EXAMPLES OF RAY TRACING IN ION INFLECTORS

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Summary

A computer code has been written to calculate the transfer of particles through ion inflectors in cyclotron centres, using the general theory presented in an accompanying contribution to this conference. The code is based on multiplication of elementary transfer and coordinate transformation matrices. Effects taken into account include transit time differences for different particles, variable values of the inflector strength, acceleration and deceleration at the edges, etc. Examples of these effects will be given.

Introduction

The transfer of particles through an ion inflector depends on various parameters such as the extraction voltage of the ion source, the voltages on the inflector electrodes, the cyclotron magnetic field, the inflector geometry, the type of ion to be accelerated, the beam emittances and momentum spread. In experimental or operational circumstances one wants to adjust some parameters, e.g. source and inflector voltage and magnetic field, in order to obtain maximum transmission. However, in general the effect of these changes is not obvious. This paper describes a computer program ("INFLECTOR") which calculates the transfer in these different situations. The computer program is based on the theory developed in an accompanying contribution to this conference [1]). The code multiplies elementary matrices and has the time as independent parameter. This paper discusses several items. First fomulas for the design setting ("Müller conditions") of the hyperboloid inflector [2]) will be given. Effects due to the geometrical finiteness of the inflector, causing transfer time differences for different initial conditions, will also be discussed. Acceleration and deceleration effects of particles entering and leaving the inflector have to be taken into account in actual transfer calculations from ion source to cyclotron median plane. A short discussion on this will be given. Furthermore the effective inflector field may have a longer or shorter extension than the design value, due to the extension of the fringe field. The effect of a longer or shorter inflector field can trivially be studied with the computer program. Finally, as mentioned, variations in the inflector voltage, or other parameters, which cause deviations in the inflector strength parameter k^2 from the Müller value $k^2 = 1/6$ will be considered. The results of the calculations may be expressed in cyclotron circle and centre coordinates [3]), or in TRANSPORT notation [4]). The last possibility makes the program attractive for obtaining inflector transfer matrices to be incorporated in ray- or beam codes like TRANSPORT. Experimental data on inflector performance have e.g. been obtained for the JULIC cyclotron [5]).
The program INFLECTOR runs on a VAX at our university, and is a self-contained unit of 20k storage (Fortran version).
In this paper we will use the nomenclature and the coordinate system of the first paper.

Design setting

Here we first discuss the general formulas which define the design setting of the inflector. The geometry of the inflector is determined by specifying the inflector radius and electrode separations. The design setting is the setting which causes a particle initially moving along the cyclotron axis, to be moving in the median plane of the cyclotron after inflection without any change in velocity.

The hyperboloid inflector potential is given by

$$V(x,y,z) = \tfrac{1}{2}k'^2z^2 - \tfrac{1}{4}k'^2(x^2 + y^2), \tag{1}$$

where k'^2 determines the strength of the inflector voltage. It has the dimension of (V/m^2). Applying voltages V_1 and V_2 on the inner and outer inflector electrodes, i.e. the electrodes being more and less far away from the z-axis, gives a k'^2:

$$V_2 - V_1 = V_I = -\tfrac{1}{2}\, k'^2\, d\, R\ , \tag{2}$$

where R is the initial distance to the z-axis of the midpoint between the inflector electrodes, and where d is the initial separation between inner and outer electrode. The voltage V_2 must be negative with respect to V_1, which means that positively charged particles experience a force directed radially outward. The separation d varies according to:

$$d(:)(7-3\cos t/3)^{-1},$$

where t is the scaled time: $0 \leqslant t \leqslant \pi\sqrt{3}/2$ between entrance and exit from the inflector.

