A FORMULATION OF THE MASS OF CYCLOTRON MAGNETS IN TERMS OF ION CHARACTERISTICS AND MAXIMUM FIELD

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Magnetic field characteristics of cyclotrons and design criteria of electromagnets are examined in order to determine the amount of magnet steel required for such accelerators in terms of ion characteristics and the chosen field strength (orbit frequency) at extraction. A simple formulation of the mass of cyclotron magnets is presented, and its validity is discussed by comparing the results obtained for a number of well-documented isochronous cyclotrons and synchrocyclotrons with the actual mass of their magnets. Some implications for the design of cyclotron magnets are pointed out.

Introduction

The amount of magnet steel required for a cyclotron is an important parameter in evaluating design alternatives and the feasibility (as well as the cost) of such accelerators. A suitable formulation of this quantity in terms of characteristic cyclotron parameters would thus not only allow one to compare and appraise existing cyclotron magnets, but could also be used as a valuable aid in the design and development of future ones. With the evolution of isochronous cyclotrons, however, the beam properties and design features of such accelerators have become remarkably diverse.[1,2] This is particularly apparent from the characteristics of their magnets and leaves the impression that a design parameter such as the amount of magnet steel depends on so many different considerations for a particular cyclotron that they cannot be taken into account by a simple formula.

The demand for higher ion energies and beam intensities in association with better beam quality and extraction efficiency has been met by separated-sector (ring) cyclotrons for instance, where the classical H-type electromagnet of solid-pole cyclotrons is replaced by a complex configuration of separate C-shaped magnets. Several have pole geometries with spiral edges to improve the vertical focusing of the beam, and some of them are also flared radially in order to contribute to the required relativistic field increase. The ions accelerated and energies achieved in ring cyclotrons are comparable with those of synchrocyclotrons, but the question arises whether the different magnet characteristics have a significant influence on the amount of steel required for such machines.

Separated-sector cyclotrons are also used as exclusively heavy-ion accelerators on the other hand, an application in which they are now successfully challenged by the first superconducting cyclotrons. The latter machines feature what could be called 'pot' magnets with extremely efficient electromagnetic design characteristics, because the yoke completely encloses the superconducting coils which are arranged around the pole geometry as in a solid-pole cyclotron. Such magnets operate at much higher field levels (~5 tesla) and are considerably smaller than conventional electromagnets for the same ion energy per nucleon. The magnetization of the steel is completely saturated under these conditions and the field contributed in the beam region by the coils is higher than that due to the polarization of the material. To some extent the yoke acts also as a magnetic shield for the environment of such cyclotrons. It seems plausible, therefore, that design considerations influencing the amount of steel required for superconducting cyclotron magnets should drastically differ from those for the sector magnets of ring cyclotrons as well as from those for conventional solid-pole magnets.

The aim of this paper is to examine essential magnetic field requirements of cyclotrons and basic design criteria of electromagnets in order to establish how they together determine the amount of magnet steel approximately required in such accelerators. For this purpose the total magnetic flux and other characteristics of the magnetic circuit are expressed in terms of the maximum magnetic rigidity required at extraction (type and energy of ion) and the chosen field strength (orbit frequency). This leads to a simple formulation of the mass of cyclotron magnets in terms of ion characteristics and maximum field. The validity of the resulting formulation is examined for a number of well-documented isochronous cyclotrons and synchrocyclotrons.[3] Some implications for the design of cyclotron magnets are pointed out and discussed in the process.

Essential Magnetic Field Requirements

In cyclotrons the magnetic field has to ensure that the orbit frequency of the circulating ions remains in tune with the rf-acceleration voltage over the whole radial range of the machine. Simultaneously the field must provide suitable focusing forces for the ion beam. While the radial focusing action can be regarded as essentially inherent in the magnetic field required, special provisions have to be made for the vertical focusing of the beam. Either radially decreasing fields with low gradients (classical and synchrocyclotrons) or azimuthal field modulations (isochronous cyclotrons) are suitable for this purpose.

