ORBIT STUDIES FOR THE NAC SEPARATED-SECTOR CYCLOTRON

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Summary. Numerical simulations have been performed to study several aspects of beam dynamics in the separated-sector cyclotron of the NAC. Results of computer calculations, using measured magnetic field data, are presented. Focusing properties of the beam are compared with those obtained previously for calculated fields. The behaviour of accelerated particles during the first few turns has been investigated to obtain the conditions which are required to satisfy injection matching of beams. A transmission matrix method is used to obtain values of the correlation factor. Accelerated beams have been simulated from injection to extraction to study the effect of beam widths, energy spread and phase compression on beam quality, taking the rf magnetic field into account with a modified impulse formula. Orbit calculations have been made for known positional errors of sector magnets to determine whether further adjustments in sector positions were required. The acceleration of polarized proton and deuteron beams has been simulated. A depolarizing resonance, which exists for some of the beams, is discussed.

Introduction

The NAC separated-sector cyclotron (SSC) is a variable-energy light and heavy ion accelerator, to be used for nuclear research, isotope production and medical therapy.[1] Its maximum design energy is 200 MeV for protons, at a resonator frequency of 26 MHz, operating on harmonic number h=4. At lower energy and for the acceleration of heavier ions harmonic numbers h=4 and h=12 will be used (the minimum rf frequency is 6 MHz). The SSC consists of four $34°$ sector magnets producing fields up to 1.27 T. Field profiles are adjusted for variable-energy operation with 29 pairs of trim-coils in each of the sector magnets. Acceleration is provided by two half wave delta resonators situated in opposite valleys. The average injection and extraction radii are R_{in} = 0.952 m and R_{ex} = 4.156 m, respectively. The first successful operation and beam extraction at the maximum energy E_p = 200 MeV, and at the therapy energy E_p = 66 MeV, are described in another paper in these proceedings.[2]

In this paper various computer simulations of the stability of motion and acceleration of particles in the SSC are discussed:

- A recalculation of focusing properties in measured fields has been performed, after completion of the mapping of the magnetic field, to determine operating lines in (ν_r, ν_z) - space.

- Orbit calculations have been made for known errors in the position of the sector magnets, in order to determine whether further adjustments in the position of the magnets were required.

- Orbit calculations have been made to prepare for the first beams to be accelerated during the commissioning of the machine, in order to establish machine parameters and to determine parameters related to injection matching. During this period a large part of the calculational effort went into the isochronization of sector fields.

- Extended simulations of accelerated beams of particles have been undertaken, using measured isochronized fields, to determine beam properties as a function of injection conditions. The injection phase width accepted by the SSC is limited (single-turn extraction is required), because the amount of phase compression provided by the radial dependence of the accelerating voltage is small.

- Polarized ions will be injected into the SSC from the second solid-pole injector cyclotron.[3] A numerical study has been made to study the behaviour of polarized ions during acceleration in the SSC.

Orbit Code

A computer code COC (Cyclotron Orbit Code) has been developed to perform orbit calculations in measured magnetic field maps, which contain median plane field values stored on a polar grid (Δr = 0.020 m and $\Delta\theta$ = 0.25°). The program is structured so that it is easy to change the number of coupled differential equations and also to redefine the equations integrated. Several versions of the program have been produced, viz. to integrate with either azimuthal angle or time as independent variable, to integrate the polarization equations or to integrate orbits in magnet faults (displaced and/or rotated sector magnets). Also available are options to calculate equilibrium orbits and focusing frequencies, to centre accelerated orbits, and to do calculations for beams of particles (central particle plus non-central particles).

Acceleration effects are treated by the code in an impulse approximation. The rf field of an accelerating gap is expressed in the form of a Fourier transform which satisfies the wave equation and boundary conditions. For a symmetrical gap with voltage V(x) dependent on the radial position x along the gap, the electric field component in a direction s, perpendicular to the gap in the median plane, is given by:[4]

$$E_s(s,x,z,t) = \frac{\cos(\omega_{rf}t+\phi)}{2\pi} \int_{-\infty}^{\infty} dk\, T(k)\, \cos ks \sum_{m=0} V_m(x)\, \cosh k_{mz}z$$

where z is the axial coordinate,

$$k_{mz} = \sqrt{k^2+k_{mx}^2-\frac{\omega_{rf}^2}{c^2}}$$

and where each of the functions $V_m(x)$ is harmonic in the coordinate x with wave number k_{mx}. The functions $V_m(x)$ are normalized so that:

$$\sum_{m=0} V_m(x) = V(x)$$

Similar equations are obtained for the vertical component B_z of the rf field and for other field components. Integration of the equations of motion is performed using perturbation methods to obtain analytical expressions for the gap effect in an impulse approximation. In the code these expressions are used,

taking into account the position and angle at which the gap is crossed, to obtain the change in energy and radial and axial impulses at gap crossings.

