FRINGEFIELD CORRECTIONS OF SHORT QUADRUPOLES WITH A LARGE APERTURE

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ABSTRACT

In quadrupole lenses with a length to aperture ratio of less than two the effective length as function of radius changes significantly, if no special fringefield corrections are applied. The relative large length of the fringefield also effects the multipole content of the integrated field gradient.

To study the relative merits of different corrections three dimensional calculations have been performed for different pole-end shapes. As to correct for the change in effective length as function of radius they have about the same effect and one gives a very good result. There is however a large difference in multipole content as generated by the different pole-end shapes. The reason why certain pole end shapes are to be preferred above others is discussed using an analytic approach.

1. INTRODUCTION.

As soon as the ratio of length to aperture of quadrupoles becomes smaller than 3, l/d<3, the fringefield region gets a significant influence. It adds not only additional multipoles but also a quadrupole term which changes with radius to the third power. In our case we needed quadrupoles with a l/d=1.9 and l/d=1.7. In addition the multipole content at 80% of aperture and the change in effective length as function of radius should be smaller than .5% . This with a pole-tip field up to .9 T. The quadrupole in this study has an aperture of 13 cm and an iron length of 21.11 cm.

The hyperbolic pole and the taper of the pole in the transverse directions, x and y, is choosen such that, using the program POISSON[1]), the 12-pole and 20-pole amplitude is about .1% . It was then confirmed that an electrostatic model of the pole gave the same field distribution within the aperture, and a few cm outside between the poles, up to .9 T, again using POISSON. At .9 T the pole-root was saturated up to 2 T. (A test run simulating a lower quality iron, C10 equivalent, shifted only the region of saturation without a significant change in field distribution.) This showed that up to .9 T pole-tip field the coils do not have a significant influence on the transverse field distribution.

In the longitudinal, z, direction the presence of the coil will change the fringefield of the pole proper slightly. How to cope with that is discussed in the section on magnetic quads.

These preparations allow us to use the program RELAX3D[2]) to study the influence of the pole-end corrections. The program RELAX3D has the advantage that it is easy to use, does not use excessive cpu time and gives accurate results. A 3D magnet program is not likely to compete with this and is expensive.

2. THEORY AND REMARKS.

The problems one runs into with a short quad are clearly shown by a series expansion of the field[3])

$$B_r = -2*r*a_2*\cos(2*\theta) - \frac{1}{4}*r^3*a_2''*\cos(2*\theta) - 6*r^5*a_6*\cos(6*\theta) - \frac{1}{8}*r^7*a_6''*\cos(6*\theta) - \ldots\ldots \quad (1)$$

$$B_z = -r^2*a_2'*\cos(2*\theta) - r^6*a_6'*\cos(6*\theta) - \ldots\ldots \quad (2)$$

B_r is the radial field component, B_z the longitudinal component, r the radius and θ the angle, being zero at the pole tip. a' means the first derivative of a to z (da(z)/dz) etc.

Formula (1) shows that not only additional multipoles are added by the fringefield but also terms e.g. $\frac{1}{4} * r^3 * a''_2 * \cos(2*\theta)$. If one term in (2) dominates then the corresponding a" in (1) becomes proportional to $B_2 - B_1$ when B_r is integrated from $z_1 - z_2$. When a mirrorplane is present then $B_2 = B_1$ when integrated from both sides at equal distance. This means that the corresponding term has no influence on a parallel beam. Formula (2) shows that the behaviour of B_z is important. It also shows that the region where B_z is large should be as short as possible to get the positive and negative derivatives close together. As a result the usual correction of end effects by a chamfer of the pole-ends to compensate either for the 12-pole or for the 20-pole is not likely to be the best solution to compensate for the additional nonlinear terms. These terms can be represented as a change in effective length as function of radius. This length can be easily partitioned to account for local effects and it can be accurately measured to determine the quality of a magnetic quad. (for an overview of pole compensation see e.g. [4])) The examples show that the r^3 quad term degrades the linearity more than the multipole terms.