The Müller conditions are:

$$k^2 = 1/6$$
$$R^2 = 24\, V_o^2 \tag{3}$$

where k^2 is the dimensionless inflector strength parameter, and V_o the inital velocity scaled by the cyclotron angular frequency. For an ion characterized by the charge Q = Ze and the mass M = Am, where e and m are the charge and mass of the proton respectively, the angular cyclotorn frequency is $\omega_o = (Z/A)\,(e/m)B$ with B the magnetic field. The first Müller condition means:

$$k'^2 = \frac{1}{6}\,\omega_o^2\,(Z/A)^{-1}(e/m)^{-1}$$

or, together with eq. (2):

$$\frac{V_I}{d} = \frac{1}{12}\,\left(\frac{Z}{A}\right)^{-1}\left(\frac{e}{m}\right)^{-1}\,\omega_o^2 R \tag{4}$$

Remembering that $V_o = \rho$: the orbit radius of a particle in a magnetic field B equals its scaled velocity, and using the non-relativistic approximation for the kinetic energy T:

$$T = \tfrac{1}{2}\,(Ze)(Z/A)(e/m)B^2\rho^2 \tag{5}$$

one obtains for the second Müller condition:

$$T = \frac{1}{48}\,\left(\frac{Z}{A}\right)^{-1}\,\left(\frac{e}{m}\right)^{-1}\,\omega_o^2\,R^2 \tag{6}$$

This then gives the kinetic energy of the ion moving on the central hyperboloid of the inflector. When the central hyperboloid is on ground potential, i.e. $V_2 = -V_1$, T/Ze corresponds to the extraction voltage of the ion source V_s. In this case:

$$\frac{V_I}{V_s} = 4\,\frac{d}{R} \tag{7}$$

Furthermore only in this case the final orbit radius in the cyclotron is connected to the inflector radius via $\rho^2 = R^2/24$. For example for the situation with a positive inner electrode and grounded outer electrode the central particle is accelerated by half the inflector voltage V_I when leaving the inflector, so eq. (5) has to be used for obtaining ρ, with energy $T + V_I/2$, and where T is obtained from eq. (6): the energy in the inflector. It is useful to have (4) and (6) in numerical form:

$$T = \left\{0.19956 \text{ keV cm}^{-2} \text{ T}^{-2}\right\} Z(Z/A)B^2R^2$$

$$V_I/d = \left\{0.79825 \text{ kV cm}^{-2} \text{ T}^{-2}\right\}(Z/A)B^2R \qquad (8)$$

These relations fix the voltage of inflector and ion source for a given inflector in a given magnetic field and for a given ion type: they provide the proper setting for $k^2 = 1/6$.
For the case of varying voltages and magnetic field the inflector strength parameter k^2 and the initial velocity V_o will deviate from the Müller conditions. In general:

$$k^2 = 2(Z/A)(e/m)\omega_o^{-2}V_I/(dR)$$

$$v = \omega_o^{-1}\left[2(Z/A)(e/m)V_s\right]^{\frac{1}{2}} \qquad (9)$$

i.e. for given magnetic field and varying V_I one merely changes k^2; varying V_s changes the velocity, however, when the voltages are fixed and B is used as an optimization parameter the result is rather complicated: both the scaled potential strength and the scaled incoming velocity are changed.

Program description

The program INFLECTOR is an interactive program based on the calculation of the matrix product $S = M^{-1}\, T^{-1}\, F\, T\, M$ [1]), with F the transfer matrix in transformed coordinates, T the coordinate transformation matrix, and M the matrix linking kinetic and canonical momenta.
The following gives the main flow line of the program INFLECTOR:

Input	: give R and d
Input	: set the number of time intervals (the transit time of the reference particle is given by $kt = \pi/2$; one chooses e.g. 8 selected times at which the calculation is to be done)
Input	: give k^2 (this can also be done by specifying actual physical parameters)
Calculate:	the transfer matrices F at the selected times, and the total matrix S at these times
Input	: give the initial vector (either in TRANSPORT coordinates or in inflector coordinates)
Calculate:	the transformed vector at the selected times
Check	: on azimuth to be 20.23 deg, otherwise
Adjust	: exit time such that θ = 20.23 deg
Output	: the final vector (in inflector coordinates and in TRANSPORT coordinates and in circle and centre coordinates).