Isochronous Cyclotrons

The average field B of isochronous cyclotrons (ISC) increases with radius R to compensate for the relativistic mass increase of the ions such that

$$B(R) = B_o \times \gamma(R) \qquad (1)$$

where $\gamma = (1-\beta^2)^{-\frac{1}{2}} = m/m_o = 1 + T/E_o$ and $\beta = v/c$. T and $E_o = m_o c^2$ are the kinetic energy and rest energy of the ion respectively, v denotes the ion velocity, c the velocity of light, m the relativistic and m_o the rest mass of the ion. The ion velocity can be expressed in terms of the chosen revolution frequency ω_o at any radius R by $v = R\omega_o$ in order to determine $\gamma(R)$. B_o is given by $B_o = m_o\omega_o/q$, where q denotes the charge of the ion.

A conservative estimate of the total magnetic flux Φ which must be provided in an ISC for a particular ion and energy can be obtained by integrating $d\Phi = B(R)2\pi RdR$ from injection to extraction. Using (1) and assuming that injection takes place very near the cyclotron centre (which is realistic in most practical cases even for ring cyclotrons) the integration of $d\Phi$ between R=0 and R_e leads to

$$\Phi_e = 2\pi B_e R_e^2/(\gamma_e + 1), \qquad (2)$$

where the subscript e refers to extraction. This equation takes neither the fringe field nor any stray flux into consideration.

For a particular ISC with given R_e the maximum magnetic flux is obtained for the maximum value of $B_e/(\gamma_e + 1)$. In multi-particle variable-energy machines this is usually the case when heavy ions are accelerated which require the maximum field at extraction, but at the same time have the smallest relativistic mass increase. For low energies per nucleon ($\gamma_e \approx 1$) the magnetic flux may be approximated simply by $\Phi_e \approx \pi R_e^2 B_e$. The magnetic rigidity $B_e R_e = (m_o c/q)\gamma_e \beta_e$ required at extraction can be used to eliminate R_e from (2). Taking into account that $\gamma^2\beta^2 = \gamma^2 - 1$ this gives the relation

$$\Phi_e = 2\pi\ (m_o c/q)^2(\gamma_e - 1)/B_e \qquad (3)$$

which shows that for the same ion and energy the magnetic flux in different isochronous cyclotrons is inversely proportional to the field at extraction. Clearly this is the main reason why superconducting cyclotron magnets are much smaller than conventional ISC magnets.

Equation (3) is based only on the field requirements for isochronism and does not take those for vertical focusing into account. Such considerations become more important as γ_e increases, because the azimuthal field modulation has then to be stronger or more complicated in order to compensate for the increased vertical defocusing effect of the higher radial gradient of the average field. One can expect, therefore, that this influence is particularly significant for ring cyclotrons.

Classical and Synchrocyclotrons

The magnetic field of classical and synchro-cyclotrons decreases radially to provide the vertical focusing forces required for the beam. This can be expressed in the form

$$B(R) = B_e(R/R_e)^n \qquad (4)$$

where the field index $n = (R/B)(dB/dR)$ is related to the field gradient and must be negative, but may change with R. The field gradient is always small, however, either to avoid an excessive phase slip of the beam (classical cyclotrons) or to keep the range of frequency modulation within acceptable limits (synchrocyclotrons). Due to the difference in radial field shape the magnetic flux in such cyclotrons is always higher than for an ISC with the same extraction radius R_e and field B_e (see fig. 1). Using (4) and assuming a constant field index to facilitate the integration of $d\Phi = 2\pi B(R)RdR$ between R=0 and R_e one obtains

$$\Phi_e = 2\pi\ B_e R_e^2/(2 + n) \qquad (5)$$

where $n < 0$ and $|n| \ll 1$. If n is not constant, but changes with R, a representative value can be used in (5) instead of n.