3. MAGNETIC QUADS.

The field distribution of a magnetic quad differs from the electrostatic equivalent case. This is mainly due to the presence of coils and possible saturation effects inside the pole of a magnetic quad. Electrostatic calculations are useful when the effect of a correction has to be predicted. But assumptions have to be made: the influence of the coil remains the same with or without the correction and the correction does not have a significant influence on the magnetization of the pole. Then one first measures the effective length as function of radius of the uncorrected quad. The net result of a correction can than be calculated as a difference effect.

4. NUMERICAL PROCEDURE.

To obtain the results shown, we used the programs RELAX3D [2]), SPEAKEASY [5]) and SKETCH [6]). RELAX3D calculates a potential distribution on a grid in three dimensions. This distribution obeys the discretized poisson equation. SPEAKEASY is used to process the data supplied by RELAX3D. SKETCH is used to transform the results into pictures.

To calculate the field produced by a quadrupole magnet, the quadrupole is represented by a limited amount of discrete grid points. These points have a fixed potential value. We made use of symmetry planes present in the quadrupole to reduce the number of points needed. Appropiate boundary conditions are set on some of these planes; i.e. the Dirichlet condition on the x=0 and y=0 planes and the Neumann condition at the mid-plane (z=28.9). At z=0 the potential is fixed to zero.

4.1 The Models.

Each evaluated quad has an iron length of 21.11 cm and an aperture of 13 cm (r_0=6.5 cm). A two dimensional cut of the models at z=28.9 is shown in figure 1. The quads only differ in their pole-end correction.

The first case has no correction. The pole-end is just a xy plane.

The 35° correction is a cut of the pole-tip under 35° with respect to the z-axis. The original hyperbolic pole profile (fig. 1) starts at 3 cm from the pole-end.

For the 45° and 55° cases this profile is reached respectively within 1.5 and 2 cm.

Grooves in the next two cases are made in the pole-end parallel to the line x=y. Each groove extends .11 cm in the z-direction. In the first case (grve 1) four grooves are made: resp. 2.82, 2.35, 1.89 and 0.94 cm wide. The second case (grv2) has 7 grooves: 6.13, 4.24, 3.29, 2.82, 2.35, 1.89 and 0.94 cm wide.

The "35° + slab" case has a 35° cut similar to the previous one and a slab of .33 cm, equivalent to the quadrupole profile, placed at the entrance.

The last case is the same as the 55° case with a mirror plate placed 8.7 cm before the pole-end of the quad.

In each case the correction is made symmetric with respect to the x=y plane.

4.2 Discussion.

The harmonic components of B_r are calculated at $R/r_0 = 0.82$ at 16 xy-planes at different z-positions. The potential values are differentiated, interpolated and then analysed. The results for the various pole-end shapes are shown in figures 2 and 3.

The effective length depends on the radius (eq.2). Calculated behaviour in two planes (between the poles and the x=y-z plane) is shown in figures 4 and 5. The lengths presented in these figures do not match exactly the value's tabulated in tabel 1. This is due to a different calculation procedure.

All the examples were optimised to correct for the effective length as function of radius in at least one transverse plane. The results summarized in table 1, show three good types of corrections: the 35° chamfer, 35° chamfer with slab and grvel. A difference beteen these cases remains: the size and sign of the integrated 12-pole component.

The harmonic components as function of z show that the 35° cut has the most smooth behaviour (if the quadrupole has to be used as a strongly diverging optical component this fact has to be considered). This type of correction could be preferred despite the relative large change in the effective field length.

The 12-pole component remains present in the middle of the quad (fig 2). This fact can be explained by the discrete grid points used for the description of the pole profile. As shown in figure 1 a bump is present on the pole surface. This bump results in a 12-pole component with a negative sign. In fact this adds about -.2% to the integrated 12-pole.

The "slab" and grvel cases result in a negative integrated 12-pole component. This number can be corrected for the effect described above. The result is a remarkable good quadrupole[7]). One has to consider the effect of saturation: the magnetic equivalent of the "slab" example will not be so perfect as the calculated one. However, it is a simple addition especially in the case of a laminated quad and probably works with the groove type of correction as well.

The last example, 55° cut with mirror plate shows an effect present in a quadrupole doublet. When the doublet is made of two similar quadrupoles which are used in opposite mode the situation is not influenced by the placement of a mirror plate between these quads. The effective field length and the multipole content change significantly (see figures 2,3,5). A more complicated beveling is needed to keep the derivatives of B_z small.

