MEASUREMENTS OF QUADRUPOLE MAGNETS

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Summary. Measurements carried out on quadrupole magnets using a long asymmetric rotating coil are described. Although the method itself is fairly well-known, the introduction of microprocessors has made this once-tedious technique into a useful and simple method of evaluating quadrupole magnets. The rotating-coil device and a variety of coil sizes are now commercially available. The coil contains a large number of extremely fine wires, embedded in a carefully balanced fibre-glass rotor, resulting in a reasonable induced voltage when the coil is rotated. A digital harmonic analyser is then used to obtain the integrated multipole content of the waveform, while the coil is rotating. By integrating over time, one can average out random noise and increase the reliability and repeatability of the measurements. Because the harmonic analysis is done in real time, the method is quick, easy and accurate, and has been extended to locate the precise magnetic centre of the quadrupole magnet by adjusting its position relative to the coil axis so as to minimize the dipole content of the output waveform. Results of these measurements are compared with those obtained with an optical method using a suspension of magnetite. The observed light pattern is explained analytically.

Introduction

Measurements of multipole content, magnetic axis and effective length have been carried out on more than a hundred quadrupole magnets in use at the NAC. These were necessary, firstly as a check on whether the magnets met their specifications, secondly to enable us to align the quadrupoles along the beamlines, and thirdly to allow us to use the multipole content of the quadrupole magnets in suitable beam transport programs.[1-2] For measuring the effective lengths we used a conventional set-up with a Hall probe on an arm and an x,y-table controlled by a microprocessor. However, for the measurement of multipole content and magnetic axis we used a rotating-coil system.[3] This proved to be extremely simple and effective, and is probably less prone to errors than a system in which a Hall probe is rotated around the axis or moved on an x,y,z-table.

A recent SLAC publication[4] gives a survey of methods for measuring the harmonic content of magnets, and refers to the use of various types of rotating coil, including that reported here.

Measurement of Multipoles

The radial and tangential components of the magnetic field in a quadrupole magnet can be expressed as Fourier series:

$$B(r) = \sum_{n=1}^{\infty} n\, A_n\, r^{n-1} \cos(n\theta+\theta_n),$$

and $$B(\theta) = \sum_{n=1}^{\infty} n\, A_n\, r^{n-1} \sin(n\theta+\theta_n).$$

For an ideal quadrupole only the A_2 components can exist; however, end effects and mechanical imperfections in the construction of the magnet, including such things as non-identical coil configurations, lead to the existence of other terms in the series. If perfect quadrupole symmetry is achieved, then only terms with n = 2,6,10.. can exist.[5]

An updated version[3] of the method suggested by Cobb et al.[6] is used to determine the coefficients A_n: an asymmetric coil is rotated at a constant angular speed in such a way that one leg of the coil lies on the symmetry axis of the quadrupole. The induced voltage waveform is then analysed by means of a commercial spectrum analyser. Modern spectrum analysers incorporate dedicated microprocessors which perform the fast Fourier transform and display the harmonic content of the waveform on a screen in real time. In addition, the process can be repeated any number of times in such a way that the result is integrated over many rotations of the coil. This averages out the random noise, and the calculation converges towards a result which is very reproducible in subsequent measurements. A block diagram of the system is shown in Fig. 1.

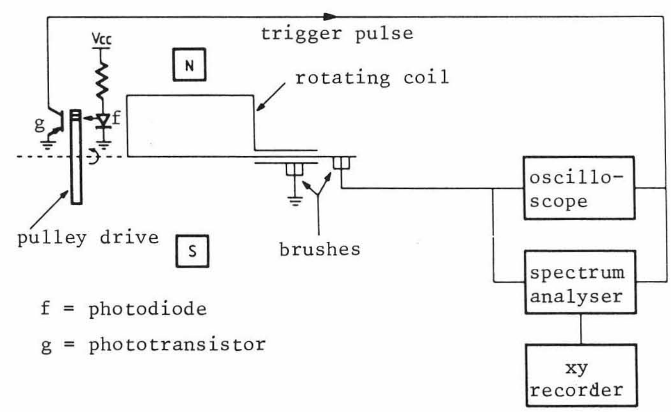

Fig. 1. The harmonic analysis system.

