AN ANALYTICAL METHOD TO DETERMINE THE GEOMETRY OF THE SECTORS.

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1. Introduction

For the calculations of average field and flutter, NEU and WERNER[1] have used conformal transformations to set up emperical formulae. When valley gap is twice the hill gap, it was interesting to observe a very exact analysis. The present method is based on the same idea and gives rise to a cubic euqation, from which the expression for $B_z(r,\theta)$ has been obtained. This, in turn, gives rise to subsequent expressions for determining the design parameters, $\langle B \rangle$, F, ν_r, ν_z and sector geometry. The parameters thus computed have been compared with those obtained from emperically established formulae. Moreover, the fringing field effects due to outer edge have also been incorporated. The analysis and the results are presented here in the following sections.

2. The Procedure

The method is based on the assumption that the permeability of the iron is infinite. Hence the geometrical surface of the pole serves as equipotential surface. The scaler magnetostatic potential V, satisfies the Leplace equation.

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Along constant r and constant θ this equation can be separated in two, 2-dimensional equations.

$$\frac{\partial^2 V}{\partial s^2} + \frac{\partial^2 V}{\partial z^2} = 0\,,\quad s = r\theta \text{ ----------------(1)}$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{\partial^2 V}{\partial z^2} = 0\,,\quad r \to \infty \text{ --- ------------(2)}$$

Equation (1) can be rewritten as

$$\frac{\partial V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0\,,\quad \begin{matrix} x = s \\ y = z \end{matrix} \text{ ---- -----------(3)}$$

Along a radius, the pole contour from the middle of the hill to the middle of immediate valley has been picturised in figure 1. Then through two conformal transformations[2-5] the 2D-Laplace equation are solved to analyse the potential and hence the fields. We get the conformal transformations as follows,

$$\frac{dz}{dw} = \frac{A_1}{(w-0)(w-1)^{-1/2}(w-a)^{1/2}}$$

$$\frac{d\chi}{dw} = \frac{B_1}{w}$$

$$\chi = \frac{iV_o}{\pi}\log w + V_o$$

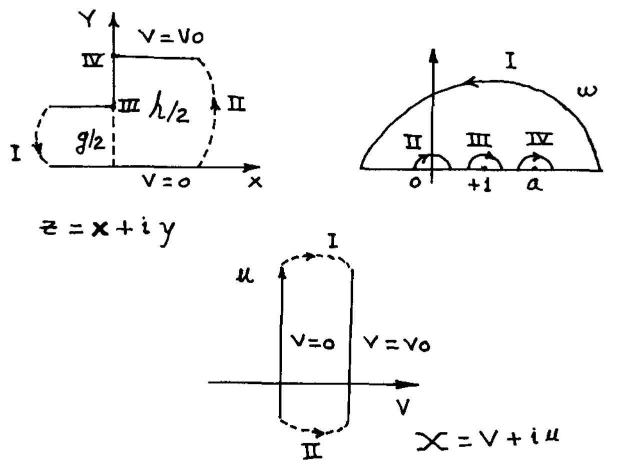

Fig.1: Transformations in different planes.

where g is the hill gap, h is the valley gap $H = h/g$, $A_1 = -g/2\pi$, $a = \frac{1}{H^2}$, $B_1 = \frac{iV_o}{\pi}$

By introducing

$$Q = \left(\frac{w-1}{w-a}\right)^{1/2}$$

we find the vertical component of field in the median plane,

$$B_y(x, y=0) = |d\chi/dz|$$

$$B_y(x, y=0) = B_o/Q$$

where B_o is maximum hill filed given by

$$B_o = \frac{2V_o}{g}.$$

For median plane we set up,

$$\frac{2x}{g} + i\cdot 0 = \frac{H}{\pi}\log\frac{H+Q}{H-Q} - \frac{1}{\pi}\log\frac{Q+1}{Q-1} \text{ ---------(4)}$$

where $1 < Q < H$

For H=2, equation (4) gives rise to a cubic equation.