In the program the general formulas from the preceeding section can also be evaluated. Moreover provisions have been made to allow for larger or smaller inflector field extensions than the design value, or the incorporation of accelerating effects at the entrance and exit. Listing 1 gives an example of the initial screen output while running INFLECTOR. The inflector coordinates are all in (mm). Cylindrical coordinates are given by the combination r, theta (deg) and z. The quantity r_2z represents the value of $(x^2+y^2-2z^2)^{\frac{1}{2}}$. For the central hyperboloid it equals R. For undisturbed transmission a particle must always have $R-d/2 \leqslant r2z \leqslant R + d/2$, with d the initial distance of the inflector hyperboloid electrodes; otherwise the particle is lost.

Transformation of unit vectors

Here we give the transformation of unit vectors using canonical variables for the time belonging to the transit time of the design particle, and for $k^2 = 1/6$. The transfer is completely linear, i.e. the transfer is expressed exactly by a first order matrix. Fig. 1 shows the transformation of the unit vectors in the hyperboloid inflector field for $kt = \pi/2$, to circle and centre coordinates. It can be checked that the total 4 dimensional phase space area spanned by these vectors remains equal to unity, demionstrating Liouville's theorem. One observes however that origianlly decoupled motion is coupled after transformation.

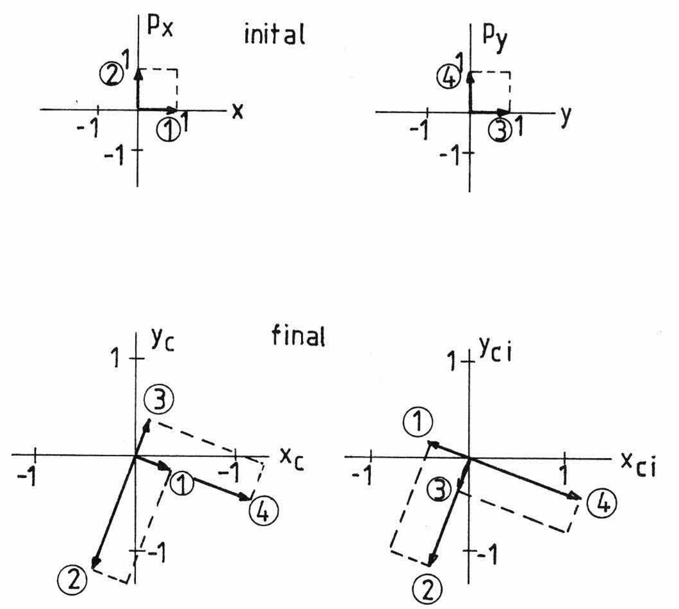

Fig. 1. Transformation of unit vectors

Transit time

In figure 2 three particles starting at the same azimuth $\theta = 0$, are shown with different energies and running on three different equipotential planes. The energy difference causes height differences at the exit. In this picture the angle θ equals 20.23 deg at the exit for all three particles, to give zero vertical velocity. The particles all have equal transit time. From this picture it is also apparent that particles starting at the same radii but with a different azimuth $\Delta\theta$ will have excactly the same trajectories rotated however by the angle $\Delta\theta$. This is because there is rotational symmetry. Since the inflector field is finite, i.e. for the design particle θ_{exit} = 20.23 deg, it follows that these particles stay longer or shorter in the inflector field than the design particle.
The most prominent effects are a linear change of the transit time: $\Delta t/t = 0.079$ for $\Delta\theta$ = 50 mrad, corresponding to y = 5 mm in an inflector with R = 100 mm, and a linear change in the vertical velocity or vertical angle in the cyclotron median plane: $\phi = Z'_{final}/Z'_{initial} = -123$ mrad also for y = + 5 mm. The last one is quite an enormous effect, leading to a large excursion from the median plane after e.g. half a revolution in the cyclotron. In the Jülich inflector test facility off-centered beams were observed 180° deg downstream of the inflector exit of about 3 mm, where the orbit radius is 1.5 cm [5]). This also corresponds to about 100 mrad at the inflector exit. Thus: steering errors in the beam line through the cyclotron yoke easily give rise to off-centered beams at the inflector entrance of several

