

BEAM DYNAMICS FOR THE IPCR SSC

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Abstract. - Several codes for the orbit calculation have been developed and orbit analysis has been done. Isochronous fields were formed, and focusing properties were examined for various particles/energies. Injection parameters to avoid the deterioration of emittances in a six-dimensional phase space were found out, and beam tracing calculations were made during acceleration. The influence of resonance, and the effect of imperfection of an isochronous field were also examined numerically.

1. Introduction. - Construction of a separated sector cyclotron (SSC) has begun at the Institute of Physical and Chemical Research (IPCR). The general features and present status are described elsewhere.¹⁾ In advance of construction, a number of computer codes have been developed in parallel with model studies of sector magnets and an RF cavity. The brief description for two typical computer codes, and a few results are presented here.

2. Magnetic field. - A 1/4 scale model sector magnet, whose sector angle is 50 degs., was constructed, and the magnetic fields were measured with polar grid over the excitation range from 8 to 17.5 kG. The measured area covered the angular range of 45 degs. from the center of an open valley (0 deg.) to that of a sector magnet. Since the model magnet, however, is not exactly of a 1/4 scale in the radial direction, the data have to be not only scaled up but also expanded to those of the full scale magnet.

Two kinds of isochronous fields, which were formed with the translated field and its Fourier-expanded field, were compared, and then no significant difference between them was recognized. Thus the translated magnetic field was adopted in subsequent calculations.

3. Computer codes. - 3.1 Isochronous field and equilibrium orbit. A code EQUIOT has been developed to form the isochronous field and to find out equilibrium orbits using measured magnetic fields. An isochronous field is formed by Garren's method. The fractional change $\sigma(r) = \Delta L / 2\pi r$ in the length of equilibrium orbit from that of the circle of averaged radius r is given approximately by

$$\sigma \approx -\frac{1}{(1+G)(N^2-1)} \left(F^2 + \frac{RdF^2}{2dR} \right)$$

where F , N , and G are the flutter, the number of sectors and the field index, respectively. Then the isochronous field B_s is²⁾

$$B_s(r) = \{1 + \sigma(r)\} \{1 - r^2(1 + \sigma(r))\}^{-\frac{1}{2}}$$

The approximation is not good for an SSC. However, this is available as a starting field in searching more accurate field.

An equilibrium orbit is sought out repeatedly by changing the initial values of radius $r(0)$ and radial momentum $p_r(0)$, until we get

$$|r(0) - r(\frac{\pi}{2})| + |p_r(0) - p_r(\frac{\pi}{2})| < \epsilon, \quad \epsilon \text{ being a given small value.}$$

In this manner, equilibrium orbits are obtained successively over the whole region of the usable

magnetic field. These equilibrium orbits, however, do not always satisfy the condition of isochronism. Accordingly, above procedure are iterated until both the conditions of an equilibrium orbit and isochronism are fulfilled simultaneously. The scaling for isochronism is made so as to reserve relative field distributions along concentric circles. An additional condition for this SSC should be taken into account that the path length of the first equilibrium orbit has to be equal to the integral multiple of the length of the last drift tube of the injector RILAC. After the isochronous field is completed, thirteen differential equations, which describe variation of positions and radial momenta of the reference particle (equilibrium orbit) and auxiliary particles are solved in the field. The independent variable is taken to be an azimuthal angle.

