CYCLOTRON MASER COOLING OF ELECTRON AND ION BEAMS

Hidetsugu Ikegami
Research Center for Nuclear Physics, Osaka University
Mihogaoka 10-1, Ibaraki, Osaka 567, Japan
and
Department of Radiation Sciences, Uppsala University
S-75121 Uppsala, Sweden

ABSTRACT

A new principle, "cyclotron maser cooling" of electron and ion beams is presented. Very rapid cooling time of order of microsecond may be obtained of the transverse- and longitudinal-phase spaces of a beam independent of its energy. This cooling can be applied to either continuous or pulsed particle beams.

INTRODUCTION

Already one decade ago, a theoretical investigation was made of the effect of cooling on improving the performance of high energy proton beams.[1)] It was concluded by the authors that the addition of a cooling mechanism such as electron cooling could be useful to increase stability and luminosity of beams in high energy proton storage rings or synchrotrons such as the ones at CERN and Fermilab. Cooling times of 100~1000 seconds seem feasible based on their finding which has, however, never reached the stage of practical use. Another cooling scheme i.e. the so- called stochastic damping method[2)] was first successfully achieved at CERN to cool an antiproton beam. However cooling times are not short enough and the scheme is not suitable for rapid-cycling accelerators and new cooler storage rings in which internal targets are employed.

In electron rings, cooling the longitudinal energy spread occurs naturally as a consequence of the emission of synchrotron radiation (SR). There have, however, been continuous demands also for very small transverse energy or emittance of electron beams in SR rings. The newly proposed monochromatic γ-ray source, "free positoronium annihilation radiation" requires merged beams of electrons and positrons of the best quality.[3)] So far any conventional cooling has not given the desired qualities of circulating electron beams.

In this paper, the author describes a new principle, "cyclotron maser cooling" (CMC) of electrons and ions, which could lead to dramatic improvements of the qualities of high energy particle beams. The principle of CMC may be understood as a stimulated emission of cyclotron radiation.[4,5)] If a magnetic field B_0 is applied along a beam, particles generally emit, in principle, their transverse energy $T_\perp^*$ through the cyclotron radiation. The damping time τ_c of the transverse energy due to cyclotron radiation is known to be $\tau_c = 3\gamma c/[4r_p(\omega_c^*)^2]$ in the laboratory frame with the assumption that $\hbar\omega_c^* < T_\perp^*$. Here, c and r_p denote the speed of light and the classical particle radius, respectively and $\omega_c^*/2\pi(=eB_0/2\pi m_0)$ the cyclotron frequency of particles with rest mass m_0 and charge e. MKSA units will be used throughout this paper. A typical case of electrons with $\gamma = 400(T_e \approx 200$ MeV) and $B_0 = 0.5$ tesla results in a damping time of $\tau_c = 4 \times 10^3 s$. For the case of ions the damping time τ_c is still much longer because of the ${m_0}^3$ dependence of τ_c. The longitudinal phase space, which for the case of electrons is damped by SR, is hardly affected at all by cyclotron radiation. On the contrary as will be described below the principle of CMC takes advantage of an enhanced emission of cyclotron radiation stimulated by an rf field resulting in the damping time being proportional to m_0 . Very rapid cooling of both the transverse and longitudinal phase spaces of electrons and ions are produced in the proposed scheme.

PRINCIPLE OF CYCLOTRON MASER COOLING

In Fig. 1 is shown schematically a basic configuration of CMC applied to circulating particles in a beam storage ring. CMC can be achieved by sending a stimulating rf field antiparallel or parallel to a particle beam. The rf field need not be in the form of a travelling wave and a standing wave is preferrable from a practical viewpoint. In the CMC section, the distribution in the transverse energy of the particles is, however, required to peak at a nonzero energy according to Twiss's theorem on the population inversion.[4] Such a distribution can be realized by changing a part of the longitudinal energy to the transverse energy through tilting the central particle orbit with respect to the axis of the CMC section. The energy of peak position and the width of the distribution can be controlled practically by tuning the lattice magnets in the ring. I shall therefore, develop a basic discussion on the basis of a quantum mechanical approach by Schneider[4] without assuming a definite form of the distribution, which allows the simple estimate of CMC.