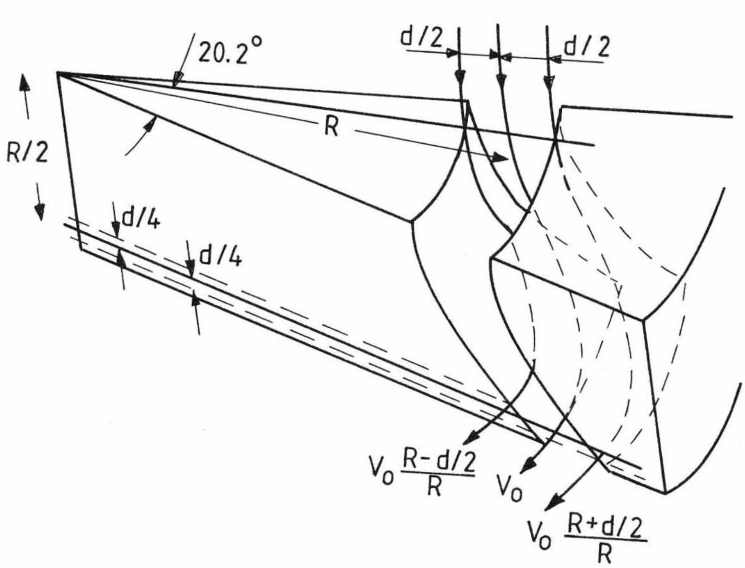

Fig. 2. Lay-out of the inflector geometry

mm, wich causes vertical angles in the cyclotron medium plane of the order of 100 mrad. Therefore it is advisable to have steering elements in last part of the beam transfer line to the cyclotron centre.

Field extension

Different field extension in the inflector than the design value due to the extent of the fringe filed cause similar vertical steering effects as discussed above. Calculations on this with INFLECTOR can easily be done by changing the reference entrance and exit time. Fig. 4 shows voltage curves for a geometry (fig. 3) in which the inflector entrance is modelled. The calculation is done with the program RELAX [6]). Figure 4 suggests that an effective field boundary extending about 0.4 d outside the entrance may be taken for the geometry shown.

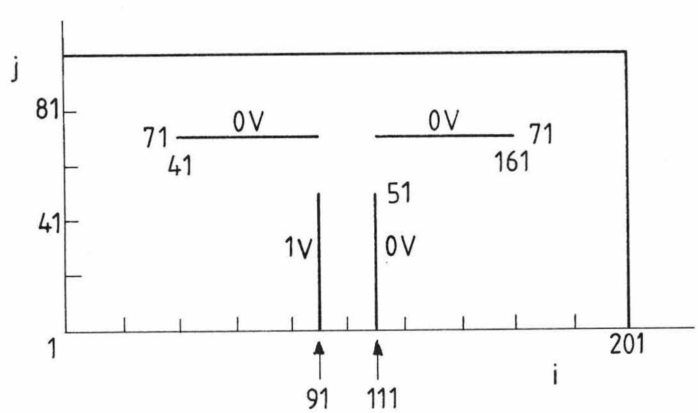

Fig. 3. Input geometry for RELAX calculation of inflector entrance

Acceleration effects

A particle entering the inflector with $x > R$ experiences an accelerating voltage > 0, for the situation with $V_2 = -V_1$. The corresponding velocity increase is:

$$\Delta V/V = 2\Delta x/R,$$

where Δx is the deviation from $x = R$. Also at the exit there is deceleration or acceleration according to $\Delta V/V = 2\Delta(r2z)/R$.
The inflector fringe field also causes a lens action, however with focal distance large compared to the dimensions of the inflector and large compared to the focal length of the first dee gap lens.
In actual transmission calculations it is important to incorporate the acceleration effects. As an example we have calculated the TRANSPORT matrix for particles traveling through the inflector including the edge deend acceleration, and without this:

$$\begin{pmatrix} x \\ \theta \\ y \\ \phi \\ l \\ \delta \end{pmatrix}_f = R \begin{pmatrix} x \\ \theta \\ y \\ \phi \\ l \\ \delta \end{pmatrix}_i$$

with R for both cases given in table 1 and 2.

