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PHASE-SPACE MATCHING BETWEEN PRE-ACCELERATORS AND CYCLOTRONS

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Abstract.- The properties of the six-dimensional eigenellipsoid of the circulating pulses on a cyclotron orbit are derived. Design features for beam line systems are presented which match a pre-accelerated beam of ions to this phase space. Possibilities are discussed for optimizing the setting parameters.

1. Introduction.- In a cyclotron with internal source the beam normally is accelerated with a rather broad phase width. Although it is possible to use phase selecting slits, in general only multiturn extraction out of the cyclotron can be achieved - with a rather moderate beam quality.

When injecting a pre-accelerated beam into a booster cyclotron, it is possible to match the beam in such a way that it fills an optimal region of the phase space acceptance of the cyclotron, resulting in a high beam quality with well separated turns during acceleration and single turn extraction. In this article first the phase space conditions for optimal acceleration in the cyclotron are derived. Afterwards we ask for criterions for beam line systems between the pre-accelerator and the cyclotron which achieve these conditions. Only radial injection is considered. In the third part we look for possibilities for checking the correct phase space matching.

2. Phase space conditions for acceleration in a booster-cyclotron.- When injecting a pre-accelerated beam of ions into a cyclotron, we must define a location along the beam path where we consider the injection process as finished and the acceleration in the cyclotron just starting. At this breakpoint the phase space of the particles should be matched properly. It is reasonable to use for that a symmetry point of the cyclotron orbit behind the last injection element (e.g. one of the first mid-valley points after injection).
The central trajectory of the beam at this breakpoint is defined by a particle on the static equilibrium orbit with the central momentum p_o, just passing the symmetry point*). In vertical direction it is positioned in the midplane of the cyclotron. Particles deviating from the central trajectory are described by the six-dimensional vector

$$\vec{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x \\ \theta \\ y \\ \varphi \\ z \\ \delta \end{bmatrix} \begin{array}{l} \text{radial (horizontal) displacement} \\ \text{radial (") angle} \\ \text{vertical displacement} \\ \text{vertical angle} \\ \text{longitudinal displacement} \\ \text{longitudinal angle = longitudinal momentum deviation } \dfrac{\Delta p_z}{p_o} \end{array} \tag{1}$$

The associated coordinate system is right-handed rectangular with its origin coinciding with the central particle. We choose clockwise motion in the cyclotron, so that the positive x-axis points radially outwards, the positive y-axis upwards, and the z-axis has the direction of the central momentum p_o. This is illustrated in Fig. 1.

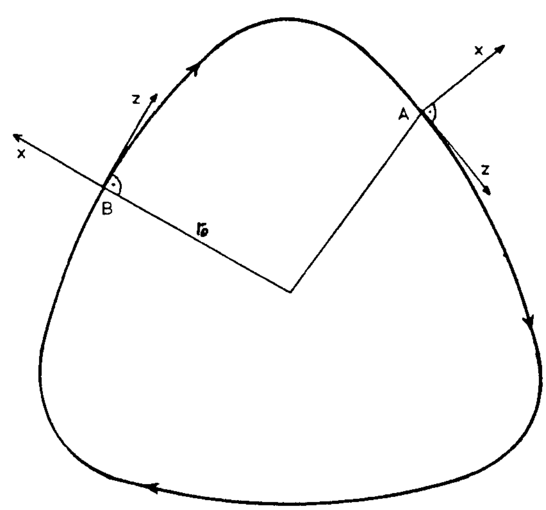

Fig. 1 : The coordinate system for the particle motion around a static equilibrium orbit. At point B the x-axis is in line with the radius which is referred to being a symmetry point.

*) In the limit of small turn separation compared with the injection radius the matching conditions for the accelerated orbits are practically the same as those for the closed static equilibrium orbit.

With this convention we adopt the description of TRANSPORT[1], which will be used also for the beam line system between the pre-accelerator and the cyclotron.
We now want to look for the correlations between the six coordinates of any particle in the beam inside an isochronous cyclotron. We consider only linear correlations, since the size of the beam is normally very small compared to the orbit radius.

For particles oscillating around the static equilibrium orbit the cyclotron is just a magnetic beam line system with midplane symmetry. The general form of the transformation matrix for such a system is given by

$$R = \begin{bmatrix} r_{11} & r_{12} & 0 & 0 & 0 & r_{16} \\ r_{21} & r_{22} & 0 & 0 & 0 & r_{26} \\ 0 & 0 & r_{33} & r_{34} & 0 & 0 \\ 0 & 0 & r_{34} & r_{44} & 0 & 0 \\ r_{51} & r_{52} & 0 & 0 & 1 & r_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

and the particle vectors are transformed through the system

by $$\vec{X}\,(1) = R \cdot \vec{X}\,(0) \qquad (3)$$

According to Liouville's theorem the determinant of the total matrix R equals one. Caused by the midplane symmetry and conservation of momentum in magnetic fields the determinants of the horizontal and vertical submatrices are also one:

$$\begin{aligned} \det\ R &= 1 \\ r_{11}\, r_{22} - r_{12}\, r_{21} &= 1 \\ r_{33}\, r_{44} - r_{34}\, r_{43} &= 1 \end{aligned} \qquad (4)$$