In the presence of the external rf field of angular frequency ω^* characterized by a Poynting vector of energy flow density $I^*(\omega^*)$, particles gyrating in the

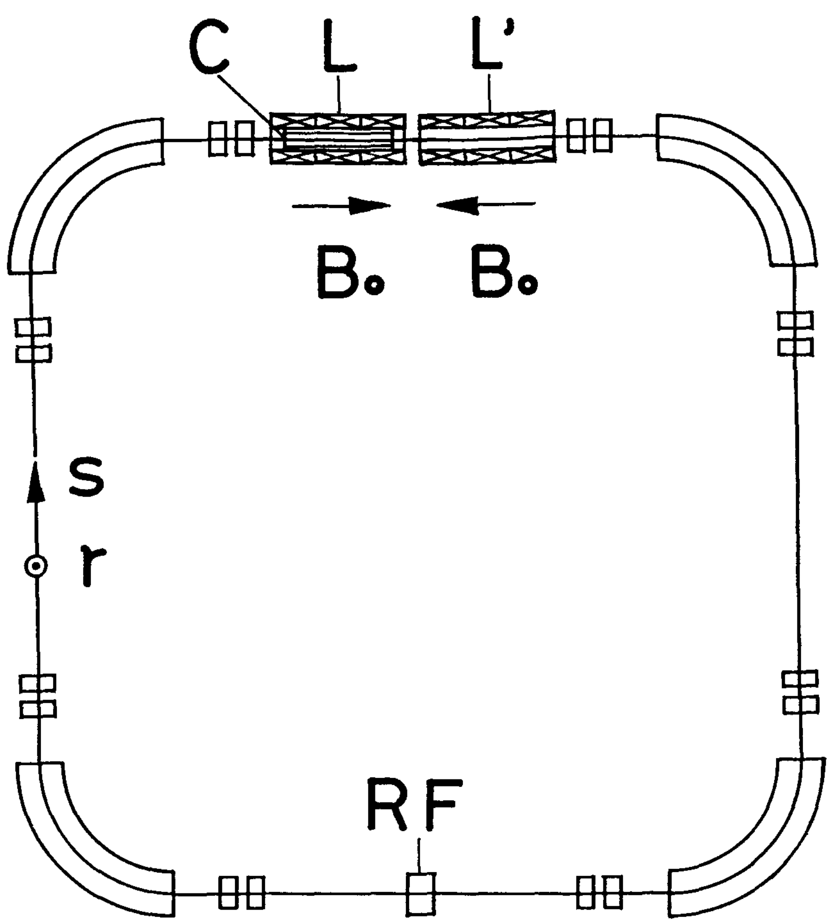

Fig. 1. A basis configuration of the cyclotron maser cooling facility. C: Stimulating rf cavity, L: Solenoidal magnet, L': Compensating solenoidal magnet, RF: Acceleration rf system. Elements without abbreviation are lattice magnets.

solenoidal magnetic field undergo free-free electronic radiative transitions leading to a change of the transverse energy with time t^*:

$$\frac{dT_\perp^*}{dt^*} = \frac{cr_p}{2\pi\nu^*} \cdot I^*(\omega^*)\left[\frac{1}{1+x^2} + \frac{2ax}{(1+x^2)^2}\right], \qquad (1)$$

$$a \equiv \frac{\omega_c^*}{2\pi\nu^*} \cdot \frac{T_\perp^*}{m_0c^2} \ ,$$

$$x \equiv \frac{\omega_c^*}{2\pi\nu^*}\left(1 - \frac{\omega^*}{\omega_c^*} - \frac{T_\perp^*}{m_0c^2}\right) \ .$$

Quantities with a star refer to the particle rest frame. Here, ν^* denotes the frequency of phase debunching on a coherent cyclotron gyration which can be sustained until interrupted by any debunching action.