$$B^3 - \frac{3}{4}B_o^2 B + \tanh\left(\frac{\pi r\theta}{g}\right)\frac{B_o^3}{4} = 0$$

where $B_z(r,\theta,z=0) = B_y(x,y=0) = B$

of which the following is the physically acceptable solution.

$$B_z(r,\theta) = -B_o\left[\cos\left(\frac{1}{3}\cos^{-1}\left(\tanh\frac{\pi r\theta}{g}\right) + 2\pi/3\right)\right] \text{-----(5)}$$

Equation (5) would determine the azimuthal field variation, for H=2 machines, from middle of the hill to the middle of the immediate valley.

3. Cyclotron Parameters

All the design parameters of the sector magnet viz. $\langle B\rangle$, F, ν_z, ν_r, $k(r)$ etc. can analytically be obtained from equation (5).

(a) Average Magnetic Field: The integration of equation (5) yields $\langle B\rangle$ as given below.

$$\langle B\rangle = \frac{3gB_o}{2\pi^2 r}\Big[\log(4\sin^2 x_2 - 3\sin^2 x_2 \operatorname{Cosec}^2 x_1) - \log(4\sin^2 x_2 - 3)\Big] \text{ ------------(6)}$$

where

$$x_i = \tfrac{1}{3}\cos^{-1}\left(\tanh\left(\tfrac{\pi r\theta_i}{g}\right)\right) + 2\pi/3,$$

$\theta_1 = \theta_H$, $\theta_2 = \frac{\pi}{N} + \theta_H$, $i = 1,2$.

θ_H is the half angular width of sectors, N is the number of sectors.

(b) Flutter: The exact formulae for flutter can be had as follows.

$$F(r) = \frac{\langle B_z^2(r,\theta)\rangle}{\langle B_z(r,\theta)\rangle^2} - 1.$$

where

$$\langle B_z^2(r,\theta)\rangle = \frac{9B_o^2 g}{\pi^2 r}\left[\frac{-1}{4}\log\frac{\tan x_2/2}{\tan x_1/2} - \frac{1}{12} * \log\frac{\tan 3x_2/2}{\tan 3x_1/2}\right] \text{ ---------(7)}$$

(c) Sector Geometry: Conversely, the desired width of sectors for the given field profile can also be calculated by solving the equation (6). This equation for the desired $\langle B\rangle$ profile would reflect the implicit dependence of $|\theta_H|$ over r. Newton Bisection method has been employed to solve the equation. The contours of the sectors can be obtained for any desired $\psi_o(r)$, spiral angle variation,

$$\tan\psi_1(r) = \tan\psi_o(r) - |\theta_H|$$

$$\tan\psi_2(r) = \tan\psi_o(r) + |\theta_H|$$

where $\psi_o(r)$ is chosen for the required $\nu_z(r)$.

4. Fringing Field Effects

So far, the analysis has assumed that the radii of the sectors are infinite in extent. The finiteness of the dimensions of the sectors modifies the field in the outer region. Figure 2 represents the pole shape of the outer edge. The resultant transformations for this can be written as

$$z = \frac{g}{\pi}\left[\sqrt{\omega+1} - \frac{1}{2}\log\frac{\left[(\omega+1)^{1/2}+1\right]^2}{\omega}\right]$$

$$\chi = -\frac{iV_o}{\pi}\log\omega$$

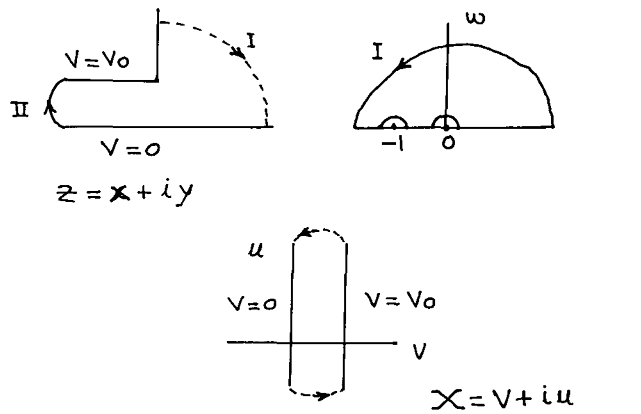

Fig.2: Transformations for outer boundary.

