AXIAL INFLECTORS AND THE CORRELATION BETWEEN PHASE SPACES

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Summary

A correlation between phase spaces may originate from an ion source with a longitudinal magnetic field like an ECR source. This correlation gives an apparent enhancement of the transversal beam emittances which can be avoided using skew quadrupoles. The effect of the electromagnetic fields of inflectors in the cyclotron central region on the correlation or decorrelation of phase spaces will be investigated.

1. INTRODUCTION

Axial magnetic fields may give strong correlations between the transverse phase spaces. This occurs when ions are started in the magnetic field or are guided from a field free region towards a magnetic field region. These situations arise in e.g. ECR sources and in cyclotron axial injection systems [1) 2) 3)]. The result of the correlation is an apparent enhancement of the emittances.

The transverse phase spaces can be partially or completely decoupled by relatively simple means in the injection beam lines using skew quadrupoles [2)]. In this paper we will shortly treat the optics in longitudinal magnetic fields, the beam emittances of an ECR source and apply the results to different inflector systems in the cyclotron centre [4) 5) 6)].

2. ION OPTICS IN CYLINDRICALLY SYMMETRICAL MAGNETIC FIELDS

As long as ions remain in cylindrically symmetrical geometrics the azimuthal canonical momentum p_ϑ remains constant. The relation between the kinetic momentum p_ϑ^* and p_ϑ is given by

$$p_\vartheta^*/r = p_\vartheta/r - eA_\vartheta$$

where A_ϑ is the azimuthal component of the vector potential. For a magnetic field pointing in the positive z direction $A_\vartheta = Br/2$ (see Fig. 1).

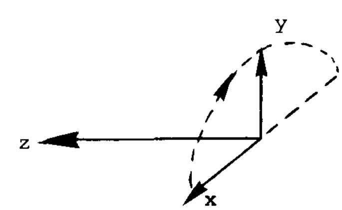

Fig. 1. The coordinate system

If p_ϑ^* inside the magnetic field equals zero it thus follows that outside it equals $p_\vartheta^* = 1/2eBr^2$. This statement shows immediately the coupling between phase spaces.

To treat the ion optical behaviour of longitudinal fields it is convenient to use the cartesian components of the canonical momenta instead of the normally used variables $x' = dx/ds$ and $y' = dy/ds$. For the ray vectors (x,p_x) and (y,p_y) the the longitudinal magnetic field acts as a positive lens of equal size and sign in both transversal directions [1) 4)]. There is, however, a rotation of the coordinates given by

$$\Delta\phi = \int_o^s \frac{1}{2}\frac{eB}{p}\,ds = \int_o^s \frac{1}{2R}\,ds$$

where p is the total kinetic momentum, $R = R(s)$ is the radius of curvature in the magnetic field $B = B(s)$. The size of $\Delta\phi$ can be adjusted to rather arbitrary values by dividing the field in two regions where B points into opposite directions. The lens strength then remains the same.

The cartesian components of the vector potential A are

$$A_x = -\frac{1}{2}By, \quad A_y = \frac{1}{2}Bx$$

The transformation from (xx'yy') to $(x\pi_x,\ y\pi_y)$ is given by the correlation matrix C. The variables π_x and π_y are the relative canonical momenta:

$$\pi_x = \frac{p_x}{p_o} = x' - \frac{1}{2}\frac{eB}{p_o}y = x' - \frac{y}{2R}$$

$$\pi_y = \frac{p_y}{p_o} = y' + \frac{1}{2}\frac{eB}{p_o}x = y' + \frac{x}{2R}$$

The matrix C is given by

$$\begin{pmatrix} x \\ \pi_x \\ y \\ \pi_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{-1}{2R} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2R} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$$