In order to familiarise ourselves with the theory and operation of the harmonic analysis system for determining multipole content[6], we constructed a small working model of the rotating-coil device. This simple model, shown in Fig. 2, has a single-turn coil, again with one leg coincident with the axis of rotation. (We used a single piece of stiff enamelled copper wire soldered to a piece of copper pipe). The coil is driven by a motor and gearbox at about 7.5 Hz, and the induced signal is collected by primitive brushes (copper shim) from slip-rings. We simulated a quadrupole field using four bar-magnets, as shown. A useful feature of this is that it is a rather poor quadrupole, and is hence rich in the harmonics which interest us, i.e. those which are normally undesirable. The measured voltage waveform from the primitive model was analysed and the harmonic content obtained.

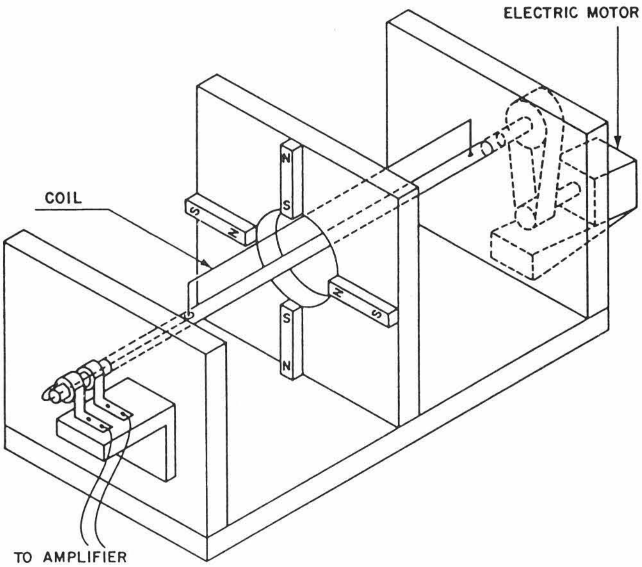

Fig. 2 Primitive model of rotating-coil device for testing harmonic analyser.

The amplitudes of the various harmonics were then inserted into the Fourier series, and the waveform was reconstructed using a computer. The comparison between the original and reconstructed waveforms demonstrated the success of the analysis. It is interesting to note that even this simple device was able to demonstrate the presence of a small dipole component, i.e. that of the earth's magnetic field!

The measurements on the real quadrupole magnets were performed using a much more sophisticated rotating coil device which was manufactured for us, and which is now commercially available. This device can accept magnets of up to one tonne in mass, and has a built-in table with full 3-dimensional positional and rotational adjustment. We also needed coils of various sizes to permit us to measure quadrupoles with apertures of 75, 100 and 150 mm respectively. The completed device is shown in Fig. 3. The asymmetric coil is embedded in a rotor made of glass-fibre reinforced epoxy, and each is dynamically balanced to minimise vibration. The coil is constructed using very fine wires in a multi-core cable, joined in such a way that 40 turns are created. A trigger signal for the harmonic analyser is also provided once per revolution by a disc and photodiode device attached to the rotor.

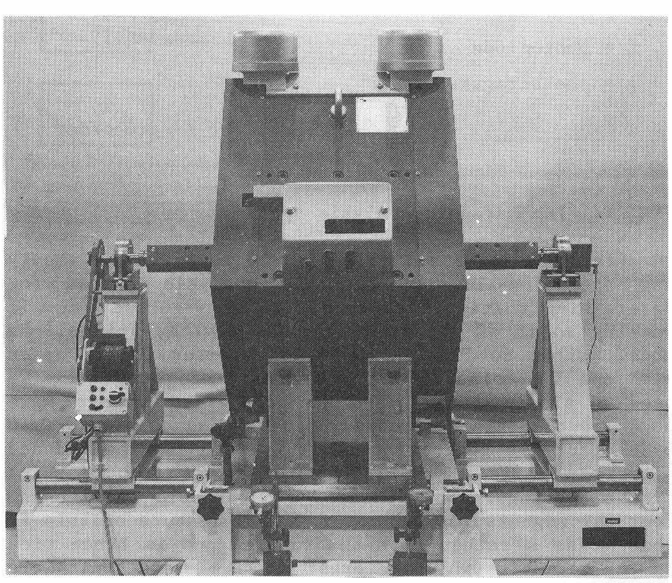

Fig. 3 A quadrupole magnet mounted on the rotating-coil device for harmonic analysis.

Results

The presence of higher-order multipole components is acceptable only as long as the effect of these multipoles on the beam is less than the effects of chromatic aberration. We examined these effects using TURTLE[2] and established the following criteria for typical combinations of quadrupoles in use at the NAC:

1. The amplitudes of the individual multipoles, measured with a long coil at 75% of the radius of the aperture should not exceed 0.5% of the integrated quadrupole amplitude at the same radius.

2. The sum of all multipoles higher than quadrupole, measured as above, should not exceed 1.5% of the integrated quadrupole amplitude.