3.2 Accelerated-beam orbit. A code ACCELP has been developed for the purpose of the calculation of accelerated-beam properties. The accelerating gap is designed to be constant (10 cm) and the edge is straight in the radial direction. Because of the fundamental harmonic number $h=9$, the dee angle of a delta-shape cavity is taken to be 20 degs. In this code are introduced the radial position and frequency dependences of accelerating voltage. However, the radially constant voltage of 250 kV is adopted, at

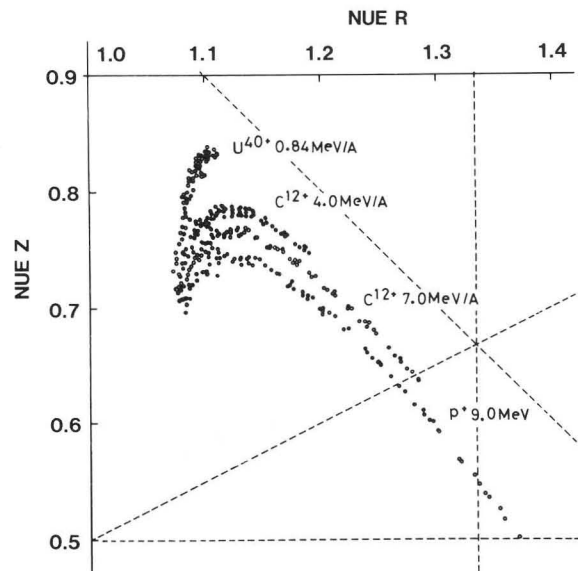


Fig. 1 Betatron frequencies for typical ions.

present. The ray tracing calculation can be done for the maximum of 10 particles. One is a reference particle, and others are auxiliary particles with different initial values with respect to the reference particle in a six-dimensional phase space.

4. Results- 4.1 Betatron frequency. At the beginning of this project, the sector angle was set to be 50 degs. owing to focusing frequencies for some typical particles in analytical magnetic fields. Afterwards, focusing properties have been examined with EQUIOT. Betatron frequencies in a $v_r - v_z$ space are shown in

figure 1 for typical particles of 0.84 MeV/u U^{40+} , 4 MeV/u and 7 MeV/u C^{12+} , and 9 MeV p^+ . The first two are injected from RILAC, and the last two from an injector cyclotron. The innermost equilibrium orbit was given from the previously-mentioned condition for the path length, and meanwhile the outermost one from the limitation that the axial betatron frequency of 9 MeV p^+ did not fall below 0.5.

It is desirable that the operating points are apart from resonant lines as possible, because of the fluctuation due to the imperfection of an isochronous field, instabilities of the magnetic field and the accelerating voltage, and so forth.

4.2 Injection condition. The acceleration in a well-centred orbit is achieved under the condition that a reference particle must be injected onto the equilibrium orbit with a certain radial gradient and with an appropriate injection phase. In that case are minimized the coherent radial oscillation amplitude and the phase slip of the accelerated beam. For instance, in case of 0.84 MeV/u U^{40+} ions the condition requires the radial gradient of $r_0' = 31.78$ mrad and the injection phase $\phi_0 = -14.0$ to the phase corresponding to the minimum RF voltage. Figure 2 shows the turn separation and phase slip at an azimuthal angle of 90 degs. Upper and lower figures correspond to the cases of off-centred and well-centred particles, respectively. A new approach to achieve a well-centred orbit is given in this

proceedings³), but above parameters were found out with one of modified ACCELP which searched automatically these quantities.

The quality of an accelerated beam is evaluated by ray tracing calculations in a six-dimensional phase space: (r, r') , (z, z') , and $(\Delta E/E, \Delta \phi)$ spaces. The matching at injection among phase parameters plays an important role in finishing these requirements⁴). The correct matching between radial and longitudinal spaces is specially significant: $\Delta \phi = -h r' R_{in} / \bar{R}$ and $r = R_{in} \Delta E / (2E)$, where h , R_{in} and \bar{R} are the harmonic