The transit time factor $T(k)$, the Fourier transform of the field component E_s, is approximated by the Fourier transform of an ideal gap (constant field in gap of width g, zero elsewhere):

$$T(k) = \frac{\sin \frac{kg}{2}}{\frac{kg}{2}}$$

In the impulse approximation the effective wave number k is calculated for the velocity with which the particle crosses the gap. Since these wave numbers are usually small, the effective gap width g is selected to yield a good approximation to the Fourier transform for these values of k (see Fig. 1).

The dependence of accelerating voltage on distance along the gap, with resonator frequency as parameter, is given for the SSC resonators in Fig. 2. The orbit code uses a normalized accelerating voltage:

$$V_N(x) = 1 - V_{1x} [1 - \sin(k_{1x}x + \phi_{1x})]$$

Values of k_{1x} and ϕ_{1x} and of V_{1x} as a function of the resonator frequency, have been obtained from a least-squares fit to the data of Fig. 2.
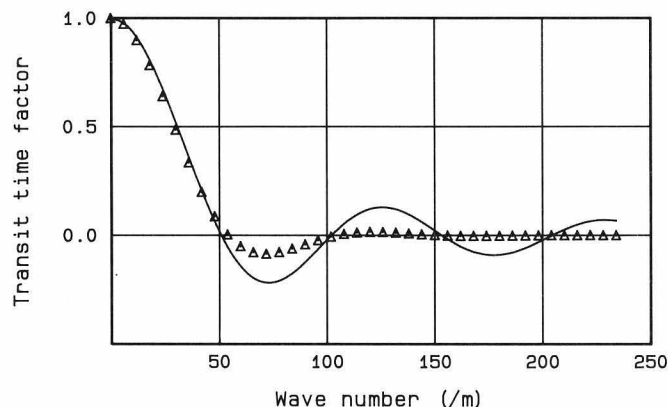

Fig. 1 Effective gap width used in transit time factor of ideal gap (solid line) is selected to fit Fourier transform of real accelerating field (symbols) at small wave numbers required by impulse approximation.
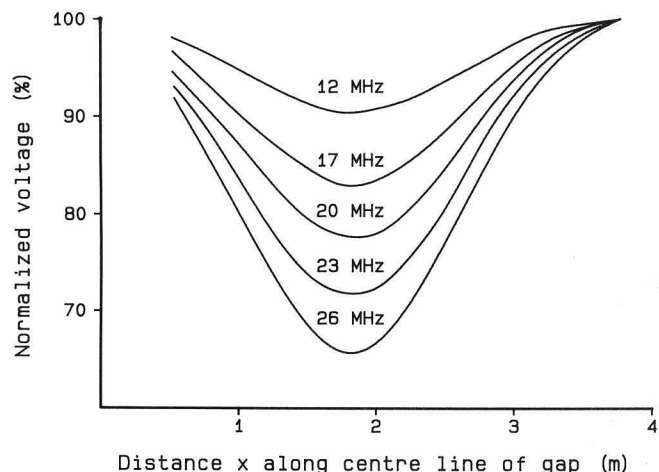

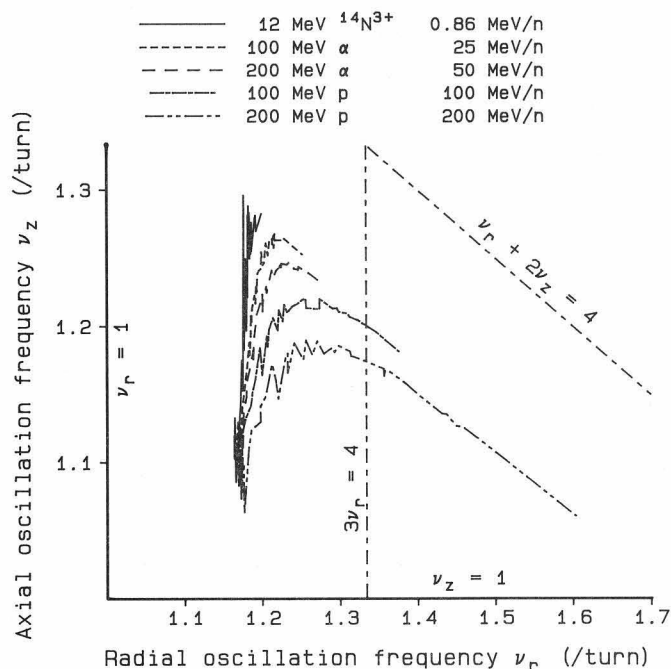Fig. 2 Normalized voltage distribution along gaps of SSC resonators.