In practice the coils extend well beyond the pole-pieces of the quadrupole, and, especially where mirror-plates are used, are effectively "at infinity". A typical spectrum of harmonics observed with one of our quadrupole magnets is shown in Fig. 4. The table below reflects the harmonics, measured at 75% of full aperture, expressed as percentages of the quadrupole content, as well as the sum of all harmonics higher than quadrupole, expressed similarly. The values are the averages for all quadrupoles of the respective type:

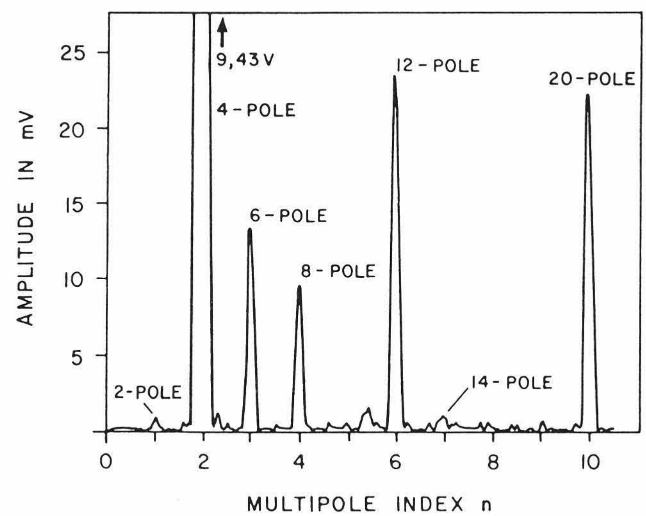

Fig. 4 Harmonic amplitudes obtained from the spectrum analyser for a typical quadrupole magnet.

Quad type*	$\frac{B_3}{B_2}$ %	$\frac{B_4}{B_2}$ %	$\frac{B_5}{B_2}$ %	$\frac{B_6}{B_2}$ %	$\frac{B_{10}}{B_2}$ %	$\sum_{n>2}^{10} \frac{B_n}{B_2}$ %
Q75S	0,123	0,083	0,055	0,099	0,213	0,578
Q75L	0,020	0,044	0,072	0,207	0,207	0,382
Q75M	0,058	0,010	0,029	0,186	0,022	0,329
Q75H	0,062	0,045	0,052	0,513	0,071	0,762
Q100	0,331	0,042	0,018	0,167	0,187	0,745
Q100S	0,179	0,014	0,014	0,238	0,040	0,485
Q150S	0,138	0,080	0,024	0,203	0,193	0,647

* A suffix S indicates a short quadrupole.

Accuracy

The two most important sources of error in these measurements are (a) the accuracy with which the fast Fourier transform is performed by the spectrum analyser (given as 3% by the manufacturer) and (b) the precision with which the measuring coil is constructed. The fractional error δC_n in the emf amplitude C_n of a given multipole n caused by a variation δr in the (average) radius r of the coil is given to first order in r by:

$$\frac{\delta C_n}{C_n} = \frac{n\,\delta r}{r} .$$

In practice it is the fractional error in the ratio of the given multipole amplitude to that of the quadrupole amplitude which is important. To first order:

$$\frac{\delta(C_n/C_2)}{C_n/C_2} = (n-2)\,\frac{\delta r}{r} .$$

If we assume an error of 0.1 mm in the average radius of the coil, then this fractional error varies in our case from 0.17% for the sextupole (n=3) in the case of a 60 mm coil radius, up to 2.5% for n=10 and a 32 mm coil radius. Combining this with the inherent inaccuracy of the analyser, we find that we have an error of approximately 5.5% in the worst case, i.e. for the 20-pole component in the smaller magnets.

False Multipoles

An expression has been derived for the false multipole components induced in an asymmetric coil of radius r which is displaced by an amount ρr from the magnetic symmetry axis of a quadrupole of effective length L. The assumptions are identical to those of Fyvie and Lobb[7], but the result differs slightly. The expression gives the emf induced in a single-turn coil rotating with angular velocity ω:

$$\text{emf} = \omega L \sum_{n=1}^{\infty} n A_n r^n \sum_{m=0}^{n-1} \left\{ \frac{(n-1)!}{(n-1-m)!\,m!} \rho^m \right\} \sin[(n-m)\omega t + m\phi + \theta_n]$$

where ϕ is the angle between the displaced axis and the magnetic axis. If we evaluate the expression in curly brackets, we have the ratio of the emf amplitudes of the false harmonics F_{n-m} to that of the source harmonic S_n in each case:

n	m	$(n-m)\omega$	F_{n-m}/S_n
2	0	2ω	1
	1	ω	ρ
3	0	3ω	1
	1	2ω	2ρ
	2	ω	ρ^2
10	0	10ω	1
	1	9ω	9ρ
	2	8ω	$36\rho^2$
	3	7ω	$4\rho^3$
	:	:	:

This table agrees with that of Fyvie and Lobb[7] and indicates the origin of the dipole component observed when the quadrupole is not correctly aligned with the axis of the rotating coil. This can be very useful for aligning the quadrupole magnet. (See below).