These transformations subsequently give

$$f_H(r) = \tanh\left[\frac{1}{f_H(r)} - \frac{\pi(r-r_o)}{g}\right] \text{ ----------(8)}$$

where r_o is the max. cyclotron radius, $f_H(r)$ is dimensionless factor accounting for field fringing in hill region at the outer edge. In equation (8), if g is replaced by h, we obtain $f_V(r)$, the field fringing factor for Valley. This implicit equation has been solved by NEWTON BISECTION method. The field distribution function of equation (5) gets modified as;

$$B_z(r,\theta) = -f(r)\left[\cos\left(\tfrac{1}{3}\cos^{-1}\left(\tanh\tfrac{\pi r\theta}{g}\right) + \tfrac{2\pi}{3}\right)\right]B_o \text{ --(9)}$$

where

$$f(r) = \begin{matrix} f_H(r) \\ f_V(r) \end{matrix} \text{ for } \begin{matrix} \theta_o \leq \theta \leq \theta_1 \\ \theta_1 < \theta \leq \theta_2 \end{matrix}$$

The modified expression for average field would be written as;

$$\langle B\rangle = \frac{3gB_o}{2\pi^2 r}\Big[f_H(r)\left[\log(4\sin^2 x_o - 3\sin^2 x_o \operatorname{Cosec}^2 x_1) - \log(4\sin^2 x_o - 3)\right] + f_V(r)\left[\log(4\sin^2 x_2 - 3\sin^2 x_2 \operatorname{Cosec}^2 x_o) - \log(4\sin^2 x_2 - 3)\right]\Big] \text{....(10)}$$

where

$$x_i = \tfrac{1}{3}\cos^{-1}\left(\tanh\tfrac{\pi r\theta_i}{g}\right) + 2\pi/3,$$

$\theta_o = 0$, $\theta_1 = \theta_H$, $\theta_2 = \pi/N + \theta_H$, $i = 0,1,2$.

Similarly the $f_H(r)$ and $f_V(r)$ factors would modify flutter also.

5. Comparison

The emperical formulae derived by NEU and WERNER for the case H=2, are quoted below.

$$\langle B\rangle = \frac{B_o}{2}\left[1 + \left(\sigma + .2291\frac{g}{r}\right)\right], \quad F = f^2/2.$$

$$f = \frac{1}{(1+\delta)}\left[2\delta(1-\delta) - .4297\frac{g}{r}\right]^{1/2} \text{ ....--(11)}$$

$$\delta = \sigma + .2387\frac{g}{r}, \quad \sigma = 3|\theta_H|/\pi$$

According to these authors these formulae are valid for $r > 15g/2\pi$. For the chosen geometry, $\langle B \rangle$, F have been calculated by using equation (12). The figure 3 displays these results by dotted lines. And the results based on our method, are represented by solid lines. Both the results should be compared after 38 cm radius. The maximum deviations have been found to be less than 1%.

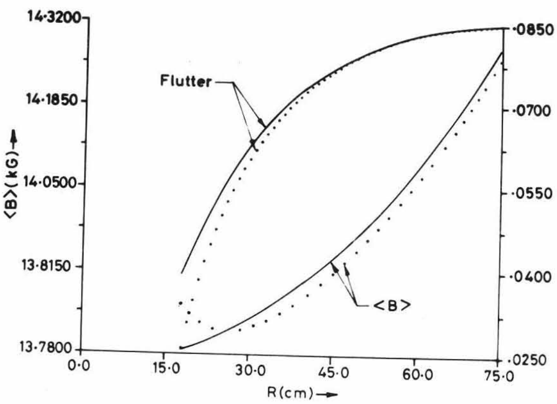

Fig.3: Comparison of results.