The second of the two terms in the bracket of the rate formula gives the cyclotron maser effect which is explained by considering the phase of cyclotron gyration with respect to the rf field. For a system of gyrating particles, originally distributed randomly in phase, coherent emission or absorption occurs only if there exists a phase bunching mechanism. Such a mechanism was shown classically to arise from the frequency change due to the relativistic mass effect on the cyclotron gyration.[5] Even for very slight relativistic cyclotron gyration of angular frequency $\omega_c^*[1-(T_\perp^*/m_0c^2)]$, this bunching does in fact occur. Particles absorbing energy of the rf field will become more massive and slip back in phase, while particles emitting radiation will become less massive and advance in phase resulting in a relativistic phase bunching as seen in Fig. 2.

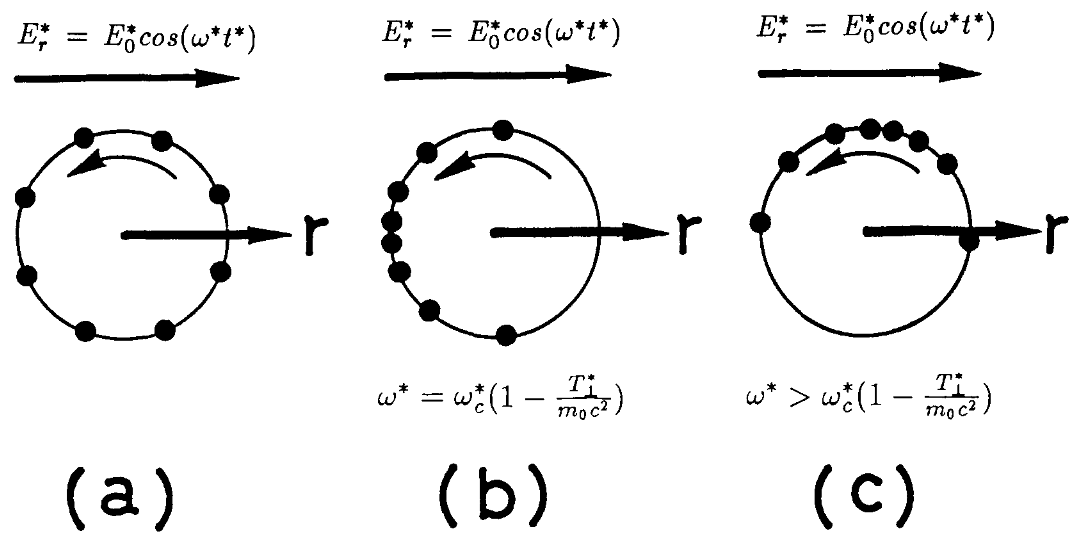

Fig. 2. Distribution of phase angles of gyrating particles with positive charge illustrating the cyclotron maser mechanism under the presence of an rf field, $E_r^* = E_0^* cos(\omega^* t^*)$. (a) Initial distribution of phase angles. Particles with phase angle above the r-axis will lose energy becoming less massive and tend to advance in phase. Similarly the particles with phase angle below the r-axis will become more massive and slip back in phase. This results in a relativistic phase bunching as seen in (b). (b) Phase angle distribution of bunched particles after several gyrations under an rf field with the angular frequency, $\omega^* = \omega_c^*[1-(T_\perp^*/m_0c^2)]$. (c) Phase angle distribution of bunched particles after several gyrations under an rf field with the angular frequency, $\omega^* > \omega_c^*[1-(T_\perp^*/m_0c^2)]$. The bunched particles lose their energy by the coherent emission of photons of energy $\hbar\omega^*$.

For $T_\perp^* = 0$ and thus $a = 0$, the rate formula gives us a familiar cyclotron resonance absorption rate. However, if the conditions $a > 1$ and $-a - \sqrt{a^2-1} < x < -a + \sqrt{a^2-1}$, are fulfilled, the rate becomes negative corresponding to a stimulated emission of radiation resulting in a damping of the particle transverse energy. The phase debunching frequency ν^* must therefore be sufficiently small so that the available energy $T_\perp^*$ can be emitted in a time shorter than $(\nu^*)^{-1}$.