In addition there exist two correlations between the path-length elements r_{51} and r_{52} and the dispersive elements r_{16} and r_{26}:

$$\begin{aligned} r_{51} &= r_{21} \cdot r_{16} - r_{11} \cdot r_{26} \\ r_{52} &= r_{22} \cdot r_{16} - r_{12} \cdot r_{26} \end{aligned} \qquad (5)$$

or, resolved for r_{16} and r_{26}, with help of the eq. (4):

$$\begin{aligned} r_{16} &= -r_{12} \cdot r_{51} + r_{11} \cdot r_{52} \\ r_{26} &= -r_{22} \cdot r_{51} + r_{21} \cdot r_{52} \end{aligned} \qquad (5')$$

Equation (5) can be found when looking for the integral form of the matrix elements[2]. We now consider R being the transfer-matrix for one orbit, starting at a symmetry point of the cyclotron. If the symmetry of the cyclotron is N, there are 2N such symmetry points on the orbit where the radius vector is rectangular to the momentum of the central particle, and the x-axis of the associated coordinate system is in line with the radius. In case the cyclotron has no spiral angle (straight sector cyclotron), one total orbit can be interpreted as the path through a magnetic system for half an orbit, followed by the mirror reflected system. As a consequence of this symmetry property we get for zero spiral angle the additional relation[3]:

$$r_{11} = r_{22} \quad \text{and} \quad r_{33} = r_{44} \qquad (6)$$

Let us now use the fact that we have an isochronous orbit. The radius of the central particle at the azimuth of the choosen symmetry point is r_o, its momentum p_o. For a particle with a slightly higher momentum

$$p = p_o\,(1 + \frac{\Delta p}{p_o}) = p_o\,(1+\delta) \qquad (7)$$

there exists also a closed orbit (static equilibrium orbit) at the radius

$$\begin{aligned} r &= r_o + p_o \cdot \frac{\partial r}{\partial p}\Big|_{p_o} \cdot \frac{\Delta p}{p_o} \\ &= r_o + \tilde{r} \cdot \delta \end{aligned} \qquad (8)$$

with the reduced radius*)

$$\tilde{r} = r_o \cdot \frac{\gamma}{\gamma + \beta^2 \gamma^3} \qquad (9)$$

$$\beta = \frac{v(r_o)}{c} \;;\quad \gamma = (1 - \beta^2)^{-1/2}$$

This particle, deviating from the central particle only in its momentum and radial

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*) This equation can be derived using the relations:

$$\tilde{r} := p_o \cdot \frac{\partial r}{\partial p}\Big|_{p_o}$$

$$\frac{\partial r}{\partial p} = \frac{\partial r}{\partial v} \Big/ \frac{\partial p}{\partial v} \;;\; \frac{\partial r}{\partial v} = \frac{r}{v}$$ in an isochronous field.

$$\frac{\partial p}{\partial v} = \frac{\partial}{\partial v}\,(m_o \cdot \gamma(v) \cdot v) = m_o\,(\gamma + v \cdot \frac{\partial \gamma}{\partial v})$$

$$v \cdot \frac{\partial \gamma}{\partial v} = \beta \cdot \frac{\partial \gamma}{\partial \beta} = \beta^2 \cdot \gamma^3$$

position coordinates, must be transfered identically by the matrix R:

$$R \cdot \begin{bmatrix} \tilde{r}\,\delta \\ 0 \\ 0 \\ 0 \\ 0 \\ \delta \end{bmatrix} = \begin{bmatrix} \tilde{r}\,\delta \\ 0 \\ 0 \\ 0 \\ 0 \\ \delta \end{bmatrix} \tag{10}$$

The coordinate $x_5 = z$ remains zero because the orbit shall be isochronous. Inserting eq (2) and using the relations (4) and (5) we get the general form of the transfer-matrix for an isochronous cyclotron orbit (now r_{ij} replaced by c_{ij}):*)

$$C = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 & \tilde{r}(1-c_{11}) \\ c_{21} & c_{22} & 0 & 0 & 0 & -\tilde{r}c_{21} \\ 0 & 0 & c_{33} & c_{34} & 0 & 0 \\ 0 & 0 & c_{34} & c_{44} & 0 & 0 \\ \tilde{r}c_{21} & -\tilde{r}(1-c_{22}) & 0 & 0 & 1 & -\tilde{r}^2\, c_{21} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

with $c_{11}\, c_{22} - c_{12}\, c_{21} = 1$ and $c_{33}\, c_{44} - c_{34}\, c_{43} = 1$

For zero spiral angle we have in addition

$$c_{11} = c_{22} \quad \text{and} \quad c_{33} = c_{44} \tag{12}$$

2.1. The uncoupled subspaces

There are no elements in the matrix C which couple the vertical coordinates $x_3 = y$ and $x_4 = \varphi$ to any other element of the particle vector. This means that the vertical motion of the particles is (to first order) independent from the horizontal and the longitudinal motion. Let us for the moment only consider particles with the central momentum p_0 ($\delta = 0$). Then also the horizontal motion is governed by its own submatrix c_{ij}, $i = 1,2;\ j = 1,2$ (although the longitudinal coordinate z is influenced by horizontal matrix elements!)