The magnetic field points in the direction of z (Fig. 1). The inverse matrix is found by changing the signs of the elements C_{23}, C_{41}. For a magnetic field pointing in the opposite direction the sign of these elements also have to be reversed. It is further convenient to multiply the momenta by R so that all elements in the matrix C^* become dimensionless and the vector components all have the same dimension (m). The determinant of C or C^* equals unity. Therefore a given volume in (xx'yy') space is conserved in $(x\pi_x y\pi_y)$ and the total 4-dimensional phase space volume of an ECR source is given by

$$\epsilon_4^* = \pi r_h^2 \cdot \frac{4kT_s}{qeV_{extr}} R^2,$$

in which r_h is the radius of the source hole, T_s the ion temperature in the source and V_{extr} the extraction voltage of the puller. The phase space can be decoupled by a special structure in the beam guiding system after the source [2)] such that

$$\epsilon_1^* = \pi r_h^2 \quad \text{and} \quad \epsilon_2^* = \frac{4kT_s}{qeV_{ext}} R^2 \qquad (mm^2)$$

or in mmmrad

$$\epsilon_1 = 10^3 \frac{\pi r^2}{R} \quad \text{and} \quad \epsilon_2 = 10^3 \frac{4kT_s}{qeV_{ext}} R \qquad (mmmrad)$$

(r and R in mm).

Taking some numerical values ($kT = 1$ eV, $qeV_{extr} = 210^4$ eV, $r_n = 4$ mm, $q = 2$, $R = 29$ mm, $B = 1T$) we get $\epsilon_1 = 1700$ mmmrad, $\epsilon_2 = 6$ mmmrad. For a n times lower magnetic induction one gets $\frac{\epsilon_1}{n}$, $\epsilon_2 \cdot n$. Without a decoupling device both emmitances have the same apparent value $\epsilon = \epsilon_1 = \epsilon_2 = 10^3\pi r^2/2R = 850$ mmmrad. An ion source without axial magnetic An ion source wihtout axial magnetic field with the same exit hole diameter and ion temperature will have roughly 100 mmmrad in both directions.

Correlation and decorrelation effects may become important at the point where the ions are injected into a cyclotron via the axial magnetic field and the inflector. Only the hyperbolic inflector shows no extra correlation effect [2)3)6)]. All other types of inflector do give extra correlation effects. One therefore has to match practically in all cases in a 6-dimensional phase space [7)].

3. THE HYPERBOLIC INFLECTOR

The transfer matrix through the hyperbolic flector, using the hamiltonian theory and taking geometrical end effects into account [8)] is given by

$$\begin{pmatrix} \Delta R \\ \Delta\phi_{cp} \\ y_c \\ x_c \\ z \\ z' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{2}{b(1+\frac{1}{2}b)R_h} & -\left(\frac{1+b}{2R_c} + \frac{1}{b(1+\frac{1}{2}b)R_4}\right) & 0 & 1 & 0 \\ \frac{-\sin\alpha}{1+b} & 0 & +\left(\frac{1+b}{2}\right)\cos\alpha & \frac{-2\sin\alpha}{1+b} & 0 & +\sin\alpha \\ \frac{+\cos\alpha}{1+b} & 0 & +\left(\frac{1+b}{2}\right)\sin\alpha & \frac{+2\cos\alpha}{1+b} & 0 & -\cos\alpha \\ 1 & 0 & 0 & 0 & 0 & \frac{2}{1+b} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ x_i' \\ y_i \\ y_i' \\ \Delta\tau_i \\ \Delta z_i' \end{pmatrix}$$

where R_h is the characteristic radius of the inflector, $1+b = \sqrt{2/3}$, $\alpha = 20.2^o$, R_c the cyclotron radius, x_i' and y_i' are multiplied by R_c, $\Delta\tau_i$ is the initial time in radians corresponding to the cyclotron revolution frequency,

$\Delta z_i' = \frac{p - p_o}{p_o} \cdot R_c$, with $p - p_o$ the deviation from the nominal kinetic momentum, ΔR and $\Delta\phi_{cp}$ are the central position coordinates, y_c and x_c the orbit centre coordinates and z and z' the axial coordinates [2)9)] (see Fig. 2).

