# OPTIMUM DESIGN OF A BEAM TRANSPORT SYSTEM

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## Summary

The matrix method of Brown has been adopted to design the beam transport system of a OSD Magnetic Spectrometer and that of an On-line Isotope Separator. The Computer Code, `ODESSA', developed for this purpose, is capable of optimising various magnetic elements through either first- or second- order aberrations for a given objective function and uses the 'Simplex' algorithm due to Nedler and Mead.

## Introduction

Various types of aberrations (in a beam transport system) contribute significantly to the final image width. Hence, for an optimum design of any such system, one is required to optimise the image width by minimising such aberrations as far as possible. The matrix method, first introduced by Penner[1] and further developed by Brown[2] has been widely used for calculating the various parameters of any beam transport system including momentum dispersion, aberrations, beam trajectory etc. We have developed a Computer Code 'ODESSA', based on the same formalism, which has been used for designing the beam transport systems for a OSD Magnetic Spectrometer and also for an On-line Isotope Separator for our Laboratory.

## Matrix Formalism

In a beam transport system, the final vector v is a function f of (x, θ, y, φ, z, δ) of displacements in the x and y plane, momentum dispersion and path length where x,y are the displacements, θ, φ are the divergences z is the path length and δ is the dispersion of the particle concerned. This is related to the initial vector u by the expression

$$v = [T]\, u \qquad \ldots \qquad (1)$$

where T is the transferred matrix for the complete beam transport system. In the second order approximation which corresponds to an expansion of v about u upto second order in the Taylor series, eqn. (1) is rewritten as -

$$v_i = \sum_j T_{ij} u_j + \sum_{j,k} V_{ijk} u_j u_k \qquad (2)$$

In the expression, the $T_{ij}$ and $V_{ijk}$ matrices are the first and second order elements of [T] and [V] are obtained by multiplying together in correct order the matrices describing the matrix elements of the individual systems. The expression for displacement in the radial plane as given by equation (2) becomes -

$$x = T_{11}x_0 + T_{12}\theta_0 + T_{16}\delta + V_{111}x_0^2 + V_{112}x_0\theta_0 + V_{116}x_0\delta + V_{122}\theta_0^2 + V_{126}\theta_0\delta + V_{133}y_0^2 + V_{134}y_0\phi_0 + V_{144}\phi_0^2 \qquad \ldots \qquad (3)$$

Similar expressions for θ, y and φ may be deduced.

Thus one may calculate the comulative transfer matrix by multiplication of the matrices of the individual elements of a given beam transport system in the order specified. This feature enables one to solve various problems such as determination of envelope of a beam passing through a beam transport system and calculation of various aberrations focussing properties and energy resolution of analyzing magnets, magnetic spectrometers etc. without going through actual ray tracing. Earlier this method was applied by Bhattacharya et al[3] in deriving the orbit properties of an A.V.F. Cyclotron as an alternative to the conventional orbit integration procedure.

## Description of the Code

ODESSA is a FORTRAN programme requiring about 16.5 K words of memory and approximately 6 seconds for calculating each case on the IRIS-80 Computer at VECC. It comprises of a main programme and subroutines DRIFT, DIPOLE, NNEND, NNEXD, QUAD, SEXT, MULT and SIMPLEX. The main programme reads input information regarding the number of elements in the magnet system, the order of occurrence of different elements and the parameters of the magnetic elements. The main programme also transfers control to one of various subroutines DRIFT, DIPOLE, NNEND, NNEXD, OUAD or SEXT in sequence depending on whether the relevant element is a drift space, a dipole, a non-normal entrance edge to the dipole, a non-normal exit edge to the dipole, a quadrupole or a sextupole element.

Finally, an optimization procedure minimizes a given objective function such as 'double focussing', 'minimum aberration' or 'optimum resolving power' depending on the users choice and by varying given number of variable parameters, in an iterative manner.

It is also possible to minimise anyone or all of these objective functions simultaneously. All optimization methods aim at minimizing an objective function of the parameters of the system. Depending on the method used restrictions are usually imposed upon the objective function e.g., it should be a linear function of the parameter, it should have derivative existing at all points or that it should be expressible in a closed form. The method used is the 'Simplex' method of Nedler and Mead[4] which is perfectly generalized in the sense that none of these restrictions be imposed on the objective function. The only limitation being that the number of variable parameters should be less than ten in order that the convergence process may proceed efficiently.

## Application to Specific Beam Transport Systems

i) QSD Magnetic Spectrometer:

One of the facilities now under construction around VEC machine is a Magnetic Spectrometer of QSD type. The design of the spectrometer is similar to the one at LBL, Berkeley and its beam optics is available[5]. We took up the beam optics of the said spectrometer to test our code and did reproduce the results. Figure1 shows schematically the lay-out of various elements of the spectrometer.