If the orbits around the equilibrium orbit are stable, then the absolute values of the traces of the horizontal and vertical submatrices are less than two, the betatron - oscillation frequencies are real and can be calculated from the matrix elements:

$$\nu_x = n + \mathrm{sign}(c_{12})\cdot\frac{1}{2\pi}\cdot\mathrm{arc\,cos}\left\{\frac{1}{2}\cdot(c_{11} + c_{22})\right\} \tag{13}$$

$$\nu_y = m + \mathrm{sign}(c_{34})\cdot\frac{1}{2\pi}\cdot\mathrm{arc\,cos}\left\{\frac{1}{2}\cdot(c_{33} + c_{44})\right\} \tag{14}$$

The frequencies are determined only up to the integer numbers n and m.

The matrix C tells us how single particles around the equilibrium orbit are transformed. The phase space of a whole bundle of particles forming one circulating pulse in the cyclotron may well be described as a volume confined in a six-dimensional ellipsoid. Vectors $\vec{X}$ for particles on the surface of this ellipsoid are given by the equation[1)]

$$\vec{X}^T \cdot \sigma^{-1} \cdot \vec{X} = 1 \tag{15}$$

where σ is a 6 x 6 matrix describing the beam. The diagonal element σ_{ii} is the square of the maximum extension (envelope) of the beam in the coordinate x_i:

$$\sigma_{ii} = (\pm\, x_i(\max))^2 \tag{16}$$

and the off-diagonal element $\sigma_{ij} = \sigma_{ji},\ i \neq j$ represents the tilt of the phase ellipse which one gets as a projection of the ellipsoid on the $(x_i - x_j)$ plane. $\sigma_{ij} = \sigma_{ji} = 0$ represents an upright ellipse (waist) in the $(x_i - x_j)$ plane, with the half-axes $\sqrt{\sigma_{ii}}$ and $\sqrt{\sigma_{jj}}$.

In a beam line system represented by the transfer matrix R an initial beam given by the matrix $\sigma(0)$ is transformed by[1)]

$$\sigma(1) = R \cdot \sigma(0) \cdot R^T \tag{17}$$

The injected beam pulses are matched properly if their occupied phase space is the same after one revolution in the isochronous cyclotron field.

$$\sigma(1) = C \cdot \sigma(0) \cdot C^T = \sigma(0) \tag{18}$$

For the horizontal and vertical subspaces this determines the shape of the eigen-ellipses. For zero spiral angle due to eq.(12) these eigenellipses are upright, and the ratio of their half-axes can be found to be

$$(\sigma_{11}/\sigma_{22})^{1/2} = x_{max}/\Theta_{max} = (-\, c_{12}/c_{21})^{1/2} \tag{19}$$

and

$$(\sigma_{33}/\sigma_{44})^{1/2} = y_{max}/\varphi_{max} = (-c_{34}/c_{43})^{1/2} \tag{20}$$

The areas of the horizontal and vertical beam emittances are given by

*) The matrix C can be used also for a fraction of an orbit, which extends from one symmetry point to another one with the same radius r_0 for the central trajectory.

$$\varepsilon_x = \pi \cdot x_{max} \cdot \vartheta_{max} \tag{21}$$

and

$$\varepsilon_y = \pi \cdot y_{max} \cdot \varphi_{max} \tag{22}$$

In the longitudinal phase plane due to the isochronous field any particle with the coordinates (z, δ) is transformed identically, provided it is positioned on its equilibrium orbit $x = \tilde{r}\,\delta$

$$C \cdot \begin{bmatrix} \tilde{r}\,\delta \\ 0 \\ 0 \\ 0 \\ z \\ \delta \end{bmatrix} = \begin{bmatrix} \tilde{r}\,\delta \\ 0 \\ 0 \\ 0 \\ z \\ \delta \end{bmatrix} \tag{23}$$

Therefore any phase ellipse is an eigenellipse. There are, however, other criteria which determine a usefull ratio z_{max}/δ_{max} for a given longitudinal phase space area.

Let us for a moment replace the longitudinal coordinate z by the corresponding phase deviation $\varphi_{rf} \propto z$ (not to be confused with the vertical angle φ!) and the momentum coordinate by the energy deviation $\Delta E \propto \delta$. When the bunch of particles is accelerated on top of a single frequency voltage (no flat-topping), then an originally elliptic $(\varphi_{rf}, \Delta E)$ phase plane gets a banana shape at the extraction orbit as indicated in Fig. 2. If the amplitude of the accelerating voltage (more precise, the peak energy gain per turn) is constant versus radius then the maximum phase deviation and the energy spread for $\varphi_{rf} = 0$ particles both remain constant (Fig. 2a). For a radially increasing voltage we get a phase compression during acceleration, and the phase width is decreased by the ratio of the voltage amplitudes between extraction and injection[4]. The energy spread for $\varphi_{rf} = 0$ particles increases by the same factor (Fig. 2b)*). Clearly the elliptic shape of the $(\varphi_{rf}, \Delta E)$-plane at injection is best maintained by accelerating with a phase width as small as possible. For a given plane area however then the energy spread gets large and, assuming single turn extraction is still possible, one may then ask for a de-bunching system after the cyclotron. If there are no flat-topping cavities in the cyclotron, a good compromise for the shape of the longitudinal plane at extraction may be reached when the energy deviation caused by the phase width is about equal to the energy spread for $\varphi_{rf} = 0$ particles coming from the initial energy spread of the injected beam (see Fig. 2b). Knowing the transformation of the cyclotron this gives backwards an optimal ratio z_{max}/δ_{max} of the longitudinal phase ellipse at injection. The orientation of the ellipse should be upright.

