SPUNCH - A SPACE CHARGE BUNCHING COMPUTER CODE

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Abstract

Even at relatively low currents, space charge forces can significantly influence the effectiveness of a beam buncher. A computer code has been written to simulate longitudinal beam motion in the presence of space charge. This program is relatively fast because it is one-dimensional and because particle-particle forces are calculated beforehand and stored in a table.

Introduction

Many computer codes exist which more or less successfully describe particle motion in six-dimensional phase space with space charge. For long, not necessarily periodic, transport channels, such codes can sometimes be less useful in studying longitudinal motion than simple two-dimensional codes whose only transverse input is an average beam size. Two-dimensional codes for longitudinal motion exist for proton synchrotrons but these all make the approximation that the space charge force on a particle is proportional to the derivative of the local line density. This approximation is only good for motion on a scale which is large compared with the beam pipe radius.

So far, the best numerical technique for modelling the bunching of an initially dc beam in the presence of space charge is a simulation. The particles used in the code SPUNCH are uniformly charged discs. A length of beam corresponding to the bunch spacing, $\beta\lambda$, is divided equally into N (typically 100) discs. At any required moment, the force on a particular disc is calculated by adding up the forces from the other discs.

The chief advantage of using discs rather than point particles is that the electric field from a disc is non-singular; collision effects are thereby avoided. A similar technique is employed in the program PARMILA[1] but there, in order to include transverse dynamics, each disc is further subdivided into rings.

Theory

Forces between discs i and j are calculated by first using the free space, on axis formula for the electric field:

$$E_{ij} = \frac{Q}{2\pi\varepsilon_0 a^2}\left\{1 - \left[1 + \left(\frac{a}{z_{ij}}\right)^2\right]^{-1/2}\right\} \mathrm{sign}(z_{ij}) \qquad (1)$$

(the charge per disc=$Q=I/(N\nu)$, where ν is the fundamental rf frequency, a=beam radius (a constant), and $z_{ij}=z_i-z_j$, the directed distance between the discs.) This expression is then multiplied by a beam pipe shielding function which is calculated from image charges of a point charge centred in the (either circular or square) vacuum chamber. Assuming that the beam does not come near to filling the chamber completely, this is a good approximation. This formalism has the advantage of computation speed. The shielding function can be permanently stored for a given geometry because it does not depend upon beam size and can be simply scaled to the size of the beam pipe.

A more general expression for the average electric field of disc j over the surface of disc i in a circular geometry is[2]

$$E_{ij}= \frac{Q}{2\pi\varepsilon_0 a^2} \sum_{n=1}^{\infty} \exp(-k_n|z_{ij}|/b) \left[\frac{2J_1(k_n a/b)}{k_n J_1(k_n)}\right]^2 \mathrm{sign}(z_{ij}) \qquad (2)$$

(b is the beam pipe radius and k_n is the nth zero of [illegible] the J's being Bessel functions.) This formula is being incorporated into SPUNCH but the resulting improvement in accuracy is expected to be slight when compared with the error incurred by using the model of discs of equal radius in a circular beam pipe. In general neither the beam nor the pipe is of constant radius and usually the beam is not circular. In running SPUNCH, one must therefore give careful thought to a proper choice of a and b. For cases where the beam line consists of regions which differ widely in a and b, the calculation can be subdivided to cover each region in turn.

Before performing the simulation, SPUNCH calculates the forces between discs for 500 different disc separations. These forces are then stored in an array which is consulted during the simulation.

Motion is calculated in the reference frame of the central particle. The program can handle any number of bunchers at any harmonic of the fundamental rf frequency. The entire bunch is instantaneously given a velocity modulation at the time that the central particle crosses a buncher gap. Similarly, the calculation terminates at the time that the central particle reaches the injection gap of the cylcotron. The calculated final distribution of particles in phase space, $\psi(\phi,\Delta p/p)$, being a 'snapshot' at an instant in time, will always possess the symmetry $\psi(\phi,\Delta p/p)=\psi(-\phi,-\Delta p/p)$. (We define the phase ϕ of a particle as $2\pi\Delta z/(\beta\lambda)$ where Δz is its position w.r.t. the central particle.) In reality, we are interested in the distribution of the beam as a function of time at the injection gap rather than the distribution as a function of position at an instant in time. The former distribution will be asymmetric but the asymmetry is negligible in cases where $\beta\lambda$ is small compared with the total drift distance. This condition is always met for injection into a cyclotron because it ensures that $\Delta p/p \ll 1$.

Only one bunch is dealt with in the calculation. Space charge forces from neighboring bunches are calculated by assuming translational symmetry: $\psi(\phi,\Delta p/p) = \psi(\phi+2\pi,\Delta p/p)$. Again, this is an approximation which is only good when $\Delta p/p \ll 1$. Forces between particles are only significant when separated by less than ~2b. In general, therefore, the effects of neighboring bunches are very small. This is particularly true for the particles which matter, i.e. those which end up inside the cyclotron acceptance.

For most purposes, the space charge effect is sufficiently well described by N=100 discs with a time increment of one rf period. For improved resolution, up to 500 discs can be used. With an Amdahl 5850 computer, the CPU time required for a SPUNCH run is 0.60 N^2T microseconds where T is the number of time steps. Typically, T~100 so that a run requires less than a second of CPU time.

Examples

All of the following examples are based upon the TRIUMF injection line: particle type = H^-, kinetic energy = 288 keV, fundamental harmonic buncher with ν=23.06 MHz (βλ=0.32 m) located 20.9 m before the cyclotron, and a second harmonic buncher at 16.3 m before the cyclotron. The rf voltages on the two bunchers (V_1 and V_2) are referred to as if they were single gap bunchers. In fact, both bunchers are of the double-gap type so the actual voltages on the central electrode are half the quoted values.