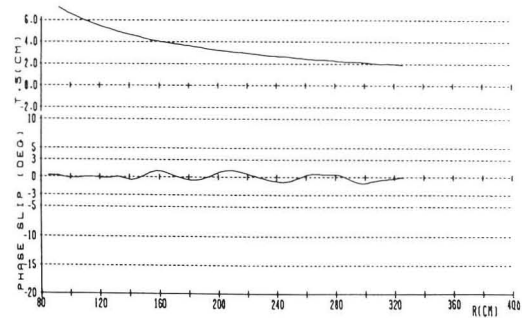
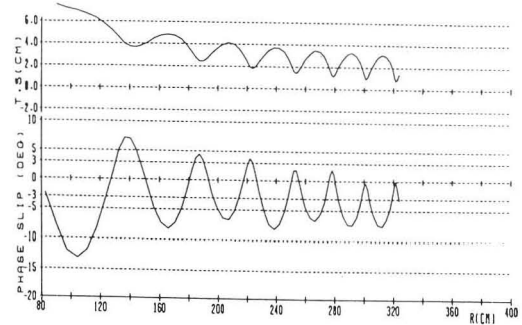


Fig. 2 Turn separation and phase slip vs. radius for 0.84 MeV/u U^{40+} ions. The upper and lower figures correspond to the cases of off-centred and well-centred beams, respectively.

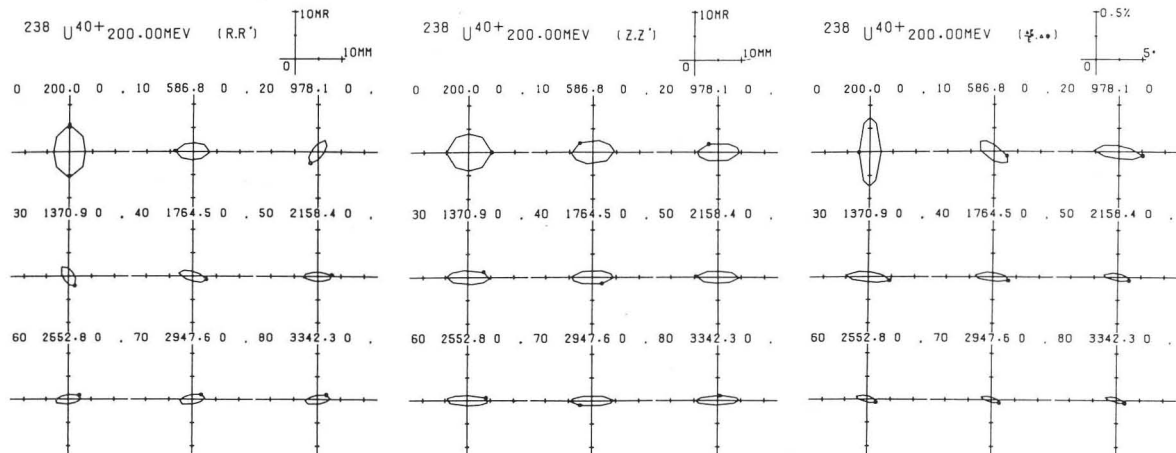


Fig. 3 The transformation of radial, axial and longitudinal phase space patterns. Numerals show a turn number, an energy and an azimuth.

number, the injection radius and the mean radius of the first equilibrium orbit, respectively.

Figure 3 shows the transformation of the phase space patterns projected on radial, axial, and longitudinal planes, respectively. The 0.84 MeV/u U^{40+} beam is employed as an example.

4.3 Resonance. As seen in figure 1, the 9 MeV p^+ beam crosses two resonant lines: $\nu_r = 2 \nu_z$ (Walkinshaw resonance) and $3 \nu_r = 4$. The former is a quadratic, non-linear, coupling resonance. The latter is a quadratic, non-linear, intrinsic resonance for the SSC composed of 4 sector magnets. The proton beam encounters these resonances at energies of nearly 128 MeV and 165 MeV. In figure 4 are shown radial and axial half widths of the beam at 0 deg. Only slight change in width is observed through the process of acceleration. The behavior of the phase space patterns