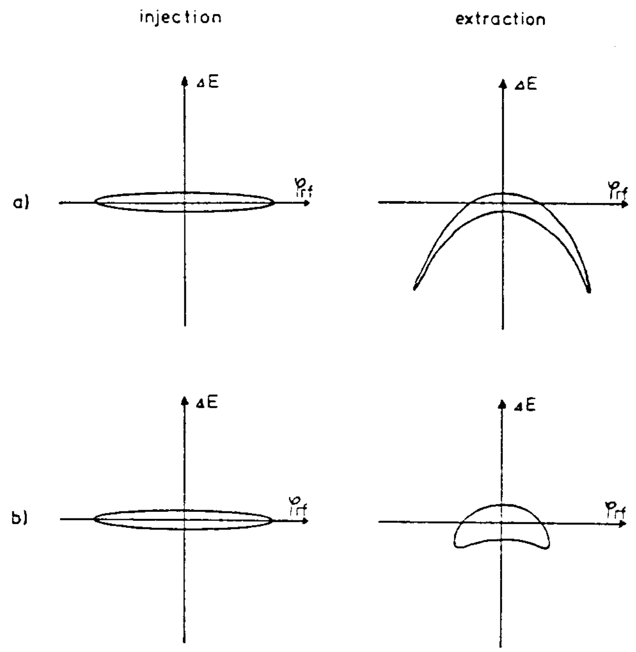

Fig. 2 : Transformation of the longitudinal phase plane through the cyclotron (schematically), a) for a constant amplitude U_o of the accelerating voltage, b) for increasing U_o.

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*) In the approximation of

$$\cos(\varphi_{rf}) \approx 1 - \tfrac{1}{2}(\varphi_{rf})^2 \text{ and } \sin(\varphi_{rf}) \approx \varphi_{rf}$$

the longitudinal phase space transformation between average radii r_1 and r_2 in an isochronous cyclotron with single turn extraction is given by:

$$\Delta E(r_2) = a \cdot \Delta E(r_1) + b \cdot (\varphi_{rf}(r_1))^2 \tag{24}$$

$$\varphi_{rf}(r_2) = \frac{1}{a} \cdot \varphi_{rf}(r_1)$$

with

$$a = \frac{U_o(r_2)}{U_o(r_1)}$$

$$b = -\frac{1}{2} \cdot E_u \cdot \left(\frac{\beta(r_2)}{r_2}\right)^2 \cdot U_o(r_2) \cdot U_o^2(r_1) \cdot \int_{r_1}^{r_2} \frac{r \cdot \gamma^3(r)}{U_o^3(r)}\, dr$$

$U = U_o(r) \cdot \cos(\varphi_{rf})$ = accelerating voltage

$$E_u = 931.5 \text{ MeV} \; ; \; \beta(r) = \frac{v(r)}{c} = \frac{\beta(r_2)}{r_2} \cdot r$$

$$\gamma = (1 - \beta^2)^{-1/2}$$

2.2. Coupling between the horizontal and longitudinal phase space.

The transformation matrix C of eqn. (11) has four non-zero coefficients which correlate the horizontal and the longitudinal motion:

$$\begin{aligned} c_{16} &= \tilde{r}(1 - c_{11}) \\ c_{26} &= -\tilde{r}\, c_{21} \\ c_{51} &= \tilde{r}\, c_{21} \\ c_{52} &= -\tilde{r}(1 - c_{22}) \end{aligned} \qquad (25)$$

Due to these matrix elements the x and Θ coordinates of a particle vector may have contributions coming from the momentum deviation δ, and the longitudinal displacement z (or the phase φ_{rf} of the particle) is influenced by the horizontal coordinates x and Θ. Let us therefore split up the particle vector $\vec{X}(0)$ in the following way:

$$\vec{X}(0) = \begin{bmatrix} x_o \\ \Theta_o \\ y_o \\ \varphi_o \\ z_o \\ \delta \end{bmatrix} = \begin{bmatrix} \bar{x}_o + k_1\delta \\ \bar{\Theta}_o + k_2\delta \\ \bar{y}_o \\ \bar{\varphi}_o \\ \bar{z}_o + k_3\bar{x}_o + k_4\bar{\Theta}_o \\ \delta \end{bmatrix} \qquad (26)$$

The coordinates indicated by bars shall be coordinates of the eigenellipses of the three uncoupled subspaces (δ=0 for the horizontal subspace; $x = \tilde{r}\cdot\delta$, Θ = 0 for the longitudinal subspace). Then the vector $\vec{X}(0)$ describes a particle on the surface of the six-dimentional eigenellipsoid, if the four coefficients k_1, k_2, k_3 and k_4 remain constant upon the transformation with the matrix C:

$$\vec{X}(1) = C\cdot\vec{X}(0) = \begin{bmatrix} \bar{x}_1 + k_1\delta \\ \bar{\Theta}_1 + k_2\delta \\ \bar{y}_1 \\ \bar{\varphi}_1 \\ \bar{z}_1 + k_3\bar{x}_1 + k_4\bar{\Theta}_1 \\ \delta \end{bmatrix} \qquad (27)$$

