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AN ORTHOGONAL QUADRUPOLE SYSTEM

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Abstract. An arrangement of six quadrupole magnets is described which provides completely independent control of the horizontal and vertical phase-space parameters in a beam of charged particles from an accelerator. In fact the system requires only two quadrupole field strengths to be altered to adjust the horizontal size of the final waist, while the vertical waist size remains completely unaffected. Another two quadrupoles control the vertical parameters similarly. This results in a considerable simplification in manual tuning of beam parameters, even when computer-controlled or computer-assisted tuning has been used. The positioning of these quadrupoles can easily be determined using analytic techniques, and the relevant formulae have been derived and evaluated for a number of specific cases. Non-coincident horizontal and vertical initial waists (slits) have also been considered. The versatility of this system is demonstrated by means of beam envelope plots derived from a program using a matrix representation of beam transport elements.

1. Introduction. In most accelerator beamlines, magnetic elements are required to enable the size and shape of the beam to be varied. Quadrupole lenses have suitable focusing properties and are widely used for this purpose. Unfortunately the action of a quadrupole is focusing in one plane, but defocusing in the other, and various combinations of quadrupoles[1] must be used to achieve overall focusing in both horizontal (x) and vertical (y) planes. As a result, the changes to the beam shape in one plane are difficult to decouple from changes in the other.

In an idealized situation it would be convenient to have these beam shaping quadrupoles allocated to a short section of beamline, probably between two sets of slits, and operating in a waist-to-waist mode. In such a section we then require a minimum of four quadrupoles to transform a double waist at the initial slit into another double waist of given size at the final slit. With four quadrupoles, however, altering the field strength of any one or more quadrupoles to vary the size of the beam in the horizontal plane unavoidably affects the beam in the vertical plane, and vice versa: i.e. the shaping in x and y is not independent, and all quadrupoles must be adjusted. However, by using an additional two quadrupoles, we can separate the shaping in x and y making them independent of each other. The quadrupoles which control the x shaping are then said to be "orthogonal" to those which control the y shaping[2]. Such an arrangement facilitates rapid tuning of the system which is often necessary even when the quadrupoles are preset under computer control.

The way in which the system operates may be seen from the ray-diagram given in figure 1. Here we represent the quadrupoles by thin lenses and the initial waist by a point source. The rays drawn are those of maximum divergence at the initial waist for x and y respectively. The quadrupoles Q_3 and Q_5 (controlling x only) are orthogonal to quadrupoles Q_2 and Q_4 (controlling y only). The quadrupoles Q_1 and Q_6 serve only to put x and y into an "orthogonal" mode. Thin lenses and point sources are of course only theroretical approximations, and we might expect only to achieve approximate orthogonality in practice. However, as we shall show, the independence of x and y is remarkably good.

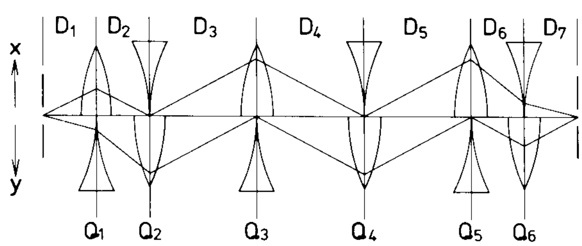

Fig.1: Six quadrupoles (represented as thin lenses) in an orthogonal arrangement.

In any system the beam should not become too large inside the quadrupoles, so that the cost of large-aperture quadrupoles can be avoided. From figure 1 we can see that the horizontal beam size becomes large in Q_3 and Q_5, and the vertical size large in Q_2 and Q_4. The optimum solution for limiting the size will be such that x at Q_3 should equal x at Q_5, this size being equal to y at Q_2 and Q_4. The placing of the quadrupoles in the system which will achieve this optimum sizing can be determined from thin-lens optical theory.

2. Thin-lens optics. We make use of approximations[3] for the focal length of a quadrupole lens:

focusing plane: $f_c = f_o + L/6$; $\quad f_o = \dfrac{a\,B\rho}{B_o L}$

defocusing plane: $f_d = -f_o + L/6$

where $B\rho$ is the magnetic rigidity of the beam, B_o the field strength at the pole tip, a the aperture radius and L the effective length of the quadrupole.

We assume a point source, so that we may apply point-to-point optics. This is a valid assumption if the waists are narrow, i.e. if x is small compared to x' (horizontal divergence) and y small compared to y' (vertical divergence). Then the only beam parameters used in the calculation are the initial divergences (x'_i, y'_i), the required final divergences (x'_f, y'_f) and the magnetic rigidity.