Magnetic Axis Determination

All multipoles higher than dipole have a magnetic axis where the field is zero. It is important for beam transport elements to be aligned so that this magnetic axis is coincident with the central axis of the beam transport system, otherwise steering effects will be observed, resulting in oscillations of the beam path about the central axis, as well as distortions caused by the misalignment of the beam. Calculations have shown that the beamline elements should be aligned to about 0.1 mm, to avoid the build-up of serious misalignment and resulting oscillations in a cyclotron. In practice, steering magnets can always rectify this problem, but their use is generally a nuisance, and steerers cannot always provide the dog-leg path needed to correct such misalignments at the right place.

Optical Method. We used an optical method[8], in which a colloidal suspension of magnetite is held between two thin glass plates 1 mm apart, and placed inside the quadrupole magnet, so that the magnetic axis passes through the glass holder. We found that the best results were obtained by using two fibre-optical light sources, placed above and to the side of the sample holder, as indicated in Fig. 5. Each light source gives rise to a visible line of light, passing through the magnetic axis, and the combination of two light sources provides a clear cross-shaped pattern (Fig.6), easily visible in daylight conditions. The intersection of the lines is the magnetic centre of the quadrupole at that point.

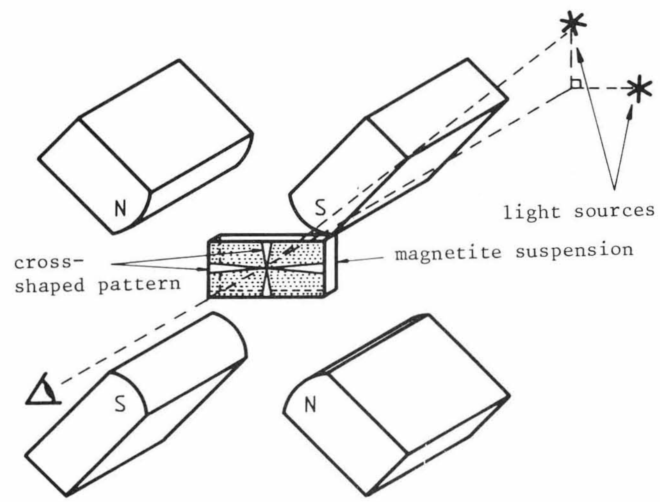

Fig. 5 Experimental arrngement for optical method of determining magnetic centre.

In practice[9] we found that a suspension of magnetite in a mixture of glycerine and water was much simpler than preparation of the colloidal suspension; the only disadvantage is the fact that the suspension settles out in a few minutes, and therefore needs to be shaken up occasionally. The pattern observed results from reflection of light from the aligned magnetite crystals, and we examined the pattern using a single light source at various positions around the axis of the quadrupole magnet. The magnetite crystals arrange themselves along the hyperbolic field lines inside the quadrupole, but the observed pattern depends on the position of both observer and light source. If we define the position of the observer (x_1,y_1,z_1), a single magnetite crystal (x_2,y_2,z_2), and the light source (x_3,y_3,z_3), we can derive an expression for the observed pattern. If we postulate that the quadrupole is arranged so that the poles lie on the x and y axes, the light can only be reflected towards the observer when:

$$(z_2-z_3)y_1y_2 + (z_3-z_1)y_2^2 + (z_1-z_2)y_3y_2 +$$
$$+ (z_3-z_1)x_1x_2 + (z_1-z_3)x_2^2 + (z_2-z_1)x_3x_2 = 0.$$

This condition can be expressed in the form

$ax_2^2 + bx_2 + c$, where

$$a = (z_1-z_3)$$
$$b = (z_2-z_1)x_3 + (z_3-z_1)x_1$$
$$c = (z_2-z_3)y_1y_2 + (z_3-z_1)y_2^2 + (z_1-z_2)y_3y_2,$$

where everything is known except x_2 and y_2. A computer program enables us to vary y_2 values, and to solve the quadratic for the corresponding x_2 values. These pairs represent the coordinates of positions in the plane of the (thin) suspension from which light is scattered to the observer, and thus joining these plots produces the observed pattern. Fig. 7 shows such patterns calculated for various orientations of the light source.