As the coordinates on the longitudinal eigenellipse do not change, we have in addition $\bar{z}_1 = \bar{z}_o$. Replacing the matrix C by eq. (11) and solving for the coefficients one finds:

$$\begin{aligned} k_1 &= \tilde{r} \;;\; k_2 = 0 \\ k_3 &= 0 \;;\; k_4 = \tilde{r} \end{aligned} \qquad (28)$$

For a cyclotron with zero spiral angle the six projections of the resulting eigenellipsoid involving the horizontal and longitudinal coordinates are sketched in Fig. 3.

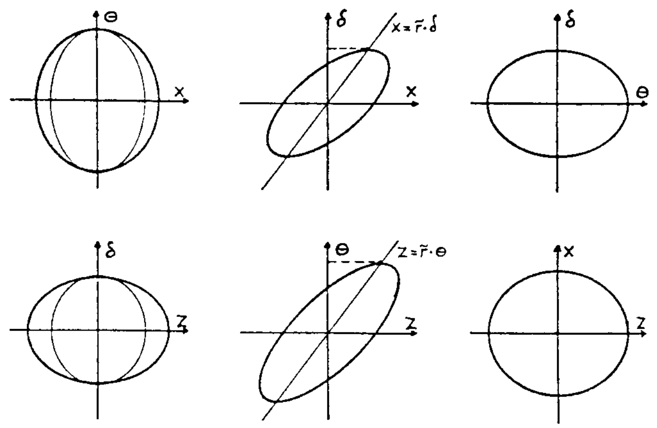

Fig. 3 : The projections of the six-dimensional eigenellipsoid involving coordinates of the horizontal and longitudinal subspace for a cyclotron with straight sectors.

The values for the coupling coefficients k_1 and k_2 have been expected already, since they make sure that particles deviating from the central trajectory only in the momentum coordinate are positioned on the corresponding equilibrium orbit. The coefficient $k_4 = \tilde{r}$ demands in addition a coupling between the horizontal angle Θ and the longitudinal displacement z. Let us explain the reason for this coupling looking on the simplest possible cyclotron orbit which is a circle. In Fig. 4 the thick lined circle indicates the central orbit. The circle marked in a thin line is an orbit with its center at M, having at point A the angle deviation Θ. The Θ - z coupling now requires that this particle shall be positioned at point B, heading the central particle by the distance $z = r\cdot\Theta = \overline{BA}$. Then, half a revolution later, the same particle will be at position B' and thus being behind the central particle by the same distance. Averaged over one orbit (over one cycle of the radial betatron cscillation in the general case) the longitudinal displacement or the phase deviation φ_{rf} is zero. If, however, the particle with the angle Θ would coincide with the central particle at point A it would be behind at point A' after half a revolution, and on the average z would be negative.
In ref. 5) the center position phase of a particle is introduced. This coordinate describes the phase deviation of a particle measured in respect to its own orbit center. Together with the energy deviation of a particle it forms a set of canonical variables. The coupling terms $k_3 = 0$ and $k_4 = \tilde{r}$ correspond to a particle with zero center position phase. The tilt of the (Θ, z) phase plane in Fig. 3 is such that for all particles their center position phase is constant.

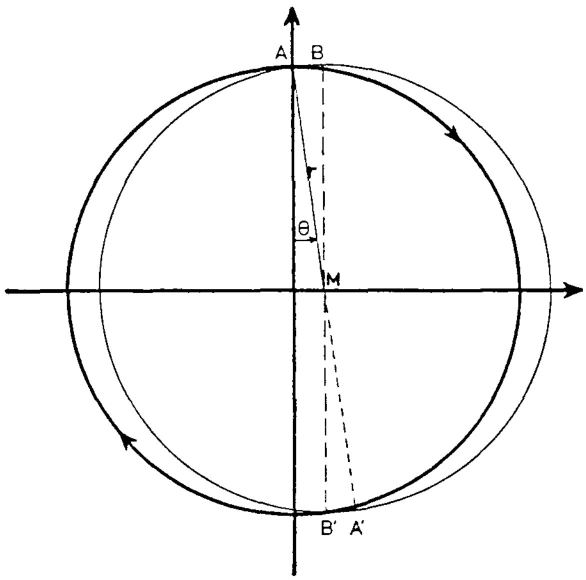

Fig. 4 : Explanation of the (θ − z) coupling, see text.

3. The beam line system between pre-accelerator and cyclotron.- In this chapter we will look for beam line systems which will match the beam from a pre-accelerator to the six-dimentional eigenellipsoid of the cyclotron orbit. The aim is to find systems with well separated functions for easy operation. They will include a stripper for increasing the charge state of the ions.

It is assumed that the time structure of the pre-accelerated beam is already such that it fits the periodicy of the accelerating voltage of the booster cyclotron. If the pre-accelerator is also a cyclotron this may be achieved by choosing its rf-frequency to a subharmonic of the frequency of the booster cyclotron. If the pre-accelerator is a dc-machine, pre-bunching is necessary.