Fig. 3 Focusing frequencies calculated for the motion of representative ions in measured magnetic fields.

Focusing properties

The initial design of the SSC and selection of the sector magnet angle had been based on magnetic fields calculated with a relaxation method. Figure 3 shows focusing frequencies which have been recalculated in the measured fields after magnetic field mapping had been completed. The vertical frequencies are a few percent lower than those calculated previously. The selection of 34^0 as sector angle gives good separation of all operating lines from the resonances $\nu_r = 1$, $\nu_z = 1$ and $\nu_r + 2\nu_z = 4$. Calculations have proved that the $3\nu_r = 4$ resonance is crossed quickly enough by the 200 MeV proton beam and that the beam quality is not degraded for normal beam widths.

Beam matching

When the beam is injected into the SSC, it should be matched in such a way that it fills an optimal region of the phase space acceptance of the cyclotron, in order that high beam quality with well separated turns during acceleration and single turn extraction be obtained.[5] The matching requirements are specified by the emittance matching of the beam and in terms of a 'reduced radius' ρ, which gives the required correlation of radial displacement with fractional momentum deviation ($x = \rho\delta$) and also of the longitudinal displacement with radial divergence ($\ell = \rho x'$). The reduced radius can be obtained from the transfer-matrix elements of an isochronous cyclotron orbit:[5]

$$c_{16} = \rho (1 - c_{11}) \qquad c_{51} = \rho c_{21}$$
$$c_{26} = -\rho c_{21} \qquad c_{52} = -\rho (1 - c_{22})$$

Orbit calculations have been performed in isochronous fields for relative motion about equilibrium orbits near injection, to obtain transfer matrix elements for protons over the range of injection energies. The reduced radius, derived from the matrix elements, has been found to be nearly constant, having the value $\rho = 0.828$ m (about 13 % lower than the approximate value R/γ^2). This value is used as a starting point in beam transport calculations and in the study of best beam quality obtainable from the SSC.

Beam quality

Orbit properties have been calculated for the acceleration of beams from injection to extraction. In order to simulate particle beams, calculations have been performed for a central particle and for 18 non-central particles divided into four groups of particles. Three groups each contain four particles on the radial phase space ellipse of the specified beam emittance, at the central particle phase and leading and lagging by the beam phase width. A group of six particles fills longitudinal phase space, at minimum and maximum energy and at the central particle phase and minimum and maximum rf phase.

The voltage distribution given in Fig. 2 initially causes dilatation and then compression of the beam phase during acceleration, to provide slight overall compression. Owing to constructional reasons, the SSC accelerating gaps are not radial, but intersect at a radius 0.225 m on the resonator valley line, and so would provide some additional phase compression had the accelerating voltage been constant. As the resultant phase compression is small, the maximum phase width at injection, which permits single-turn extraction, is an important parameter as far as beam quality is concerned (the rf phase acceptance is 8° for a 3.2π mm.mrad proton beam accelerated to 200 MeV where beam separation is 7.5 mm in the extraction valley).