Under the conditions, $a >> 1$ and $x < 0$, a time $\tau_\perp^* = T_\perp^*/(-dT_\perp^*/dt^*)$ defined in the particle rest frame can be calculated as

$$\tau_\perp^* \simeq \frac{(2\pi\nu^*)^2}{r_p\omega_c^*} \cdot \frac{m_0 c}{I^*(\omega^*)} \cdot \frac{(1+x^2)^2}{2x} \quad . \qquad (2)$$

The discussion so far has been limited to the case of particles with transverse motion only. In the presence of longitudinal energy, which is about equal to the beam energy, two kinds of relativistic effects will change the cooling rate in the laboratory. Firstly, Doppler shifts change the angular frequency and the energy flow density of the rf field. Viewed from the laboratory these quantities are $\omega = (1-\varepsilon\beta)^{-1}\gamma^{-1}\omega^*$ and $I(\omega) = (1-\varepsilon\beta)^{-1}\gamma^{-1}I^*(\omega^*)$, respectively, resulting in a change of the intensity for photon absorption accompanied by stimulated emission. Here $\varepsilon = 1$ for a stimulating rf field travelling parallel to the particle beam and $\varepsilon = -1$ for an rf field travelling antiparallel to the beam. Secondly the cooling time $\tau_\perp$ in the laboratory is dilated with respect to the cooling time in the particle rest frame by a factor γ. The optimum cooling time $\tau_\perp$ in the laboratory frame is thus

$$\tau_\perp \simeq \frac{1.5}{(1-\varepsilon\beta)} \cdot \frac{(2\pi\nu^*)^2}{r_p\omega_c^*} \cdot \frac{m_0 c}{I(\omega)} \qquad (3)$$

for the case of optimized stimulated emission which occurs for $x \simeq -0.58$.

As seen in the above formula, the relativistic effects[3] enhance CMC of $T_\perp^*$ when the stimulating rf field travels antiparallel to the beam ($\varepsilon = -1$). While, in the longitudinal phase space, the effects result in heating of the particle beam with the rate $[(\omega/\omega^*) - 1](dT_\perp^*/dt)$. However, cooling of the longitudinal phase space can also be produced at the CMC section simultaneously, as will be described later. When the stimulating rf field travels parallel to the particle beam ($\varepsilon = 1$), CMC of $T_\perp^*$ is reduced due to the relativistic effects by a factor $(1-\beta)$. Nevertheless the case of a parallel travelling to the beam may be useful in some cases, for instance, in the design and construction of small high-Q CMC cavity.

If we consider an rf field of TEM mode in a cavity of parallel plate and disregard the interaction of the rf field with the space charge of particle beam, one wave length of the stimulating rf field is obtained as

$$\lambda = 2\pi c/\omega = 2\pi\gamma c(1-\varepsilon\beta)/\omega^* \quad . \qquad (4)$$

The gyration angle ϕ_0 of particles in a cavity of length L_0 located inside a uniform magnetic field is given by

$$\phi_0 = \omega^* L_0/\beta\gamma c = 2\pi(1-\varepsilon\beta)\beta^{-1}L_0/\lambda \, . \qquad (5)$$

Extremely relativistic particles gyrate therefore about twice in a cavity of one wave length and with an rf field travelling antiparallel to the beam. This is, of course, only a conceptual estimate, since one has to generally consider the cut-off frequency and other parameters in the design of high-Q cavity.

From the discussion above, it is evident that CMC depends critically on the phase debunching frequency ν^*; it determines, together with ω_c^* and $T_\perp^*$, the condition $a >> 1$ and it affects the optimum cooling time $\tau_\perp$. The short flight time of high energy particles through the CMC section, $(\nu^*)^{-1} = L_0/\beta\gamma c$ will limit the emission of $T_\perp^*$ as compared with the case of cyclotron maser such as a gyrotron in which electrons usually gyrate some ten times. For 200 MeV electrons traversing a cavity of length, say, $L_0 = 1.4$ m, the phase debunching frequency ν^* amounts to $\simeq 10^{11}\mathrm{s}^{-1}$. This gives $a \leq 1$ for typically $T_\perp^* \simeq 25$ keV. For an ion beam, the situation is evidently worse and $a \approx 0$. However, since a particle passes the cavity many times, the effective length over which CMC takes place can be very much longer than L_0. Here the phase of cyclotron gyration can be controlled within a certain range as determined from the synchrotron phase stability of particle circulation in the presence of the stimulating rf field synchronized with an accelerating rf field. The concept of "phase locking" of cyclotron gyration will therefore be introduced.