Let us subdivide the beam line system into groups with different functions as indicated in Fig. 5. Point B shall be that location on a symmetry point of the cyclotron orbit where the beam has to be matched to its six-dimensional eigenellipsoid. At point A at the end of the preparation system the beam shall be achromatic (uncoupled subspaces; all projections of the ellipsoid involving coordinates of two different subspaces are upright ellipses). In longitudinal direction it shall be focussed such that it coverges to the desired waist on its way to point B.

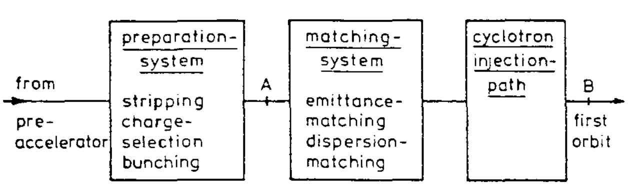

Fig. 5 : Subdivision of the beam line system in groups with specific tasks.

For convenience one may ask for a horizontal and vertical waist at point A. The beam shall have the proper charge state for acceleration in the cyclotron. Some comments on internal stripping are given later.

3.1. The matching system
As we consider only radial injection the matching system shall be a pure magnetic beam line (no buncher) with the same midplane symmetry as the cyclotron. It is represented by a matrix M with the general form of eq. (2). Neglecting a possible crossing of the accelerating gap in the injection path of the cyclotron this part of the system may also be considered as a magnetic beam line with a matrix I, giving

$$T = I \cdot M \qquad (29)$$

for the transformation from point A to B. Using eq. (5) it can be shown that the transformation by the general matrix R of eq. (2) can be written as

$$R \cdot \begin{bmatrix} x_o \\ \theta_o \\ y_o \\ \varphi_o \\ z_o \\ \delta \end{bmatrix} = \begin{bmatrix} \bar{x}_1 + r_{16}\delta \\ \bar{\theta}_1 + r_{26}\delta \\ \bar{y}_1 \\ \bar{\varphi}_1 \\ \bar{z}_1 - r_{26}\bar{x}_1 + r_{16}\bar{\theta}_1 \\ \delta \end{bmatrix} \qquad (30)$$

where the coordinates indicated by bars result from the transformation by the uncoupled submatrices. Thus, the desired coupling between the horizontal and longitudinal phase plane described by eq. (28) is completely done when the correct dispersion matching

$$t_{16} = \tilde{r} \quad \text{and} \quad t_{26} = 0 \qquad (31)$$

is accomplished.
The dispersion matching may be necessary even when the injected beam has a negligible momentum spread, since it serves for the constant center position phase of the particles. When accelerating on high harmonic numbers the pulse length may be comparable to the radial extension of the beam, and the tilt of the (θ, z) - plane in Fig. 3 becomes important. Then, without dispersion matching, oscillation of the center position phase occurs, resulting in a higher energy spread of the extracted beam.

The elements of the horizontal and vertical submatrices of T have to be matched to produce the uncoupled eigenellipses at B. Since the beam at A shall be achromatic, this emittance matching can be decoupled from the dispersion matching when splitting M into

$$M = D \cdot E \qquad (32)$$

where E is a straight beam path with at least four quadupole singlet magnets. Four parameters are necessary for matching in both planes the tilt of the ellipses at B and the ratio of their half-axes.

The injection path I normally does not produce the correct dispersion $i_{16} = \tilde{r}$ and $i_{26} = 0$. Therefore the part D must contain dipol magnets to get the necessary dispersion. A suitable bending unit for this purpose may be a combination of two dipole magnets in reflection symmetry with a quadrupole singlet in the middle.*) Together with another quadrupole singlet between this bending unit and the cyclotron entrance the dispersion matching can be accomplished using the gradients of the two quadrupoles as fit parameter. Depending on the optical properties of the injection path in the cyclotron it may be necessary to have some more quadrupoles to control the horizontal and vertical beam envelope.

3.2. The preparation system.

In most cases where heavy ions are accelerated in a booster cyclotron an increase of their charge state between the pre-accelerator and the cyclotron is necessary. Especially when a foil stripper is used for that purpose this may be accompanied by a considerable angle- and momentum straggling in the stripper. The corresponding increase of the phase space volume can be minimized when there is a sharp focus with uncoupled subspaces in all three dimensions at the location of the stripper. This is illustrated in Fig. 6.

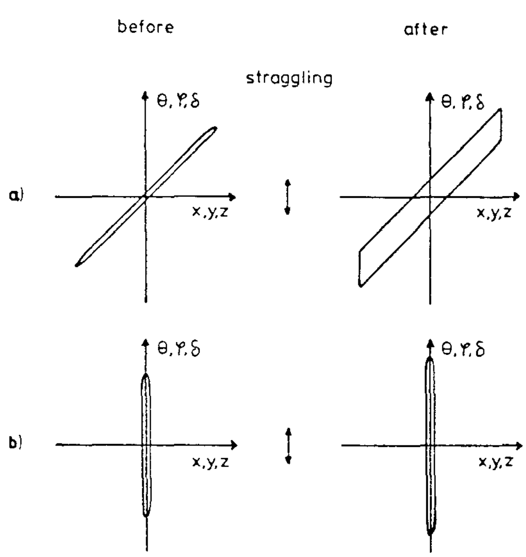

Fig. 6 : Increase of the phase space by angle- and momentum straggling a) for not properly matched ellipses and b) for a sharp focus.