6. Application

This method has been used to design the sector geometry for the Medical Cyclotron Facility[6]. The configuration of sectors has been optimised in such a way that it meets the requirements of 30 MeV proton, 7.5 MeV deuteron and 15 MeV alpha particles, by exerting minimum load on the trim coils. Figure 4 shows the requirements of field profiles.

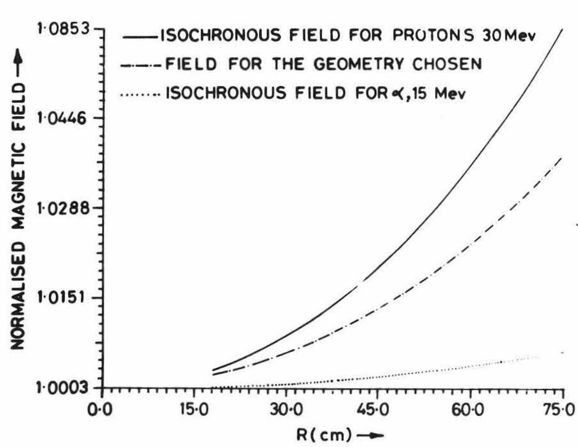

Fig.4: Magnetic field profiles.

The field profile represented by the middle curve has been chosen as the optimum curve for the design of the sectors. Figure 5 shows the computed profile of these sectors and in figure 6 is shown the plots of $B(r,\theta)$ for various radii, for the above mentioned geometry.

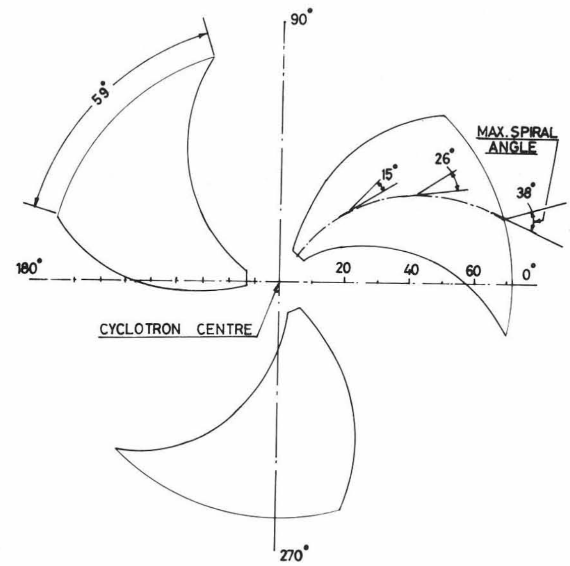

Fig.5: The sector geometry for the medical cyclotron facility.

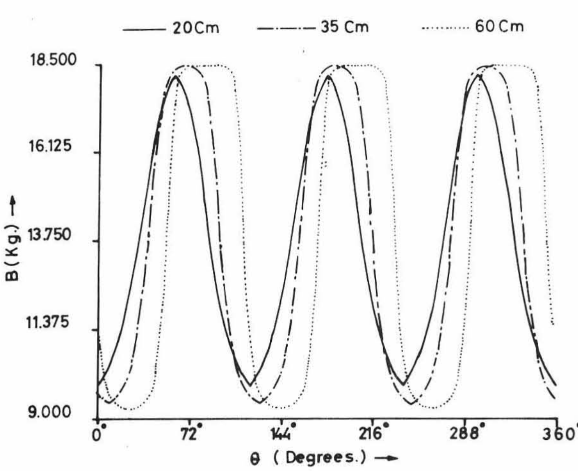

Fig.6: Azimuthal Magnetic field for various radii.

7. Acknowledgement

Authors would like to thank Dr. S.S. Kapoor for his keen interest in the work and active support.

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