Since the field gradient is inherently small, the magnetic flux of such cyclotrons can always be approximated by $\Phi_e \approx \pi R_e^2 B_e$ according to (5). In practice Φ_e is here not much larger, therefore, than the flux required for equivalent isochronous cyclotrons with $\gamma_e \approx 1$ and the same B_e and R_e. Using $B_e R_e$ to eliminate R_e from (5) as we did with (2) leads to

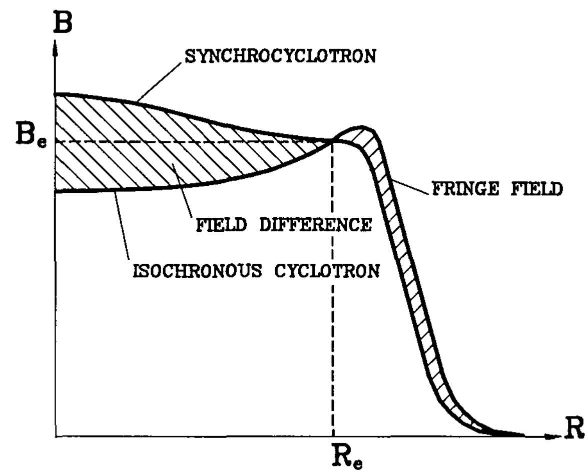

Fig. 1 Radial field characteristics in different cyclotrons with the same radius R_e and field B_e at extraction.

$$\Phi_e = 2\pi\ (m_o c/q)^2(\gamma_e^2 - 1)/B_e(2 + n) \qquad (6)$$

which differs from (3) by a factor $(\gamma_e + 1)/(2 + n)$. This factor is significantly larger than one only when the energy per nucleon is high so that γ_e becomes considerably larger than one. This applies particularly when the magnetic flux in large synchrocyclotrons is compared with that for ring cyclotrons. Otherwise the equations (3) and (6) give approximately the same result. In some synchrocyclotrons an azimuthal field variation is added to improve the vertical beam focusing. The radial field shape of such a hybrid machine falls somewhere between those indicated in fig.1, and its magnetic flux between the values defined by (3) and (6).

Strong Vertical Focusing

The field index in an ISC is positive and equal to $\gamma^2 - 1$. Its defocusing effect must be converted into a net vertical focusing force by the influence of the azimuthal field modulation. This requirement can be represented in the form

$$B_h/B = f > (\gamma^2 - 1)/(1 + 2\tan^2\psi) \qquad (7)$$

where f is the field flutter, B_h the hill field and ψ the spiral angle of the azimuthal field variation.

As γ increases it becomes more difficult to satisfy (7), so that f and ψ must be raised independently or simultaneously. For a particular ISC the upper limit is defined by the maximum γ_e at extraction, and in cyclotrons with high energies per nucleon the flutter is always quite large. The value of f is determined by the amplitude of the azimuthal field variation and increases as the azimuthal width of the hills becomes smaller. This is significant for the magnet design in that for larger f the hill field becomes a more important magnetic parameter than B. In ring cyclotrons for instance, only B_h and the azimuthal magnet width determine B, if the field between the magnets is zero. The spiral angle does not have such a magnetic significance directly. The relation (7) indicates, however, that the spiral term considerably reduces the requirement for a high flutter.

The above considerations show that according to (7) the average field B_e in (3) should be replaced by $B_e = B_{he}/f_e$ for an ISC, in particular for ring cyclotrons. In practice this is equivalent to $B_e = B_{he}/(\gamma_e + 1)$ for most ring cyclotrons with very few exceptions (TRIUMF, SIN-Injector II) as can be established from their field properties.[3] Using (3) this leads to

$$\Phi_e = 2\pi(m_oc/q)^2(\gamma_e^2-1)/B_{he} \qquad (8)$$

which is equivalent to (6) except for the factor (2+n). Equation (8) takes vertical focusing requirements into account to some extent. For the TRIUMF cyclotron Φ_e is actually much smaller than according to (8), because $f_e \ll \gamma_e+1$ applies, which only becomes possible due to the large spiral angle. For the SIN-Injector II on the other hand, f_e is considerably larger than (γ_e+1) in order to provide the increased vertical focusing required for exceptionally high beam intensities.