Table 1. Matrix R without acceleration.

$$\begin{pmatrix} -1.2 & 0 & 0.03 & -.05 & 0 & 0 \\ 0 & .01 & -20 & 0 & 0 & 0 \\ 0 & 0 & -.08 & 0 & 0 & .05 \\ 0 & 1 & -25 & 0 & -10 & 0 \\ 0 & 0 & .005 & 0 & 0 & 0 \\ 10 & .01 & .025 & 1.25 & .1 & 0 \end{pmatrix}$$

Table 2. Matrix R including acceleration

$$\begin{pmatrix} -1.2 & 0 & .01 & -.05 & 0 & 0 \\ 0 & .01 & -20 & 0 & 0 & 0 \\ 1 & 0 & -.02 & 0 & 0 & .05 \\ 0 & 1 & -25 & 0 & -20 & 0 \\ 0 & 0 & .01 & 0 & 0 & 0 \\ -.1 & 0 & -.1 & -.01 & .2 & 1 \end{pmatrix}$$

Changing k^2

The transmission can be calculated for varying values of k^2. In order to obtain transmission one also has to change the initial velocity of the reference particle, according to $z_1 = kR/2$. Table 3 shows the effect of varying k^2 for R = 100 mm. With d = 10 mm, r2z would have to be bounded by 95 mm and 105 mm, to avoid particle loss. Figure 5 shows the variation of k^2 for an inflector with R = 74 mm, d = 10 mm, for deuterium, for constant source and inflector voltage, however with varying magnetic field. This figure simultaneously shows the variations in V_s and V_I under the condition $k^2 = 1/6$.

Table 3. Effect of k^2

k^2	r2z(mm)	z'(mrad)
.12	86.8	-46
.13	90.2	-23
.14	93.3	-10.7
.15	96.0	-3.6
.16	98.5	-0.5
.17	100.7	-0.0
.18	102.9	-1.9
.19	104.9	-4.5
.20	106.8	-8.5
.21	108.6	-13.5

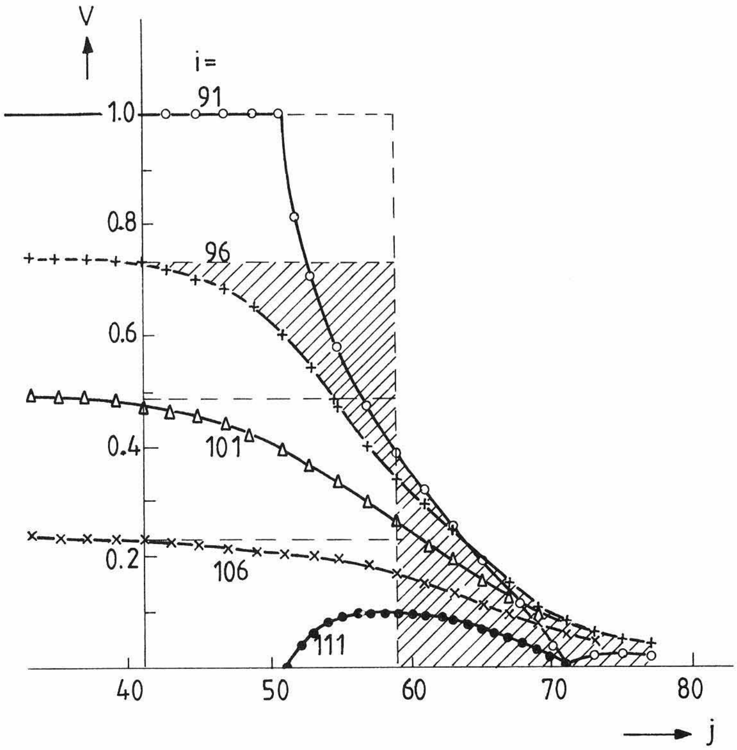

Fig. 4. Inflector entrance voltage curves

Conclusion

A computer program has been written to obtain the transfer of particles through an inflector in various situations. The preliminary results given here show the programs usefulness for interactive calculations and comparisons during experimental transmission operations with ion inflectors.