PHASE LOCKING OF CYCLOTRON GYRATION

The condition of "phase locking" is derived by considering the phase angle ϕ of cyclotron gyration per particle circulation in the ring.

$$\phi \equiv \oint \omega^*(t^*)dt^* = \frac{1}{\beta\gamma c}\oint \omega^*(s)ds$$

$$= \frac{\omega_c^*}{\beta\gamma c}(1-\frac{T_\perp^*}{m_0c^2})\frac{1}{B_0}\oint B_s(r,s)ds \ . \qquad (6)$$

Here, $\omega^*(s)$ is the angular frequency of the cyclotron gyration due to the longitudinal component of the magnetic field $B_s(r,s)$ and r and s are the transverse and longitudinal coordinates, respectively. All integrals are line integrals along the particle orbits. If $\phi = 2n\pi (n \equiv 0, 1, 2, \cdots)$ independent of orbits, the particles would recirculate in the ring without any phase debunching of the cyclotron gyration. The relativistic phase bunching due to $T_\perp^*$ would therefore be sufficient yielding the stimulated emission of the transverse energy $T_\perp^*$ in the CMC section. Any change of phase angle $\Delta\phi$ limits, however, the number of phase bunched recirculation within $\pi/\Delta\phi$ resulting in the effective length of CMC cavity being $\pi L_0/\Delta\phi$. This determines the phase debunching frequency $\nu^* = \beta\gamma c \cdot \Delta\phi/\pi L_0$ and the ratio $2\pi\nu^*/\omega_c^*$ as

$$\frac{2\pi\nu^*}{\omega_c^*} \simeq \frac{2\beta\gamma}{B_0L_0}\Delta[\frac{1}{\beta\gamma}\oint B_s(r,s)ds]$$

$$= \frac{2}{B_0L_0}\{-\frac{\Delta\gamma}{\beta^2\gamma}\oint B_s(o,s)ds + \oint \Delta B_s(r,s)ds\} \ . \quad (7)$$

The first of the two terms in the bracket vanishes independent of γ and $\Delta\gamma$ under the condition of "phase locking",

$$\oint B_s(o,s)ds = 0 \ . \quad (8)$$

This can be achieved by introducing a compensating solenoidal magnet as shown in Fig. 1. The second term denotes the variation in the fields and consists of those of solenoidal magnets $\int \Delta B_s(r,s)ds = -(1/4)(d/ds)B_s(o,s)r^2 + \cdots$, and those of lattice magnets, $\int \Delta B_s(r,s)ds = B_r(o,s)r + (1/6)(d^2/ds^2)B_r(o,s)r^3 + \cdots$. However, any beam dynamical effects do not cause the pile up of these field integrals for a ring of symmetric lattice. This means that the recirculation in a ring of symmetric lattice does not affect the phase bunching mechanism under the condition of "phase locking".

Phase debunching of the particles may occur due to the transverse impulses received in the fringing fields of solenoidal and lattice magnets. All particles receive, however, the impulses in phase within a short time duration Δt_0 which satisfies the condition, $\omega\Delta t_0 < \omega t_0 (\Delta f/f) << 1$. Here t_0 and f are the circulation period of the particles and the frequency of the accelerating rf field, respectively. And further, the sum of the impulses vanishes for each magnet.

The particles may have their longitudinal energies changed to transverse energies (*or vice versa*), during their passage through a rippled magnetic field of averaged value B_{av} which satisfies the relation $eB_{av}/m_0 \approx 2\pi\beta\gamma\ c/L$, where L is the periodicity of the field.[6] This nonadiabatic process has a resonance feature like the depolarizing resonance in the acceleration of spin polarized particles and hence, if any, correction can be achieved for the process.[7]

Intra-beam scattering may be another origin of the phase debunching. The scattering frequency is[8]

$$\nu^* \simeq 3.45 n_0^* \, r_p^2 c \cdot ln\Lambda/(T^*/m_0c^2)^{3/2} \, , \quad (9)$$

where n_0^* and T^* are the number density and the temperature of particles in the beam, respectively, and $ln\Lambda$ is the Coulomb logarithm. For an electron beam of density$n_0^* \simeq 10^{15} m^{-3}$ and temperature $T^* \simeq 0.01 m_0c^2$, ν^* is estimated to be about $0.1 s^{-1}$ implying that the phase debunching will be insignificant for the intra-beam scattering as well as the transverse impulses and the non-adiabatic process.