Characteristics of Cyclotron Magnets

Magnetic Circuit

Electromagnetic criteria used in the design of cyclotron magnets are well established and can be applied to formulate the amount of magnet steel M_{Fe} required for the magnetic circuit according to

$$M_{Fe} = \rho_{Fe} \times A_\phi \times L_\phi \qquad (9)$$

where A_ϕ and L_ϕ denote the average cross-section and representative length of the magnetic circuit respectively, and $\rho_{Fe} = 7.86$ ton/m^3 is the density of iron. A_ϕ is always proportional to the total magnetic flux Φ_m at maximum excitation and can be determined from $A_\phi \approx \Phi_m/(B_{Fe})_m$, where $(B_{Fe})_m$ denotes the representative average flux density in the magnetic circuit for maximum excitation. As a guide for the design of electromagnets one can assume that $(B_{Fe})_m$ should be chosen somewhere between the maximum field required in the pole gap and the saturation flux-density $B_s \approx 2.14$ tesla of soft-magnetic steel. A suitable way to represent this for conventional as well as superconducting magnets is found by taking the geometric average such that $(B_{Fe})_m \approx (B_sB_m)^{\frac{1}{2}}$. This leads to $A_\phi \approx \Phi_m/(B_sB_m)^{\frac{1}{2}}$, where B_m is the maximum field at extraction.

In almost all cyclotrons the magnetic circuit closes only around extraction and L_ϕ is then proportional to the radius of curvature ρ_e of the extraction orbit in the pole gap such that $2\pi\rho_e < L_\phi < 8\rho_e$ (average $L_\phi \approx 7\rho_e$). This gives $L_\phi \approx 7\,(m_oc/q)\gamma_e\beta_e/B_e$ in all cyclotrons with solid-pole (H-type) magnets and $L_\phi \approx 7(m_oc/q)\gamma_e\beta_e/B_{he}$ for most superconducting and ring cyclotrons. The only exceptions are ring cyclotrons with such a large injection radius that H-type magnets can be used (as proposed for kaon-factories)[4,5] and superconducting separated-orbit cyclotrons with the magnet concept of Tritron.[6] Introducing A_ϕ and L_ϕ into (9) gives

$$M_{Fe} \approx 7(\rho_{Fe}/B_s{}^{\frac{1}{2}})\,(m_oc/q)\,\gamma_e\beta_e\Phi_m/B_m{}^{3/2} \qquad (10)$$

where $B_m = B_e$ for solid-pole and synchrocyclotrons, but $B_m = B_{he}$ for superconducting and ring cyclotrons.

Radial Fringe Field

The gap width G of a cyclotron magnet has an important influence on the total magnetic flux of the magnetic circuit. G should be as small as possible in order to reduce the additional flux Φ_a due to the fringe field outside the extraction radius, as well as to reduce the required magnetomotive force. When the rf electrodes are positioned in the pole gap, the minimum gap width (hills) has to be much larger than in cyclotrons where the resonators are arranged between adjoining hills (superconducting and ring cyclotrons), in order to achieve the required peak-voltage and resonance frequency without problems.

The additional flux Φ_a is essentially determined by the characteristics of the radial fringe field of the pole gap. In order to avoid an excessive field reduction at the extraction radius, the pole radius has to be chosen roughly as $(R_e + G_e)$, and the effective magnetic field boundary can then be found at a radius of about $(R_e + 2G_e)$. This gives an additional flux of approximately $\Phi_a \approx \pi B_e\left[(R_e + 2G_e)^2 - R_e{}^2\right]$ which is equal to Φ_e for $G_e \approx R_e/4.8$. In most synchrocyclotrons this is indeed a good but slightly high approximation for G_e, because the first part of the fringe field can still be used to provide the negative field index required. This allows a somewhat smaller pole radius than $R_e + G_e$ and can be taken into account by assuming that $\Phi_a \approx \Phi_e\,(1 + n)$ with $n < 0$, $|n| \ll 1$. The total magnetic flux for synchrocyclotron magnets is then given approximately by $\Phi_e(2 + n)$ with Φ_e as formulated in (6).