REFERENCES

1) POISSON, Acc. Group, LANL, New Mexico, USA
2) RELAX3D, Triumf, UBC, Canada.
3) H.L.Hagedoorn, F.Schutte, "Lectures on particle accelerators", T.U. Eindhoven, Netherlands.
4) G.Parsen, "Magnetic fields for transporting charged beams", BNL 50536, jan 1976.
5) SPEAKEASY, Speakeasy Computing Corporation, Chicago, USA.
6) SKETCH, J.Hinnefeld, KVI, Netherlands.
7) R.E.Pollock, IUCF, private communication.

Table 1

pole end	mark	ΔL x=y cm.	ΔL y=0 cm.	4-pole	12-pole (%)	20-pole (%)
no cor.	●	0.28	0.13	14.16	-0.573	0.481
35° chamf	o	0.12	0.25	13.37	0.290	0.295
45° chamf	x	0.14	0.22	13.53	0.022	0.397
55° chamf	*	0.13	0.20	13.19	-0.208	0.382
grve1	+	0.17	0.18	13.96	-0.198	0.338
grve2	·	0.14	0.22	13.80	-0.043	0.306
35° +slab	□	0.18	0.18	14.19	-0.267	0.380
55° +mir.	■	0.35	0.30	12.67	-0.375	0.400

mark : mark used in figures
ΔL : eff. length at R/ro = .75 - eff. length at R/ro = .15
4-pole : half of integrated quadcomponent
12-pole, 20-pole: integrated component divided by integrated quadrupole component.
y=0 : xz plane between the poles
If ΔL is corrected for the downwards dip below 2 cm (see fig.4) it becomes .08 for case 5 and 7.

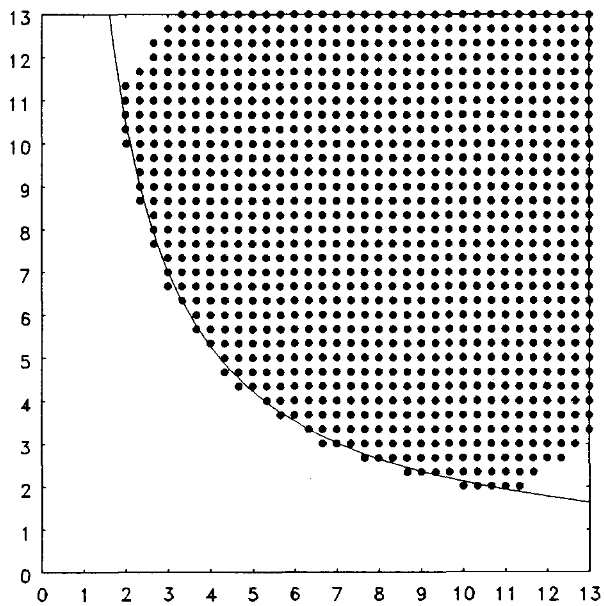

fig.1 Two dimensional cut of the quad.
dots: fixed potential points in RELAX3D
line: $2xy=(6.5)^2$ ideal hyperbolic surface

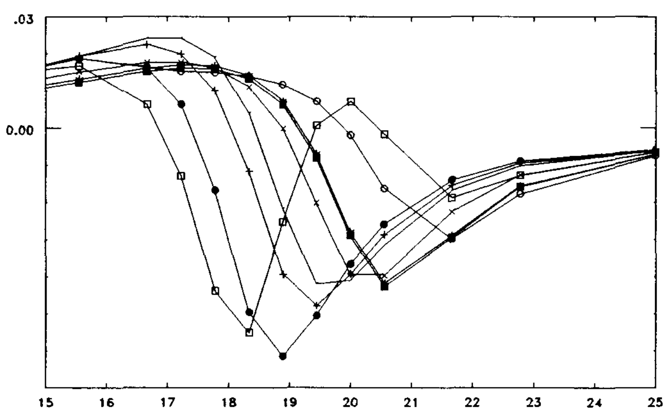

fig.2 The twelve-pole component (%) versus z (cm)
Various corrections are indicated by marks.
The pole-end is situated at z=18.44 cm,
z=28.9 is the mid-plane of the quad.