The dependence of beam quality on proper injection matching of the beam has been investigated. For this purpose the previously discussed radial–longitudinal correlations have been introduced into the simulation of beam properties. An example of the beam quality obtainable with a 200 MeV beam is given in Fig. 4, where
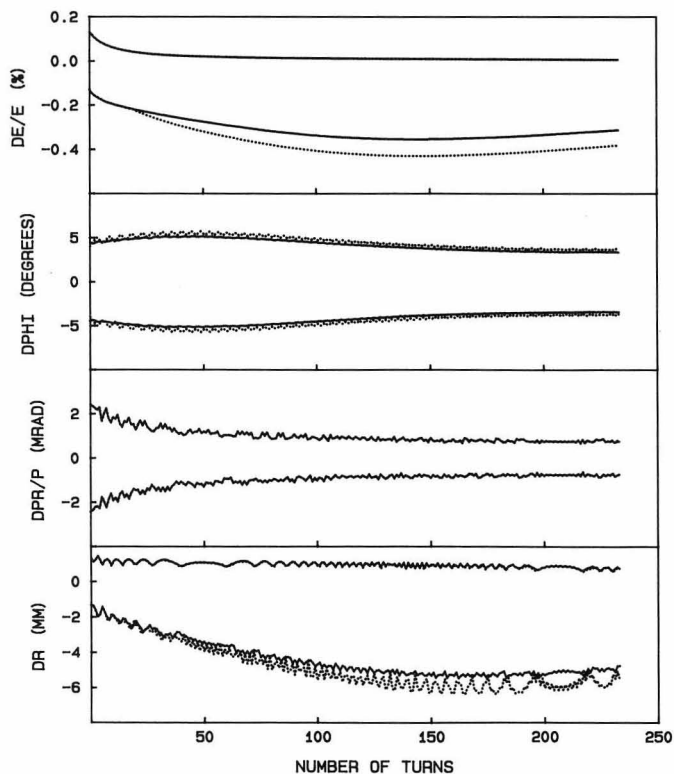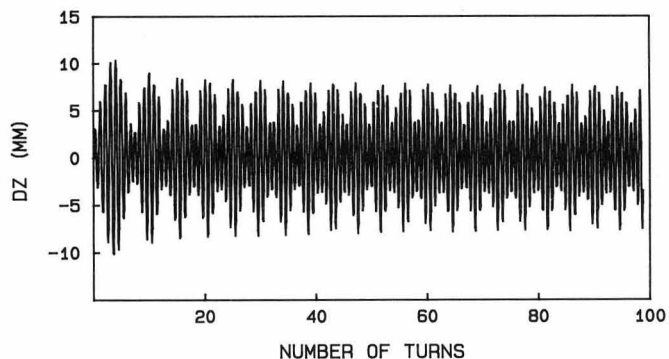

Fig. 5 Calculated vertical displacement of 66 MeV proton beam in field defect caused by lifting nose of a sector magnet by 1.2 mm. Large increase in beam height is due to the $\nu_z = 1$ resonance, change in beat rate of oscillations is caused by variation in ν_z from 1.08 to 1.23.

two beams are compared, one without any injection correlations, the other injected with the proper correlations (the radial emittances of both beams are matched). In the envelopes of the uncorrelated beam an oscillatory pattern is evident, having at minimum amplitude the same amplitude as the matched beam. The beam width increase is approximately 1 mm on a correlated width of 6 mm at beam extraction, for a 200 MeV beam having an initial rf phase width of 8°.

Effects of magnet displacements

Tolerances on the placement of sector magnets have been calculated for the separated–sector cyclotron. The orbit code has been modified to perform calculations for an SSC in which one of the sector magnets is displaced and/or rotated about a given axis. The calculation is performed by transforming the coordinates of a particle, when it is in the displaced sector, from the laboratory to the sector's system of coordinates and by calculating the field in the latter system. The field is then transformed back to the laboratory system for use by the orbit code.

An example of such a calculation, for a sector magnet having its tip raised by 1.2 mm, is given in Fig. 5. This displacement produces horizontal field components in the median plane right through the sector and the $\nu_z=1$ resonance would cause a completely unacceptable increase of 20 mm in the beam height. The tolerance obtained is 0.048 mm of pole tip tilt per mm increase in beam height.

Beam depolarization

Polarized beams will be injected into the SSC from the second solid–pole injector cyclotron SPC2.[3] A study has been undertaken to determine the degree of depolarization which takes place when beams of vertically polarized particles are accelerated in the cyclotrons of the NAC and transported through the beam lines.

In a separated–sector cyclotron, the only component of the magnetic field which does not vanish when averaged over a particle trajectory is the vertical component. The spin of a particle moving in this field with spin polarized approximately in the vertical direction, precesses about the mean field and the vertical polarization is not affected. All other field components are oscillatory with zero mean value and do not contribute to depolarization effects unless there is a resonance between the frequency of precession about the mean field and the frequency of one of the oscillatory depolarizing fields.

Fig. 4 Calculated beam envelopes for acceleration of protons to 200 MeV. Solid lines are for matched radial–longitudinal coupling at injection, dotted lines for no correlation. Radial emittance of 3.2π mm.mrad is matched at injection valley in both cases. Initial phase width of beam is 8°. Reference values 0 are with respect to central particle.