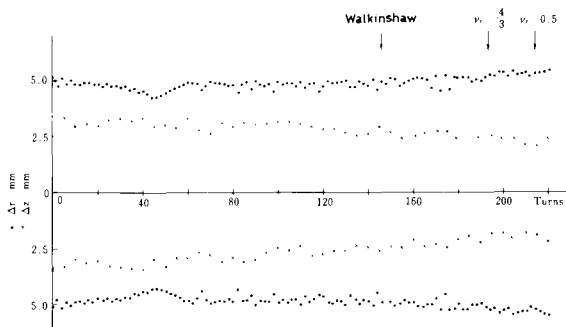


Fig. 4 Radial(●) and axial(×) half widths of 9 MeV p^+ beams.

at the vicinity of the $\nu_r = 2 \nu_z$ resonance is also calculated. Harmful distortion is not observed. According to these facts, the influence of these resonances is considered to be very small at least in the case of the 20 mm mrad proton beam. It will be expected that the influence is not so large for 7.0 MeV/u C^{12+} beams, too. It is considered that the reason why the influence of resonances is small is due to a large energy gain per turn.

4.4 Imperfection of isochronous field The influence of imperfection of an isochronous field has been investigated. In order to estimate the effect of the imperfection, a simple model is introduced. It is assumed that the radial field strength on the magnet centerline can be written as

$$B(r) = B_{iso}(r) + \Delta B \sin\left\{\frac{2\pi}{\lambda}(r-r_0) + \psi_0\right\}$$
, where $B_{iso}(r)$, ΔB , λ , r_0 , and ψ_0 are an isochronous field calculated already, an additional deviation, the wave length, the starting radius and the initial phase of the wave, respectively. Two kinds of a trim-coil shape were examined for an azimuthal winding. One of them, we call a concentric type, is of a concentric circle. The other, we call a Gordon type, is of the equilibrium orbit in the hard-edge field. Energy resolutions at an extraction radius were calculated for a monoenergetic 0.84 MeV/u U^{40+} beam of a radial emittance 20 mm mrad.

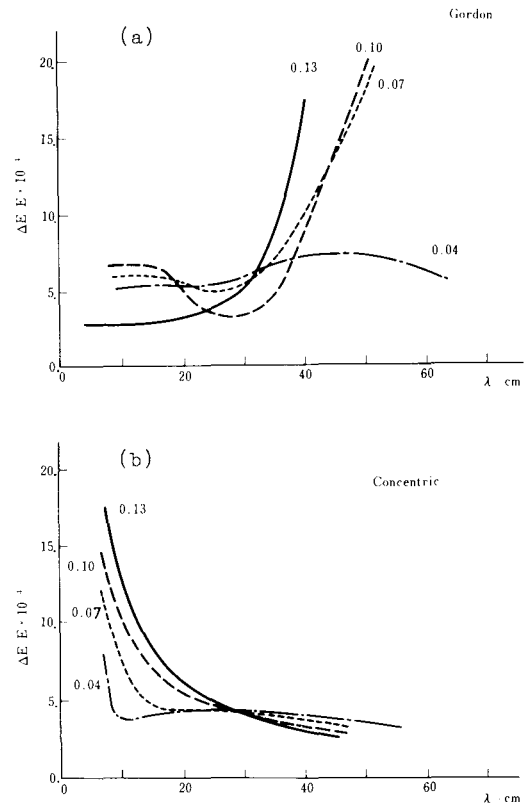


Fig. 5-a,b Resultant energy resolution at an extraction radius by trim coils of a Gordon type(a) and a concentric type(b). The parameters show field deviation in percent.

Figures 5-a,b show variation of the resultant energy resolution for the field deviations of 0.04, 0.07, 0.10 and 0.13%. As seen in the figures, there exists a clearly different trend between the two types. In addition, the phase slip of a reference particle was also calculated for each case. In this SSC, 29 pairs of a trim coil are designed to be mounted on magnet pole faces.⁵⁾ It may be guessed that the possible deviation will be of less than 0.1% and of its wave length of less than 20 cm. Thus, the trim coils of a Gordon type are considered to be suitable for this SSC.

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