All things considered, the ratio $2\pi\nu^*/\omega_c^*$ seems to be about equal to the fractional frequency widths, $\Delta\omega/\omega$ and $\Delta f/f$. The fractional widths are about 10^{-4} for conventional rf cavities, 10^{-5} for high- Q cavities and even $< 10^{-6}$ for superconducting cavities. It is thus concluded that the ratio $2\pi\nu^*/\omega_c^*$ is expected to be of order of 10^{-5} for a well constructed CMC ring. This means that electrons and ions, say, protons can be cooled down to $T_\perp^* \simeq 5$ eV and 9 keV, respectively, since the ultimate value of cooled energy is expected to be $T_\perp^* \simeq (2\pi\nu^*/\omega_c^*) m_0c^2$ which corresponds to $a = 1$ independent of particle longitudinal energy. In an ideal case of $2\pi\nu^*/\omega_c^* \simeq 10^{-6}$, respective values are even 0.5 eV and 0.9 keV.

EXAMPLES OF CYCLOTRON MASER COOLING

For electrons with $\gamma = 400$ and $B_0 = 6.0$ tesla, it is found that the frequency of required antiparallel stimulating rf field is $\omega/2\pi = 210$ MHz. The electrons gyrate about twice during traversing a cavity of $L_0 = \lambda \simeq 1.4$ m. Using a conservative value of $2\pi\nu^*/\omega_c^* \simeq 10^{-4}$ for this case, shows that a stimulating rf field of only $I(\omega) \simeq 11$ kW$\cdot$m^{-2} is needed to achieve a very rapid cooling time $\eta^{-1}\tau \simeq 1$ μs with an assumption of $\eta \simeq 0.07$ where a fraction η of the ring circumference is occupied by the rf cavity.

Another case of protons with $\gamma = 2.8 (T_p \simeq 1.7$ GeV) and again $B_0 = 6.0$ tesla results in a parallel stimulating rf field of $\omega/2\pi \simeq 0.50$ GHz. Protons gyrate about π radians in a cavity of length $L_0 = 7.0\lambda \simeq 4.2$ m. The assumptions, $2\pi\nu^*/\omega_c^* \simeq 10^{-5}$ and $\eta \simeq 0.040$, lead to a required energy flow density of the rf field $I(\omega) \simeq 1.1$ Mw $\cdot$ m^{-2} which gives also a rapid cooling of $\eta^{-1}\gamma \simeq 10\mu$ s. While an rf field of $\omega/2\pi = 17$ MHz and $I(\omega)$ =21 kW $\cdot$ m^{-2} travelling antiparallel to the beam results in about the same cooling time. And the protons gyrate about once in a cavity of $L_0 = \lambda/2 \simeq 8.9$ m.

However, the frequency ω^* and thus ω have a small dependence on the energy $T_\perp^*$ and one may sometimes have to tune the frequency ω to keep the cooling steadily as $T_\perp^*$ reduces. If any, this tuning can be achieved without change of ω by decreasing a little the value of B_0 with keeping the CMC condition at each cooling stage. The discussion so far has been limited to the case of the fundamental gyro-resonance only for the sake of the simplicity. The great CMC action upon the particle beams is also caused by the multi-harmonic stimulating rf field with the frequency $\omega^* \geq n\omega_c^*[1-(T_\perp^*/m_0c^2)](n \equiv 2, 3, \cdots)$.