For a given momentum the resulting image at the focal plane can be found by using equation (3). In order to have a first order image at the focal plane, the displacement should be independent of $\theta_0$. Hence, $T_{12} = 0$. Also since we are studying the behaviour of a beam for a given momentum $T_{16} = V_{116} = V_{126} = V_{166} = 0$. So the image width $W_m$ along the axis reduces to

$$W_m/2 = T_{11}x_0 + V_{111}x_0^2 + V_{112}x_0\theta_0 + V_{122}\theta_0^2 + V_{133}y_0^2 + V_{134}y_0\phi_0 + V_{144}\phi_0^2 \quad \ldots (4)$$

The resolution of the spectrometer is then given by

$$\frac{\Delta E}{E} = 2W_m/T_{16} \quad \ldots (5)$$

Equations (4) and (5) tell us that the second order aberrations in $\theta_0^2$, $\phi_0^2$, $y_0^2$ contribute significantly to the image width specially when the solid angle of acceptance is large. Hence in order to improve the resolving power the designer has to minimize the second order co-efficients $V_{122}$, $T_{144}$, $V_{133}$, $V_{134}$ etc. with the help of multipole elements. Since the co-efficients $T_{ijk}$'s depend on the parameters of the individual elements of the magnetic system, designing the system for minimum aberration would require calculating the $T_{ijk}$'s for an arbitary set of parameter followed by systematic optimization with a view to minimise the most important aberrations and at the same time achieving best resolution.

The object functions chosen in our case is the image width, inverse of the energy resolution and $\phi_0^2$ aberration. Taking the dipole field strength to be 8.993 kG and $\rho$ = 1.775 m, the strength of quadrupole and the length of the last drift space was allowed to vary to achieve double focussing in the first order. This led to dB/dx = 120.08 kG/meter and L = 1.382 meters. Next, the strength of the sextupole was varied to minimise $\phi_0^2$ aberration, leading $d^2B/dx^2$ = 235 kG/m$^2$. Final first and second order matrix agrees completely with those obtained in the Berkeley design[5].

ii) Gas-jet Coupled ISOL (Isotope Separator On-Line)

An ISOL facility coupled to a Gas-jet Recoil Transport (GJRT) system has been designed for VECC which will allow spectroscopic studies on short-lived nuclides far from the line of β-stability with half-lives ~100 m.sec. and production cross-section ~ 50 μb. The magnetic analyser forms a crucial component which decides the ultimate mass resolving power of the instrument, thereby assuming minimum contamination in the radio-active nuclide of interest. We chose a 90° dipole magnet with 1 m radius of curvature as the main analyzer, because it allows to plan for a near symmetric beam optics as well as less involvement in fabricational complexity. The B$\rho$ has been chosen to suit the energy of the extracted ion beam from the ion-source.

We introduced a non-normal entry (entry angle $\beta_1$ = 40.4° ) and a normal exit for the dipole. This assures double focussing without the help of quadrupoles normally used in similar systems[6].

A sextupole placed down-stream after the dipole has been incorporated in order to correct for the rotation of the focal plane perpendicular to the beam axis. As a first step towards optimization we chose 'Double Focussing' as the objective function while the object distance, image distance and entry angle $\beta_1$, were kept variable. For double focussing one has objective function as

$$F = T_{12}^2 + T_{34}^2 \quad \ldots (6)$$

since $T_{12}^2 + T_{34}^2 = 0$ only when $T_{12}=T_{34}=0$.

In the second stage we introduced the new values of $L_0$, $L_1$ and $\beta_1$. The objective function was $V_{144}$ i.e., the co-efficient of $\phi_0^2$, keeping $R_2$ as variable. The optimization procedure gave $R_2$ = - 0.197 m.

In the third stage we used the modified values as the input and tried to minimise the $\phi_0^2$ aberration and also satisfy the double focussing condition simultaneously, so that objective function is

$$F = T_{12}^2 + T_{34}^2 + V_{144}^2 \quad \ldots \quad (7)$$

while the deflection angle $\theta$ and radius of curvature $\rho$ are the variables.

In the final stage of optimization we compared image width $W_m$ in the two cases: (1) with the modified values of $\theta$ and $\rho$ and (2) without the modified values of and

$$W_m = 0.001591 \quad \text{Case (2)}$$
$$= 0.001589 \quad \text{Case (1)}$$

Since the change in image width was found to be negligible we decided to retain the previous values of $\theta$ and $\rho$ i.e., 90 and 1 meter respectively and calculated resolving power corresponding to $W_m$ = .001591.