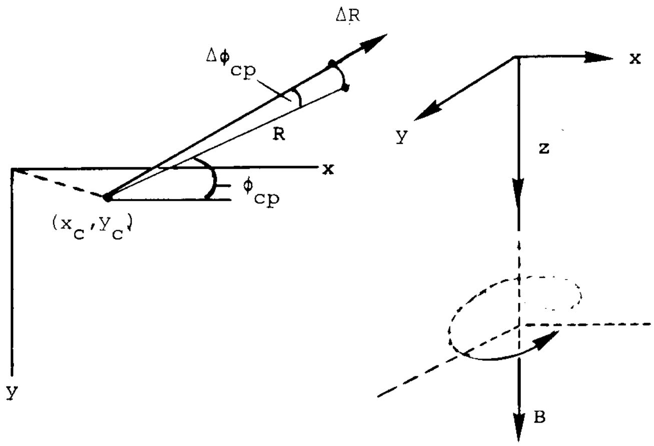

Fig. 2. The coordinate system and the definition of the central position coordinates.

This transport matrix corresponds to the one given by R. Müller. However, the input and output coordinates are different and may be for some cases more adapted to numerical and/or analytical calculations. Using the canonical variables this matrix has to be multiplied by the correlation matrix C^{*-1}:

$$C^{*-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & ½ & 0 \\ 0 & 0 & 1 & 0 \\ -½ & 0 & 0 & 1 \end{pmatrix}$$

This results in:

$$\begin{pmatrix} \Delta R \\ \Delta\phi_{cp} \\ y_c \\ x_c \\ \Delta z_f \\ z'_f \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{2}{b(1+½b)R_h} & -\frac{1+b}{2R_c} & 0 & 1 & 0 \\ 0 & 0 & +(\frac{1+b}{2})\cos\alpha & \frac{-2\sin\alpha}{1+b} & 0 & +\sin\alpha \\ 0 & 0 & +(\frac{1+b}{2})\sin\alpha & \frac{+2\cos\alpha}{1+b} & 0 & -\cos\alpha \\ 1 & 0 & 0 & 0 & 0 & \frac{2}{1+b} \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta x_i \\ \pi_{xi} \\ \Delta y_i \\ \pi_{yi} \\ \Delta\tau_i \\ \Delta z_i \end{pmatrix}$$

One clearly observes the separated behaviour for the horizontal and vertical betatron motion in the cyclotron. The emittances as given by the injection line are transported to the cyclotron centre. As a remark there is a debunching effect due to the ellements T_{22} and T_{23} which is important. To get a matched beam for $\nu_R \approx 1$, $\nu_z \approx 0.2$ one must inject into the injector broad beams with π_{xi} and π_{yi} small.

As the numerical values of T_{22} and T_{23} are $-0.24\ mm^{-1}$ and $-0.014\ mm^{-1}$ ($R_h \approx 100$ mm) it is clear that one likes π_{xi} to be small for bunched beams into the cyclotron.

An energy variation of 1% gives a change in $\Delta z'_i$ equal to 0.145 mm and in Δz_f of about •5 mm and at the same time in the orbit centre coordinate x_c, y_s of the same small order of size.

Due to the odd symmetry in the hamiltonian representation for the x,y dimensions the conclusion about the separation of the horizontal and vertical phase space remains identical if the sign of the magnetic field is reversed.

4. THE MIRROR INFLECTOR

For easiness we take the transport matrix for the mirror as given by Bellomo c.s. [5)]

$$\begin{pmatrix} x \\ x' \\ z \\ z' \\ \Delta\tau \\ \Delta p/p \end{pmatrix} = \begin{pmatrix} \cos\tau/2 & 2\sin\tau/2 & \sin\tau/2 & 0 & 0 & 0 \\ -\sin\tau/2 & \frac{\cos\tau}{\cos\tau/2} & \cos\tau/2 & 0 & 0 & tg\tau/2 \\ 0 & 0 & -tg\alpha & 0 & 0 & -\tau \\ 0 & \frac{-tg\tau/2}{tg\alpha} & 0 & \frac{-1}{tg\alpha} & 0 & \frac{1}{tg\alpha\cos\tau/2} \\ \sin\tau/2 & 0 & tg\alpha-\cos\tau/2 & -2\sin\tau/2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ \Delta\tau \\ p/p_o \end{pmatrix}$$