COOLING OF LONGITUDINAL ENERGY SPREAD

CMC is even more useful for cooling the longitudinal particle energy spread than the transverse energy. In the presence of the finite derivative of momentum dispersion D', all particles introduced into the CMC section start their coherent cyclotron gyration with the transverse momentum D'Δp where $\Delta p = m_0c\Delta\gamma/\beta$ denotes the momentum deviation with respect to the reference momentum which is chosen to be less than the minimum value of the longitudinal momentum. Orbits of the reference momentum have been adjusted to be parallel to the s-axis in the section. One may thus replace $T_\perp^*$ in the discussion above by $T_\perp^*(\Delta\gamma) \equiv (D'\Delta p)^2/2m_0 = (D' \cdot \Delta\gamma/\beta)^2 \cdot (m_0c^2/2)$ which is tunable through adjusting the value of D' so as to satisfy the condition,

$a >> 1$. From the rate formula, $m_0c^2 \cdot d(\Delta\gamma)/dt = (\omega/\omega^*)[dT_\perp^*(\Delta\gamma)/dt]$, a time $\tau_\parallel \equiv \Delta\gamma/[-d(\Delta\gamma)/dt]$ defined in the laboratory frame can be calculated as

$$\tau_\parallel \simeq 3.1\frac{(2\pi\nu^*)^2}{r_p\omega_c^*} \cdot \frac{m_0c}{I(\omega)} \cdot (\frac{\beta}{D'})^2 \cdot \frac{\gamma}{\Delta\gamma} \quad . \qquad (10)$$

It may be of great importance to note that the cooling time $\tau_\parallel$ is independent of the direction of the stimulating rf field. In this cooling scheme, the phase bunching of the cyclotron gyration implies the cooling of the transverse phase space through transferring the energy $T_\perp^*$ to the longitudinal phase space.

CONCLUSION

In summary, CMC can be applied to either continuous or pulsed electron and ion beams. Very rapid cooling may be obtained for the transverse and longitudinal phase spaces of a beam independent of its energy. Problems on CMC considering the difinite form of distributions in both the transverse and longitudinal phase spaces and the beam dynamics for the ring equipped with the CMC section are left for further studies.

ACKNOWLEDGEMENT

The author is grateful to professor S. Kullander of Uppsala University for his stimulating comments and continuous encouragements. He is also indebted to professors A. Ando, H. Ohtsubo and I. Katayama of Osaka University for their helpful discussions. Part of this work was supported by the Monbusho International Science Research Program.

REFERENCE

1. D. Cline et al., "High Energy Electron Cooling to Improve the Luminosity and Lifetime in Cooling Beam Machine", IEEE Trans. Nucl. Sci. 26, 3472 (1979) and those cited therein.
2. C. van der Meer, CERN Internal Report, CERN/ISR/72-31, August 1972, unpublished.
3. H. Ikegami, "Free Positronium Radiation", Phys. Rev. Lett. 60, 929 (1988).
4. R.Q. Twiss, "Radiation Transfer and The Possibility of Negative Absorption in Radio Astronomy", Aust. J. Phys. 11, 564 (1958);
   J. Schneider, "Stimulated Emission of Radiation by Relativistic Electrons in A Magnetic Field", Phys. Rev. Lett. 2, 504 (1959);
   A.V. Gaponov, Izv. VUZ. Radiofizika 2, 450 and 837 (1959).
5. E. Otto and W. Manheimer, "Theory of Microwave Emission by Velocity-Space Instabilities of An Intense Relativistic Electron Beam", IEEE Trans. Plasma Sci. 3, 1(1975);
   P. Sprangle and A.T. Drobot, "The Linear and Self-Consistent Non- Linear Theory of the Electron Cyclotron Maser Instability", IEEE Trans. Microwave Theory Tech. 25, 528 (1977);
   J.L. Hirshfield and V.L. Granatstein, "The Electron Cyclotron Maser - An Historical Survey", IEEE Trans. Microwave Theory Tech. 25, 522 (1977).
6. E.W. Laing and A.E. Robson, "Non-adiabatic Magnetic Traps", Plasma Phys. 3, 146 (1961).
7. H. Sato, "Depolarizing Resonance Correction in Polarized Proton Beam Acceleration up to 5.0 GeV at the KEK PS", Jpn. J. Appl. Phys. 27, 1022 (1988).
8. L. Spitzer, Jr., *Physics of Fully Ionized Gases*, (Intersciences, New York, 1962), p.113.