For isochronous solid-pole cyclotrons G_e is always somewhat larger than $R_e/4.8$, when G_e is calculated as the geometric average of the gap width for hills (G_{he}) and valleys (G_{ve}). The pole radius $(R_e + G_e)$ and Φ_a due to the fringe field are thus larger than for synchrocyclotrons with the same R_e and B_e (see fig. 1). This is mainly caused by the azimuthal gap variation which requires $G_e > G_{he}$ and $R_{ve} + G_{ve} > R_e + G_e$, whereas G_{he} defines the space available for the dees. The resulting increase in Φ_a can be represented roughly by $\Phi_a \approx \Phi_e\gamma_e$ such that the total magnetic flux becomes $\Phi_e(\gamma_e + 1)$ in such cyclotrons, where Φ_e is given by (3). In contrast, the additional flux Φ_a for superconducting and ring cyclotrons can be neglected because of $G_{he} \ll R_e/4.8$, and therefore (8) still applies for the total magnetic flux in such machines.

As a result of the additional magnetic flux from the radial fringe field of cyclotron magnets the equations (3), (6) and (8) can now be combined to formulate the maximum total magnetic flux as $\Phi_m = 2\pi(m_oc/q)^2\,\gamma_e^2\beta_e^2/B_m$, where B_m is either the average or hill field at extraction for maximum excitation. Introducing Φ_m into (10) and using MKSA units gives (rounded up)

$$M_{Fe}(\text{ton}) = 7500\left[\gamma_e\beta_e/(Q/A)\right]_m^3/B_m{}^{5/2}, \qquad (11)$$

where the factor $f_m = \left[\gamma_e\beta_e/(Q/A)\right]_m^3$ depends only on the ion characteristics at extraction (subscript m indicates maximum value for the whole bracket), and Q/A is the atomic charge-to-mass ratio. B_m (tesla) is the maximum field provided for the ions with f_m at extraction, and the azimuthal average of the maximum field must be used for classical and synchrocyclotrons, as well as isochronous solid-pole cyclotrons. Only for superconducting and ring cyclotrons does B_m denote the maximum hill field in the pole gap at extraction. It is interesting to note that this also applies to the TRIUMF magnet in spite of its large pole gap, because the radial fringe field is cut off sharply.

Results

A suitable method for comparing the actual amount of steel used for the magnets of different cyclotrons with the steel mass M_{Fe} resulting from (11) is illustrated in fig. 2, where the reduced steel mass M_{Fe}/f_m is plotted as a function of B_m. This also allows us to compare different cyclotron magnets with each other in terms of this quantity. The results shown in fig. 2 are based on cyclotron data supplied in the latest 'List of Cyclotrons' at the end of the proceedings of the previous conference.[3] Since it is not always possible to determine $f_m = [\gamma_e \beta_e/(Q/A)]_m^3$ accurately from these data, the results could be slightly different in reality. According to (11) the reduced steel mass should be determined by $M_{Fe}/f_m = 7500 / B_m^{5/2}$, but deviations up to ± 33% may easily occur due to the many assumptions which have had to be made for arriving at this equation. Representative lines for these limits are also indicated in fig. 2. The validity of the formulation (11) within these limits is evident from the graph and extends over nearly one order of magnitude in terms of the field value B_m. It is in fact only due to the development of superconducting cyclotrons that the dependence of M_{Fe} on the maximum field can be given as $B_m^{-5/2}$ with great assurance.

