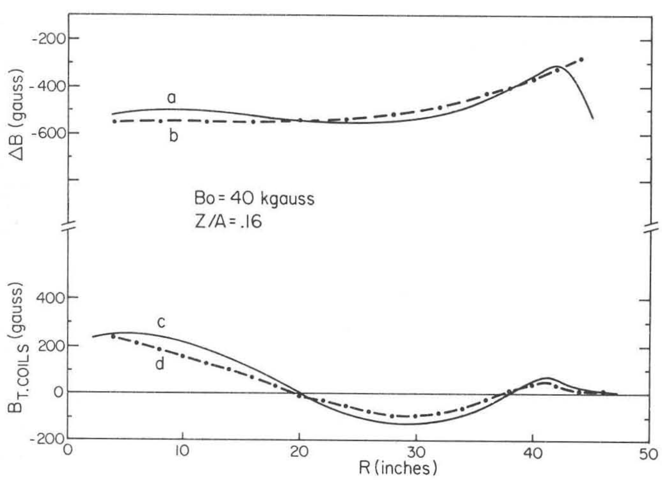

Fig. 11. B_{iron} differences (a), main coil field differences (b), and trim coil contributions (c,d) when the iron field is computed according to the appropriate coil excitation level (see text for details).

For the few cases investigated so far, power reductions of the order of up to 25%-30% have equally been observed. A more detailed analysis throughout the operating range of the machine will be needed to establish an overall validity of the results quoted here. In other words, since it seems at present that the iron field shape and level variation goes in the direction of further minimizing the necessary trim coil power, one should check whether this is actually true at all coil excitations.

We also wish to stress that given the quality of the fits (the resulting phase being very close to zero for most of the accelerating cycle), it is possible to relax somewhat on the power requirements by allowing departures from isochronism. This can be done by least squares fitting, setting a constraint on the maximum total trim coil power. Results of this technique indicate that further power reductions of 10%-20% can be tolerated without substantial departures from isochronism for the least relativistic particles.

Conclusions

These results indicate that optimization of superconducting cyclotron design with the aim of minimizing trim coil power is indeed possible with the method developed here.

Preliminary calculations indicate that the preference for high partition fractions of the main coils, i.e. 0.5 or 0.6, proved here for the K=800, retains its validity also for machines with different radius and, therefore, energies.

It is also clear that the desired operating range of the cyclotron in terms of (Bo, Z/A) should be known in considerable detail in order to properly optimize the design.

A major advantage of the present method is that it allows calculation of the B_{iron}(r) function required to produce the absolute minimum of trim coil power. Of course, every effort should then be made to effectively obtain this field in the cyclotron, since any error will reflect itself in a power increase.

References

[1] G. Bellomo, F. Resmini, to be published.

[2] F. Resmini, G. Bellomo, E. Fabrici, H.G. Blosser, and D.A. Johnson, "Design characteristics of the K=800 superconducting cyclotron at MSU," paper at this Conference.

0018-9499/79/0400-2100$00.75 © 1979 IEEE