*) Such a system delivers a variable dispersion, with r_{16} proportional to r_{26}. In Fig.7 two such systems around the quadrupoles q_2 and q_5 are used as achromatic bending units.

In cyclotrons with internal stripping this stripper in positioned normally just in front of the first orbit, where the beam should be matched to its eigenellipsoid. As we have seen this in a location where the ellipses may be not upright, and there is a coupling between the subspaces. Thus, in that case an optimal matching for the cyclotron orbit and for the stripper can not be achieved. The deterioration of the beam ellipsoid may become large if the angle- and / or momentum straggling is comparable or even much higher than the extension of the angle- and momentum coordinates of the eigenellipsoid.

If outside stripping is possible then it should take place in the preparation part of the system. Since we need a longitudinal (time-) focus at the stripper and on the first orbit of the cyclotron, normally two bunchers are needed. As the focal lengths of such lenses for the longitudinal phase space are often in the range of about ten meters, they may determine the total layout of the preparation system. Since they change the momentum coordinates of the particles they should also be positioned at an achromatic beam position where they mainly influence the longitudinal phase space.

As an example for a beam matching system the connection between the 6 MV Van de Graaff and the cyclotron for the VICKSI[6] facility in Berlin is shown in Fig. 7. The

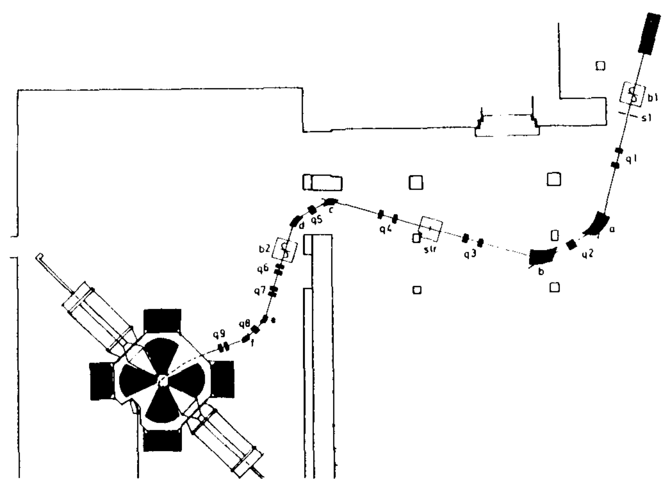

Fig. 7 : Beam matching system of the VICKI accelerator combination in Berlin.

preparation system ends immediately after the second buncher b2 which refocusses the pulses into the cyclotron. The first buncher b1 produces a time focus at the stripper str. The emittance matching part consists of the four singlets of the quadrupoles q_6 and q_7, and the dispersion matching is done with the bending magnets e and f together with the quadrupoles q_8 and q_9.

Instead of refocussing with the second buncher from the stripper into the cyclotron it is possible to design the magnetic system in between such that its effective length for the longitudinal phase space becomes zero, thus giving a unit transformation for that subspace. Since the matching system and the injection path in the cyclotron provide a positive drift (element t_{56} of the matrix in eq. (29)) one needs a "negative drift" of the same length between the stripper and the matching system.

A possible way for such a magnetic system which gives a negative effective length r_{56} for the longitudinal submatrix is sketched in Fig. 8. It consists of four doubly focusing 90 degree analysing systems. Indicated besides the central path is a path for a particle with a positive momentum deviation. In the second and third dipole magnet this particle makes a considerable detour which overcompensates the higher velocity. Therefore, when starting at the same time at the beginning of the system, it will be behind the central particle at the end. This is the opposite from what happens in a normal drift. Since particles with $\delta = 0$ have a horizontal and vertical focus on the axis between the first and second dipole magnet this is a good place to regulate the detour of the $\delta \neq 0$ particles and consequently the value of r_{56} with the indicated quadrupole lenses,

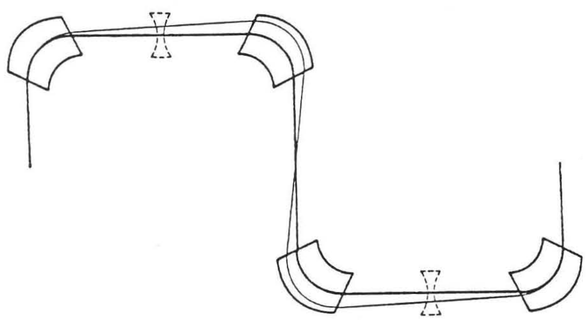

Fig. 8 : Effectively negative drift for the longitudinal phase plane; see text.

with little influence on the transversal focusing properties. The overall system is achromatic. With the help of a few more quadrupole lenses it is even possible to get for the total transformation a six-dimensional unit matrix- with the only exception of a suitable negative value of r_{56}. Such a magnetic system will be used for the SuSe accelerator combination between the stripper and the matching system.