A version COCP of the orbit code has been programmed to integrate the equations of classical spin motion in an electromagnetic field: [6]

$$\frac{d\vec{S}}{dt} = \vec{\omega} \times \vec{S}$$

where the axial vector $\vec{\omega}$ is given by:

$$\vec{\omega} = -\frac{q}{m\gamma}\left[(1+\gamma a)\vec{B} - (\gamma-1)a\frac{\vec{v}(\vec{v}.\vec{B})}{v^2}\right]$$

Here q and m are the charge and mass respectively of the particle, γ is the Lorentz energy factor, $\vec{v}$ is the velocity, $\vec{B}$ the magnetic field, $a = (g-2)/2$ the gyromagnetic anomaly and g the gyromagnetic ratio. The vector $\vec{S}$ can be considered either as the polarization of an ensemble of particles or as the classical representation of the spin of a single particle.

Calculations have been performed for polarized protons and deuterons accelerated in SPC2 and SSC. In the solid-pole cyclotron depolarization is negligible for all particles at all energies. In the separated-sector cyclotron there is no depolarization of any deuteron beam, but some resonant depolarization occurs for proton beams. The result of a calculation is given in Fig. 6, for a proton accelerated to 200 MeV, starting 1 mm above the median plane in a state of vertical polarization. A resonant depolarization of 3% occurs between turn numbers 20 and 40. Calculations have been performed for the acceleration of protons at other energies starting at various distances from the median plane. The results are summarized in Fig. 7, and show that depolarization is limited to approximately 15% in beams of half-height less than 2.5 mm.

The condition for a depolarizing resonance is: [7]

$$a\gamma = n + \ell_z\nu_z + \ell_r\nu_r$$

where ℓ_z and ℓ_r may take any integer values, positive or negative, and where a = 1.7928 for protons. For intrinsic resonances in the SSC, n is a multiple of 4. In a machine having no mid-plane errors ℓ_z is limited to odd values. The loss in polarization is proportional to the square of the vertical beam amplitude, confirming that $\ell_z = \pm 1$. The resonance responsible may well be $\ell_r = \ell_z = -1$, n=4, since the resonance condition is very nearly satisfied by these values at injection for all proton beams.

References

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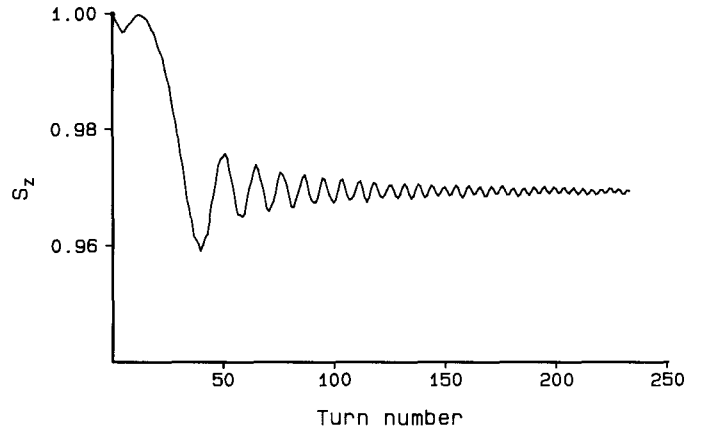
Fig. 6 Polarization of a proton accelerated to 200 MeV in the SSC. The initial polarization is $S_z = 1$ and the initial vertical displacement is z = 1 mm.
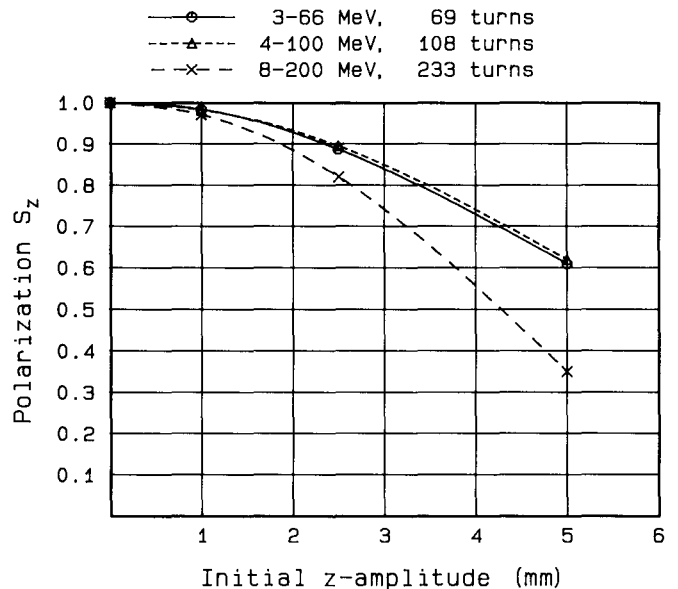

Fig. 7 Depolarization of protons accelerated in the separated-sector cyclotron. The initial polarization is $S_z = 1$, the z-axis is vertical.