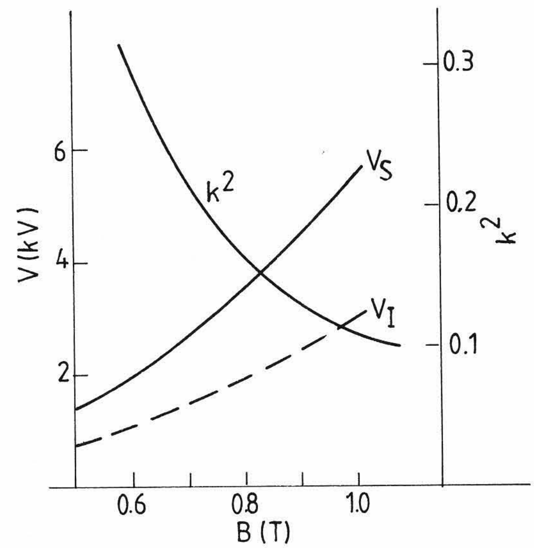

Fig. 5. Dependence of V_s and V_I on B for Z = 1 A = 2 $k^2 = 1/6$, and dependence of k^2 for V_s, V_I constant.

References

1) J.I.M. Botman, J. Reich, P. Wucherer and H.L. Hagedoorn, these proceedings
2) R.W. Müller, Nucl. Instr. Meth. 54 (1967) 29
3) W.M. Schulte and H.L. Hagedoorn, Nucl. Instr. Meth. 171 (1980) 409
4) K.L. Brown, D.C. Carey, Ch. Iselin and F. Rothacker, TRANSPORT, CERN 80-04
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6) H. Houtman, C.J. Kost, Lecture Notes in Physics 215, Springer 1985, 98.

LISTING. SCREEN OUTPUT WHILE RUNNING INFLECTOR

```
Give inflector radius and electrode separation (mm)
100 10
Give # of calculation intervals, max 20.
8
Include acceleration effects at the edges (Y=1) ?
1
Do you want to study the effect of having a wider
inflector? Specify the extra azimuthal width in mm.
0
Give particle properties (Z & A) and magn. field(T).
1 2 1
o.k.: your orbital frequency is:                7.623[MHz]
      you have a source voltage of:             9.978[kV]
      you need an inflector voltage of:         3.991[kV]
      the orbit radius in the cyclotron is:     2.041[cm]
      the height of the inflector is:           5.000[cm]
The trajectory of the reference particle is:

2kt/pi      x      x`      y      y`      z     z`      r   theta     v  v-hor   r2z

0.000 100.00   0.00   0.00   0.00   0.00 20.41 100.00   0.00 20.41   0.00 100.0
0.125 100.95   3.87  -0.15  -0.95   9.75 20.02 100.95  -0.09 20.41   3.98 100.0
0.250 103.59   6.93  -1.19  -3.61  19.13 18.86 103.60  -0.66 20.41   7.81 100.0
0.375 107.37   8.52  -3.82  -7.49  27.78 16.97 107.44  -2.04 20.41  11.34 100.0
0.500 111.48   8.26  -8.47 -11.84  35.36 14.43 111.80  -4.35 20.41  14.43 100.0
0.625 115.01   6.11 -15.16 -15.83  41.57 11.34 116.00  -7.51 20.41  16.97 100.0
0.750 117.11   2.41 -23.52 -18.70  46.19  7.81 119.45 -11.36 20.41  18.86 100.0
0.875 117.17  -2.25 -32.88 -19.89  49.04  3.98 121.70 -15.67 20.41  20.02 100.0
1.000 114.92  -7.06 -42.34 -19.15  50.00  0.00 122.47 -20.23 20.41  20.41 100.0
1.000 114.92  -7.06 -42.34 -19.15  50.00  0.00 122.47 -20.23 20.41  20.41 100.0
More particles? (Y=1), other # of intervals? (Y=2), other k? (Y=3)
```