The initial variables as well as the final ones are given in relative values : $x \rightarrow x/R$ and $x' = p^*_x/R_o$ etc., where p^*_x is the radial kinetic momentum at the exit of the inflector, R the cyclotron radius and τ the transit time. One observes the coupling of the transversal phase spaces via T_{13}, T_{23}, T_{42}. We have to keep in mind that the initial variables are still in the cyclotron magnetic field. Therefore a correlation matrix has to be applied on the right side of the transport matrix. Given the orientation of the used coordinate system and the sign of the magnetic field this last matrix has the shape

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -½ & 0 \\ 0 & 0 & 1 & 0 \\ ½ & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \pi_x \\ y \\ \pi_y \end{pmatrix}$$

Now the ray vector (x, π_x, y, π_y) can be seen as a vector for which normal ion optics are valid except for a rotation of the coordinates, which we neglect as it can be adjusted according to requirements. The axial magnetic field acts as a lens with positive and equal strength in both directions. We always can make a unit transformation from some place outside the magnetic field to the entrance of the mirror. Placing a quadrupole lens at that place gives for the left upper 4 x 4 matrix a simpler shape:

$$T_4 = \begin{pmatrix} 0 & 2\sin\tau/2 & 0 & 0 \\ -\sin\tau/2 - \dfrac{Q\cos\tau}{\cos\tau/2} & \dfrac{\cos\tau}{\cos\tau/2} & \dfrac{1}{2\cos\tau/2} & 0 \\ 0 & 0 & tg\alpha & 0 \\ 0 & \dfrac{-tg\tau/2}{tg\alpha} & \dfrac{\frac{1}{2}tg\tau/2 - Q}{tg\alpha} & \dfrac{1}{tg\alpha} \end{pmatrix}$$

$$Q = \frac{1}{2tg\alpha/2}.$$

There is still a coupling present. This coupling looks roughly like that generated by an ECR source. Therefore using an ECR source it should be possible to decouple here the total transport matrix from the source opening to the mirror exit. This should be possible by making a unit transformation in one phase space and a $\pi/2$ rotation in the other phase space. The exit of this system should lie in the middle of the earlier mentioned quadrupole. We thus get

$$T_4{}^* \times \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2\rho} & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2\rho} & 0 & 0 & 1 \end{pmatrix}$$

where ρ is the ratio $\frac{R_s}{R}$, with R_s the radius of curvature in the source field. Taking $\frac{1}{2\rho} = tg\tau/2$ one gets the total matrix

$$T_4 = \begin{pmatrix} -2\sin\tau/2 & 0 & 0 & 0 \\ \dfrac{\cos\tau}{\cos\tau/2} & -\dfrac{1}{2\sin\tau/2} & 0 & 0 \\ 0 & 0 & tg\alpha & 0 \\ 0 & 0 & \dfrac{1}{tg\alpha}(\frac{1}{2}\, tg\tau/2 - Q) & \dfrac{1}{tg\alpha} \end{pmatrix}$$

The decoupling in the transversal phase spaces is thus clearly shown. There remains still the coupling to the RF phase. In fact to study this influence in the beam dynamics in the cyclotron in more detail one has to go over to centre-position coordinates. Further the given array of transformations certainly do not need to be the only ones yielding this result.

5. CONCLUDING REMARKS

The effects described in this paper also must apply to the spiral inflector. However, up to now we were not able to fit this inflector in a clear way analytically in the hamiltonian theory. The hyperbolic inflector does not show a coupling between the two transversal phase spaces. Therefore the emittances of the ion sources are transported to the cyclotron centre. The ECR source can deliver a very small emittance in one direction. This can be used for achieving small RF phase widths in the cyclotron together with a very small axial emittance.

The mirror inflector shows an extra coupling between the two transversal phase spaces. In combination with an ion source without an axial magnetic field this may lead to rather enhanced emittances, unless decoupling systems using skew quadrupoles [2] are used to provide a certain coupling in advance. This should be investigated in more detail. In any case using an ECR source, or any source with an axial magnetic field the decoupling can be made completely with regard to the transversal phase spaces in the cyclotron. For both types of inflectors there always remains a coupling into the RF phase, which must be looked at carefully.

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