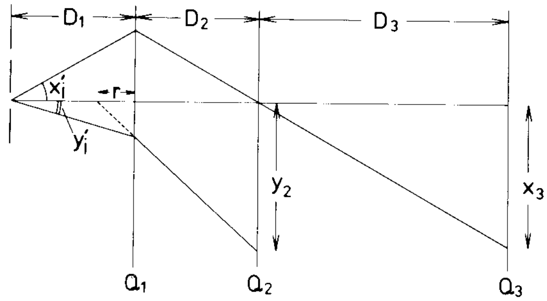

Fig. 2: Ray diagram through quadrupoles Q_1 and Q_2.

We start by assuming that lens Q_1 focuses x to a waist at the centre of Q_2, and then calculate y_2, the resulting y-size in Q_2. This is shown in figure 2. We require y_2 to be minimized, i.e. we put $dy_2/dD_1 = 0$, which results in the restriction $D_1 = D_2$. We now put x_3, y_4 and x_5 all equal to y_2, which leads to the result: $D_3 = D_4 = D_5$. The last two quadrupoles are treated similarly to the first two, but in reverse.

If the system does not begin with a double waist, we assume that D_1 is measured from the waist in x. We call P the distance between the x and y waists. The system parameters are found in a similar manner to the coincident double waist, with the following results[4]:

$$E = D_1 + D_2,$$

$$D_1 = E + \frac{ELC}{3P}, \qquad C = 1 - \left\{1 + \frac{3P}{L} + \frac{3P^2}{EL}\right\}^{\frac{1}{2}}$$

$$D_3 = D_4 = D_5 = \frac{y'_i}{x'_i}\,\frac{D_2}{D_1}\left\{2E + P + \frac{3P^2}{3P + 2LC}\right\},$$

$$D_6 = D_7 = \frac{4L + 3\frac{y'_f}{x'_f}D_3 \pm \left[16L^2 - 72\frac{y'_f}{x'_f}LD_3 + 9\left\{\frac{y'_f}{x'_f}\right\}^2 D_3^2\right]^{\frac{1}{2}}}{24},$$

where the desired length of the system is $T = \sum_i D_i$.

These equations may be solved by a short computer program. It should be noted that several solutions may be found, some of which may give unphysical or impractical values of D_1, and all solutions should be examined to find the optimal one.

3. Quadrupole polarities. The normal quadrupole polarities for any given system are those which result in minimum beam width in the quadrupoles. This implies that if $x'_i > y'_i$ then Q_1 should be positive (x-focusing). The subsequent quadrupoles always have alternate polarities (see figure 1). Conversely, if $x'_i < y'_i$ then the normal polarity for Q_1 is negative. If $x'_i = y'_i$ the normal polarity sequence is then determined in similar manner by the ratio y'_f/x'_f.

4. Results. We have used the program TRANSPORT[5] to test the validity of our approximations and the results are summarized in Table 1, for a system 13 m long, with quadrupoles of effective length L = 0.40 m.

We examine a special case where $y'_i/x'_i = (y'_f/x'_f)^{-1}$ and we choose $y'_i/x'_i = 0.75$ and thus $y'_f/x'_f = 1.33$. We calculated the distances between quadrupole centres as described in Section 2, and then used TRANSPORT to fit waists at succeeding quadrupoles to form a final double waist, with optimum values of x'_f and y'_f, i.e. for which the system is properly orthogonal.

TABLE 1 A range of final beam parameters obtainable for the special case discussed in the text.

Quadrupole polarities	x_f (mm)	y_f (mm)	y'_f/x'_f	Figure
Normal	0.64	0.52	1.33	3a
Normal	7.00	0.52	13.46	3b
Normal	0.64	0.32	2.00	3c
Reversed	0.40	0.88	0.454	-
Reversed	0.40	7.00	0.057	3d
Reversed	0.35	0.88	0.395	3e

If we alter only Q_3 and Q_5 to change x_f from its optimum value of 0.64 mm to 7.00 mm, y_f is unaffected and remains equal to 0.52 mm. This is demonstrated in figure 3b. If instead we alter Q_2 and Q_4 to change y from 0.52 mm to 0.32 mm, x_f is completely unaffected and remains equal to 0.64 mm as shown in figure 3c. In each of these two cases the final ratio y'_f/x'_f is increased relative to the thin-lens design value (refer to Table 1). The system as designed works well for $y'_f/x'_f > 1$. For $y'_f/x'_f < 1$ the system should be used with the quadrupole polarities reversed. This is illustrated in figure 3d, where y now behaves similarly to x in figure 3b; and in figure 3e where x behaves like y in figure 3c.

In all the above cases we have started with x small compared to x', and the orthogonality of the quadrupoles is very good. If we start with larger x and y (implying smaller x' and y' respectively for any given emittance) we find larger waists within quadrupoles Q_2 to Q_5 and there is thus more effect on one dimension when the other is varied (i.e. the orthogonality becomes degraded). However, this only becomes noticeable when x or y (in mm) is significantly larger than x' or y' (in mrad) respectively.