Figure 1 shows how space charge effects are already important at relatively low intensities.

Figure 2 shows the effect of beam pipe size. It is well known that for b << bunch length, the

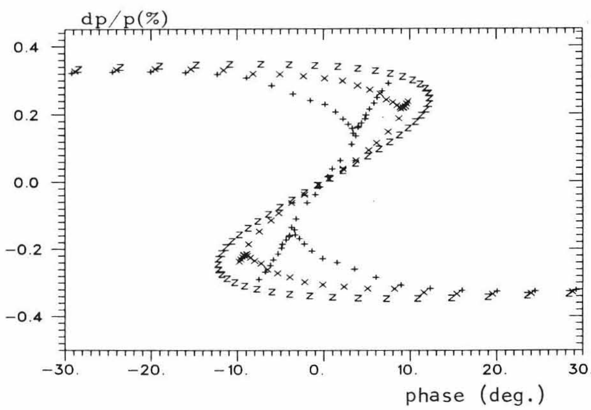

Fig. 1. Final distribution in longitudinal phase space for three different intensities: (z) 0 μA, (×) 20 μA, (+) 40 μA. Other parameters for this calculation are: fundamental harmonic buncher voltage V_1=2 kV, beam radius a = 3.8 mm, and beam pipe radius b = 54 mm. Of the 100 particles, 60 are lying inside this ±30° phase window.

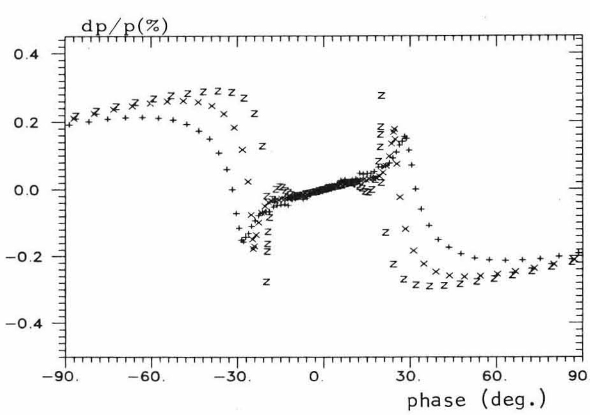

Fig. 2. Effect of three different beam pipe sizes for an average beam current of 520 μA: (z) b=9 mm, (x) b= 18 mm, (+) b=54 mm. Other parameters are as in Fig. 1.

longitudinal electric field is proportional to 1 + 2 ℓn(b/a) times the derivative of the line density. Figure 3 shows that SPUNCH calculations are consistent with this. Two calculations for different values of b give approximately the same results when the currents for the two cases have a ratio of 1 + 2 ℓn(b/a).

Figure 4 shows a run corresponding to TRIUMF's normal operating conditions. According to this calculation, 77% of the beam lies inside the longitudinal acceptance of the TRIUMF cyclotron (±15° × ±0.5% Δp/p). This is somewhat more than the observed 65% but is good agreement considering the limitations of the model. More significantly, the values for the voltages on the two bunchers which optimize the amount of beam inside the cyclotron acceptance ellipse agree with the operational settings to within ±5%.

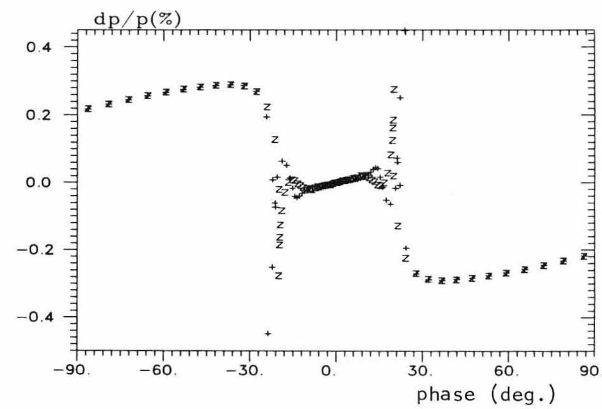

Fig. 3. Two separate cases demonstrating the 1+2ℓn(b/a) scaling. For one run (z), a=3.8 mm, b=9 mm, and I=520 mA; for the other run (+), a=3.8 mm, b=5.4 mm and I = 830 mA. Other parameters are as in Fig. 1.

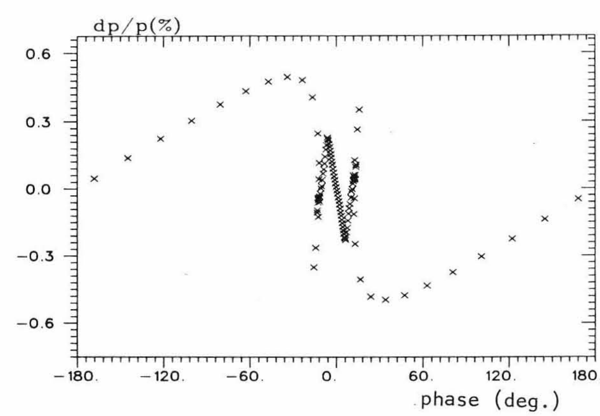

Fig. 4. SPUNCH calculation corresponding to TRIUMF's operating conditions: I=250 μA, first harmonic V_1=3.6 keV, second harmonic V_2=−1.8 kV, b=54 mm, and a=5.1 mm.

References

1. The space charge algorithm of PARMILA is described in - K.R. Crandall, LASL, MP Division Internal Report MP-4/KC-3 (1967).
2. C.B. Williams and M.H. MacGregor, IEEE Trans. NS-14, 581 (1967).