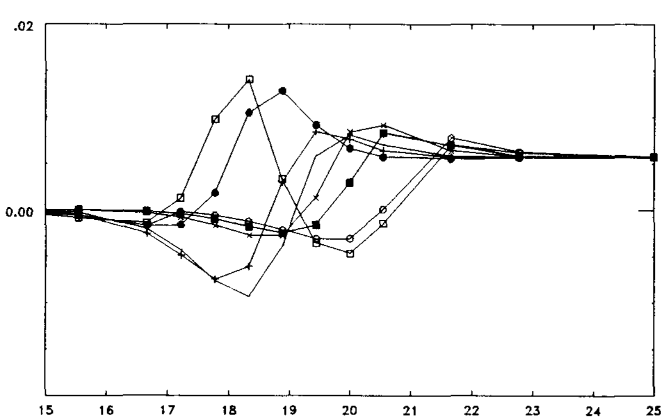

fig.3 The twenty-pole component (%) versus z (cm)

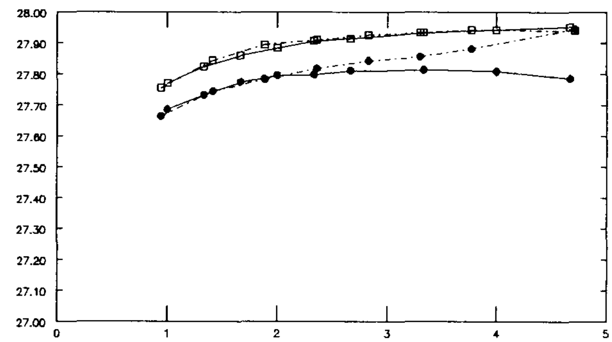

fig.4 Effective field length (cm) versus radius
for the uncorrected and "slab" case
solid line: y=0 plane between the poles
dotted line: x=y plane

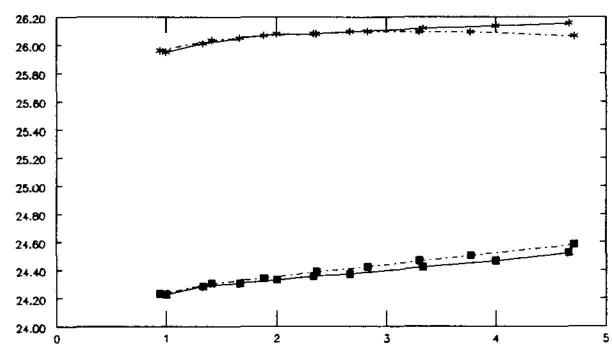

fig.5 The effect of a mirror plate on the
effective field length of the 55 case

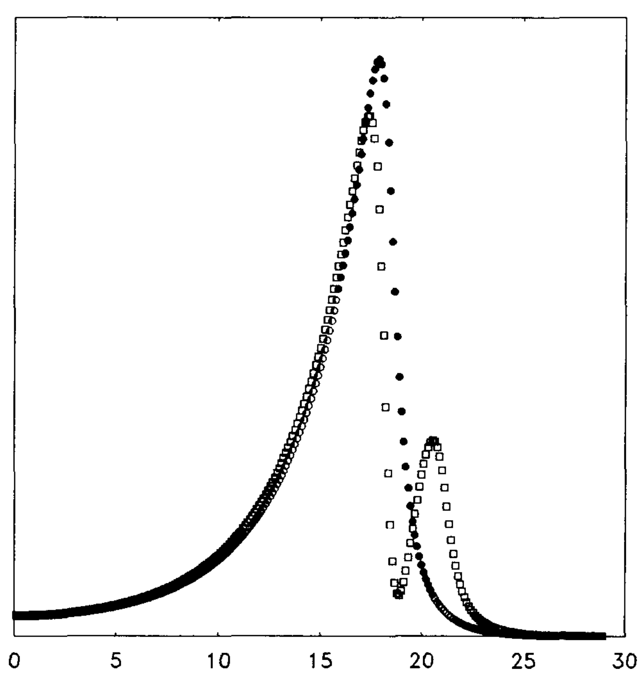

fig.6 Bz behaviour of the uncorrected quad and the
"slab" case at r=5.6 cm