5. Conclusions. We have demonstrated that the orthogonal quadrupole system gives excellent independent control over the horizontal and vertical focusing properties for both coincident and non-coincident initial waists. While it is in principle possible to accomplish the same focusing with only four quadrupoles, using a computer-linked control system and look-up tables, for example, this also requires very precise knowledge of the initial beam parameters. In practice these are rarely so well-defined, and manual tuning is almost always required.

The orthogonal quadrupole system described here is considered to provide sufficiently improved ease of operation to warrant the cost of the additional quadrupoles. Such orthogonal quadrupole systems are planned for the beamline between injector and main cyclotron and in the external beam preparation system of the National Accelerator Centre at present under construction[6].

References

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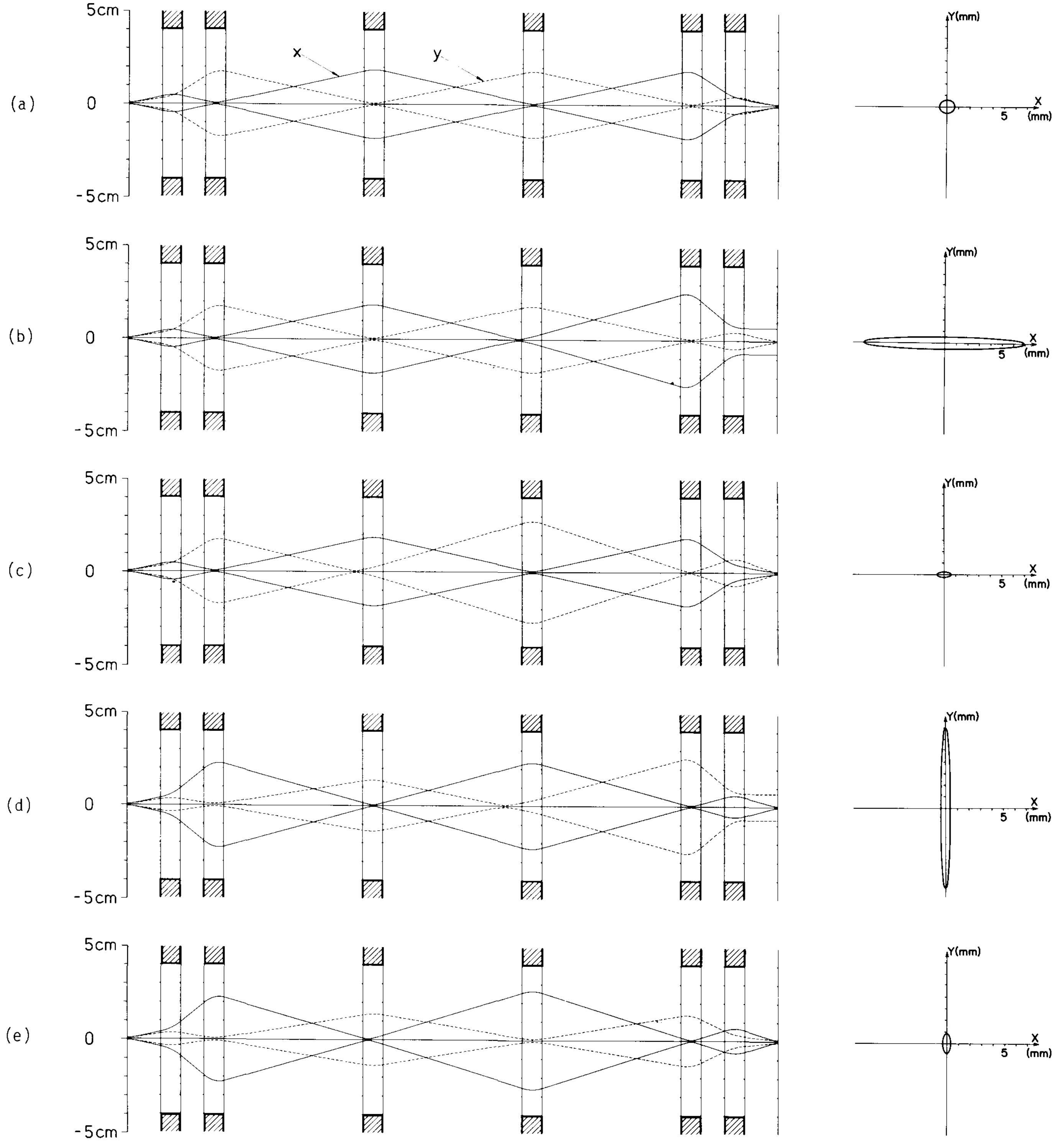

Fig. 3: Beam envelope plots showing an orthogonal quadrupole system with (a) optimum x_f and y_f, (b) x_f increased, y_f unaltered and (c) y_f decreased, x_f unaltered. Reverse quadrupole polarities give (d) y_f increased x_f unaltered and (e) x_f decreased and y_f unaltered.