When examining or applying the results it should be kept in mind that H-type magnets for conventional solid-pole cyclotrons with a small extraction radius usually have somewhat less efficient eletromagnetic design characteristics than superconducting and large conventional cyclotron magnets, because the additional magnetic flux Φ_a becomes much larger than the useful flux Φ_e under such circumstances. Also the representative length L_ϕ of the magnetic circuit is relatively longer, because it is then determined to an increasing extent by the size of the main excitation coils and by the required access to the region in and around the pole gap. The result for the TRIUMF magnet is the only one which is far (one order of magnitude) below the amount of steel predicted with (11). This indicates an extraordinary efficient magnet design and justifies a more detailed examination which might be generally useful for large ring cyclotrons.

One reason why the TRIUMF magnet is actually much lighter than predicted has its origin in the fact that the magnet steel is completely saturated although B_m remains very low. This reduces the amount of steel by a factor 2 and necessitates a large pole gap, but the effect of such a choice in terms of a large additional magnetic flux through the radial fringe field is minimized by a coil geometry which is similar to that of superconducting cyclotrons and cuts the fringe field off. The magnetic field due to the coils makes a considerable contribution to the relativistic radial field increase and total field required at extraction, thus reducing the necessary amount of flux originating from the magnetization of the steel by about another factor 2. The remaining saving in steel is achieved by keeping the length L_ϕ of the magnetic circuit as short as possible. In fact L_ϕ is considerably smaller than $7\rho_e$ for this magnet. This can be related to the large spiral angle and low flutter at extraction which allow that the pole geometry is flared strongly towards extraction.

Acknowledgements

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References

1. J.A. Martin, Proc. 10th Int. Conf. on Cycl. and their Applic. (East Lansing, 1984) p.1.

2. J.R. Richardson, ibid p.617.

3. List of Cyclotrons, ibid p.623.

4. M.K. Craddock et. al., Proc. 9th Int. Conf. on Cycl. and their Applic. (Caen, 1981) p. 209.

5. W. Joho, Proc. 10th Int. Conf. on Cycl. and their Applic. (East Lansing, 1984) p. 611.

6. U. Trinks, ibid p.608.

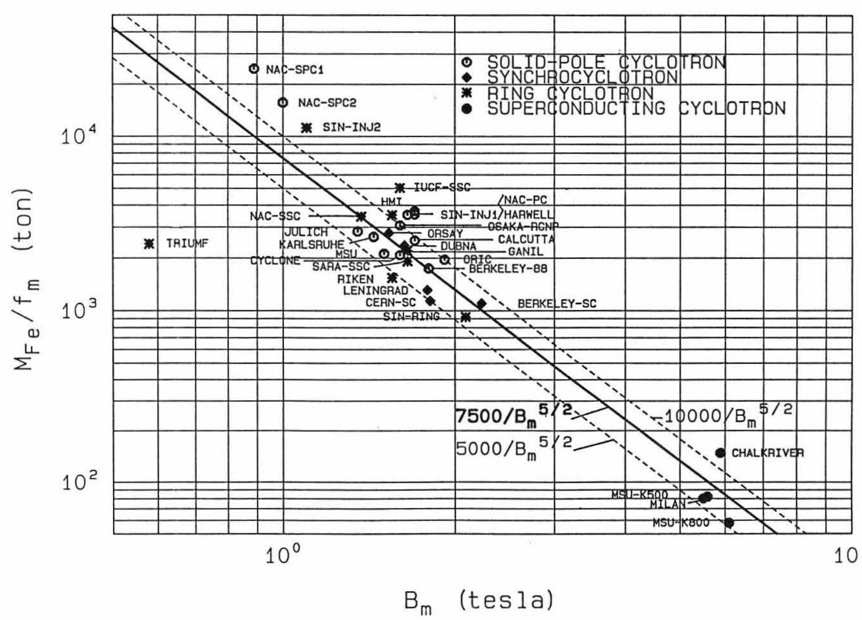

Fig. 2 The steel mass of cyclotron magnets (reduced by the factor $f_m = [\gamma_e \beta_e/(Q/A)]_m^3$ representing the ion characteristics) plotted vs. the maximum field in the pole gap at extraction.