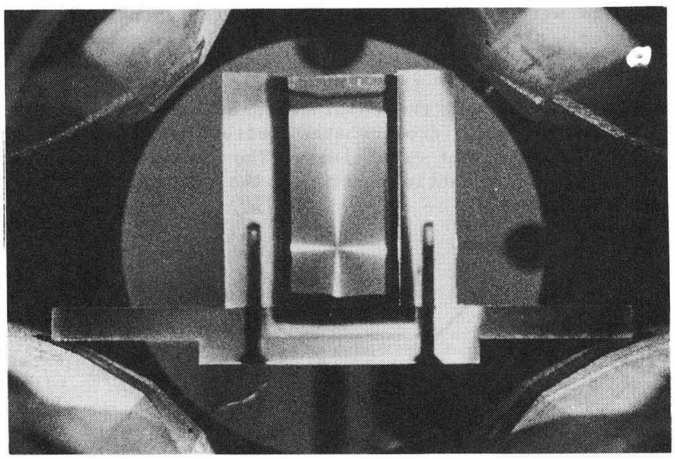

Fig. 6 Reflection pattern observed with two light sources and magnetite suspension.

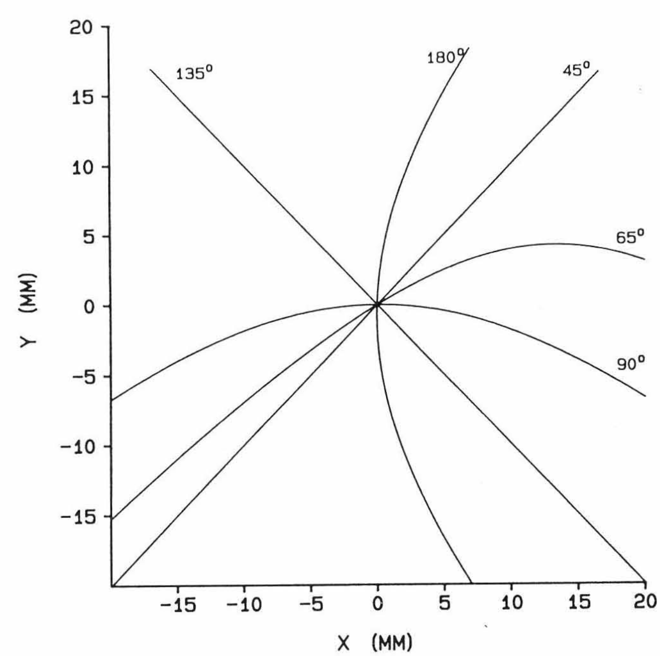

Fig. 7 Lines of maximum reflectiion for various angles between source and horizontal plane.

A theodolite is used to locate the intersection of the lines derived from two light sources, as described above. The magnetic axis is then transferred to survey markers mounted on top and at each end of the quadrupole, while a precise optical level is used to ensure that these markers are at the correct height above the magnetic axis. By displacing the quadrupole in a random way, and then repeating the process to return the magnet to the same position, we determined that the repeatability of the method was 0.04 mm.

Rotating Coil Method. The adjustment described above was carried out very conveniently on the x,y-table which is an integral part of the rotating-coil harmonic analysis device described earlier. And in fact it is also possible to use the asymmetric rotating coil device to align the magnetic symmetry axis of a quadrupole magnet to an optically defined axis in a similar way. The method proceeds along exactly the same lines as the harmonic analysis described, and the magnet is positioned so that the dipole component is reduced to zero. If the magnet is reasonably close to the correct position, then it is a simple matter to move it the last few tenths of a millimetre at each end of the magnet, using the adjusting screws provided. However, one must be aware that the method is not sensitive to pure rotation of the magnet about its centre, as the dipole components induced at each end of the quadrupole are of opposite sign, and thus cancel out in the observed waveform.

In order to use this method, a theodolite is set up on a rigid support and aligned precisely with the axis of rotation of the measuring coil. This is achieved by removing the coil, and replacing the supporting bearing by a cylindrical target, containing a light-emitting diode behind a tiny hole: the theodolite can then be brought into exact alignment with the axis of rotation of the coil.

By using the optical method first, and then using the rotating-coil method to reduce the dipole component to zero, and reading the small movements required on tracking micrometers, we found that the two methods agreed to better than 0.04 mm, although with the majority of magnets the differences amounted to between 0.02 and 0.03 mm.

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