4. Control of the correct matching.- Beam matching systems as described above can be calculated with computer codes like TRANSPORT[1]. Since there is a long way from the principal design calculation to the setting of the resulting parameters by the operator this, however, does not assure a correct matching at the end. For optimizing we need to know the influence of a change of the set values on the physical beam parameters, and we must have a possibility of measuring the beam properties.

The first one may get from the calculation itself. Let us consider only the matching part of the system. Starting from the calculated values for the "correct" eigenellipsoid one calculates different sets of quadrupole excitations, each of them yielding a change in one eigenellipsoid parameter like radial dispersion, angular dispersion, tilt of the horizontal eigenellipse, or ratio of the ellipse half-axes. Thus we get sets of combined changes of the quadrupole excitations which should produce orthogonal changes in the beam parameters.

The correct setting can be checked when measuring the turn pattern in the cyclotron with radial probes. If the matching of the coupled horizontal and longitudinal phase space is perfect, one should get turn peaks which change their height and width only smoothly with radius. If the emittance and dispersion matching is not correct, the turn peaks show oscillations in their height and width as can be seen in Fig. 9. These

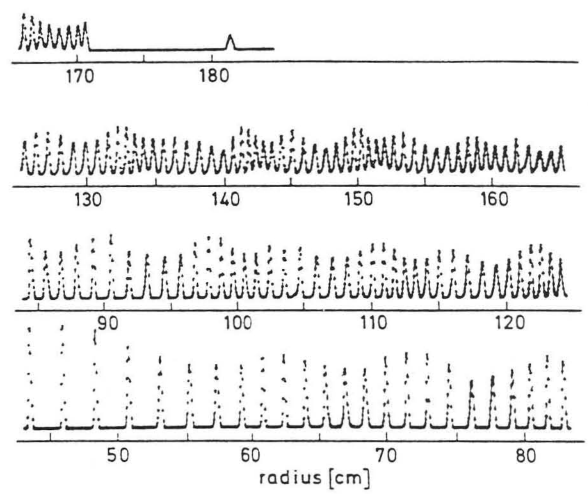

Fig. 9 : Turn pattern taken with the radial probe in the VICKSI cyclotron. The hight oscillation of the peaks show that the emittance and dispersion matching is not perfect.

oscillations contain two frequencies. If the dispersion matching is not correct we get oscillations with a period of $(\nu_r - 1)^{-1}$ turns. A wrong setting of the horizontal eigenellipse results in an oscillation with half that number of turns in one period because the rotating ellipse is transformed identically already after half a revolution.

The best way for getting an optimal matching may be applying successively the calculated orthogonal changes of the quadrupole parameters, and measuring their influence on the turn height pattern. Then a fitted linear combination of these changes should give a correct setting.

This matching optimization was tested at the VICKSI accelerator - for some beams however with only moderate success. The reason for that was found to be the very narrow phase space acceptance in the last electrostatic inflection element. This caused different beam losses when applying the parameter changes, resulting in an incorrect measuring of their influences. Thus care should be taken to have enough acceptance around the calculated beam size for playing with the focussing elements of the matching system.

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" DISCUSSION "

K. ZIEGLER : Remark : if one wants to have perfect matching as described, one has to be careful not to cut part of the beam after the matching part of the beam line, that means that the acceptance of all following elements should be large enough to transmit the full beam.

G. HINDERER : That is very true !

S. ADAM : Remark : The use of the transport program for the calculation of cyclotron orbits is highly dangerous in case of spiral angles differing substantially from zero.

J.C. CORNELL : A comment and a question. People are often rather suspicious of using TRANSPORT for cyclotron magnets, but we have used the calculations you described, by dividing the sector magnet field from valley to valley into slices, calculating transfer matrices for each and multiplying these together with TRANSPORT, to obtain eigen ellipsoids for the closed injection orbit.

" DISCUSSION " (continued)

What comments do you have on the use of TRANSPORT alone for cyclotron design ?

G. HINDERER : TRANSPORT of course does not provide for acceleration, so one would have to redefine the central orbit after acceleration. I think it is possible to use linear matrices for the transformation, since the cyclotron beam width is normally very small compared to the radius. The matrices however could also be obtained by numerical integration.
The question if the program TRANSPORT delivers the correct matrices depends on the separation of the magnetic field into sectors, since this program assumes separated elements.

M.K. CRADDOCK : Comment : Regarding the question by J.C. CORNELL, at TRIUMF we have developed a matrix multiplication code (COMA) for tracking accelerated particles through a sector cyclotron.

P. ROUSSEL : What is the amount of negative length which you can generate with the system of four magnets which you presented ? If you try to avoid the use of a second buncher between a tandem and a cyclotron, it is something like 100 meters which you need and more magnets are necessary. We present in this conference a project with 6 magnets allowing to compensate for 40 to 90 meters.

G. HINDERER : In the case for SuSe, we need about 20m. This can be achieved easily with a bending radius of the magnets of 1m, still getting telescopic optics in both planes ($r_{21} = r_{12} = r_{34} = r_{43} = 0$).

I think it is possible to get an even longer negative drift . However then, one must look at the overfocussing effect, since the r_{21} element of the transfer matrix may become large.