The optimised parameters for the mass separator along with the sextupole is given in Table 1. The final first order as well as second order matrix elements are given in Table 2. The details of the other design parameters for the Gas-jet coupled ISOL is being published elsewhere[7].

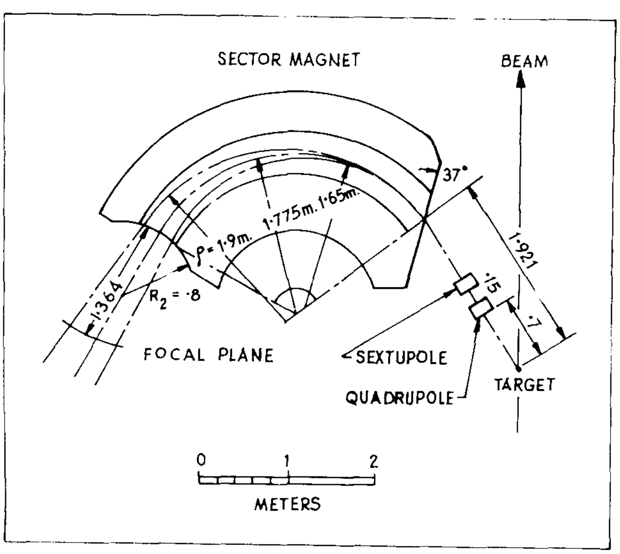


FIG.1 BEAM TRANSPORT OF MAGNETIC SPECTROMETER

Table 1

Parameters for the separator

Dipole

| | |
|---|---|
| Deflection angle ($\theta$) | 90 |
| Radius of Curvature ($\rho$) | 1 metre |
| Field Index (n) | 0 |
| Magnetic field (B) | 10 kG |
| Magnet gap (g) | 7 cm |
| Entry angle ($\beta_1$) | 40.4° |
| Exit angle ($\beta_2$) | 0° |
| Object distance ($L_0$) | 1.96 metre |
| Radius of Curvature ($R_2$) | -0.197 metre |
| Image distance ($L_1$) (from dipole end) | 1.36 metre |
| Magnification ($M_X$) | -0.5 |
| ($M_Y$) | -1.5 |
| Resolving Power ($\frac{m}{\Delta m}$) | 600 |
| (Object size: $x_0$ = 0.25mm, $y_0$ = 2.5 mm, $x_0'$ = 6.25 mrad, $y_0'$ = 45 mrad) | |
| Dispersion (D) | 2.36 metre |
| Weight (including coil) | 2.5 ton |

Sextupole

| | |
|---|---|
| Length | 15 cm |
| $\frac{d^2B}{dX^2}$ | 235 kG/metre$^2$ |

Table - 2

First and Second Order Matrix elements, Units are meters, radians.

| i \ j | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| 1 | -0.51 | 0.002 | | | 2.36 |
| 2 | -1.0 | -1.96 | | | 1.0 |
| 3 | | | -1.5 | 0.002 | |
| 4 | | | -0.851 | -0.67 | |
| 6 | | | | | 1.0 |

| ij | $V_{1\ ij}$ | $V_{2\ ij}$ | Kl | $V_{3\ kl}$ | $V_{4\ kl}$ |
|---|---|---|---|---|---|
| 11 | -3.00 | -1.84 | 13 | -2.25 | -1.12 |
| 12 | -18.07 | -12.38 | 14 | 2.27 | 2.30 |
| 16 | -3.51 | -3.32 | 23 | -13.48 | -5.63 |
| 22 | -28.02 | -20.24 | 24 | -0.51 | -5.75 |
| 26 | -12.79 | -11.58 | 36 | -2.96 | -1.20 |
| 33 | -0.65 | -0.94 | 46 | 10.31 | 7.20 |
| 34 | -4.39 | -5.51 | | | |
| 44 | -0.0009 | -1.43 | | | |
| 66 | -5.31 | -3.54 | | | |

## Conclusion

In this work, we have demonstrated the usefulness of a simple-minded code, such as ODESSA. Based on matrix method beam optics to obtain the optimum parameters of system like QSD spectrograph and magnetic analyser of an ISOL system. The code, in its present form, can handle all types of dipole magnets with non-uniform field, non-normal entry and exit angles, having finite radii of curvature, quadrupole and sextupole magnets and drift spaces. The input requirements for the code is simpler compared to the widely used code 'TRANSPORT' though it is capable of giving comparable results. It is planned to include effects due to solenoids, Einzel lens etc. in the current list of elements for completeness sake.

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