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DERIVING THE GENERAL EXPRESSIONS OF NONLINEAR TRANSPORT COEFFICIENTS OF ACCELERATOR BEAM PHASE SPACE BY ARTIFICIAL INTELLIGENCE

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ABSTRACT

The transport of accelerator beam phase space is a nonlinear theoretical problem. Recently, a new theory for the nonlinear transport of accelerator beam phase space has been developed, and a mathematical model of the nonlinear beam transport was provided. It is difficult to establish the model because of considerable and complex symbol calculations. In this paper a computer program in the logic programming language of artificial intelligence Arity/Prolog was made on IBM-PC 286 or 386 machine to derive automatically the general expressions of any order phase space transport coefficients. By this method, the concrete mathematical model of beam transport was established easily from the nonlinear transport theory. The prospect of the application of artificial intelligence can be seen, people will be further saved out from the chore of symbol calculation. Finally, the ninth order transport coefficient was given as a typical example of high order result obtained from machine.

1. INTRODUCTION

The phase space transport of accelerator beam is a nonlinear dynamic theoretical problem. Recently, a new theory has been developed for the nonlinear phase space transport. By using this new theory, the concrete mathematical model can be established for the nonlinear transport of beam phase space, but it is very difficult to establish the model by manual deriving because the mathematical derivation is a very complex work and needs a unusual carefulness and much more time. The main work of the calculation is deriving the general expressions of nonlinear transport coefficients. Now, many new techniques of artificial intelligence have been developed, symbol calculations can be done by computers with logic programming languages, such as, Lisp, Prolog etc. So, the complex manual work of mathematical calculations can become a simple computer process by using artificial intelligence, thus relieving scientists from the chore of symbol calculation. In this paper a computer program in the artificial intelligence language Arity/Prolog was made on IBM-PC 286 or 386 computer to derive automatically the general expressions of any order phase space transport coefficients.

2. MATHEMATICAL THOUGHT OF DERIVING ANY ORDER TRANSPORT COEFFICIENTS BY ARTIFICIAL INTELLIGENCE

In the new theory, the phase space transport of nonlinear dynamic system is such a mathematical process that the initial state equation of the system phase space boundary $F(X_0) = 0$ transforms to the final state boundary equation $F[X_0(X_t, t)] = f(X_t, t) = 0$. Where, the initial phase point of dynamic system $X_0 = [x_0^1, x_0^2, ..., x_0^N]^T$ is related with the final phase point $X_t = [x_t^1, x_t^2, ..., x_t^N]^T$ by the motion differential equation[1] of the system in N-dimension phase space. The solution[1] of the differential equation is

$$\begin{aligned} x_t^i = \ & R_j^i(t)[\delta_{j_1}^j x_0^{j_1} + \beta_{j_1 j_2}^j(t) x_0^{j_1} x_0^{j_2} \\ & + \beta_{j_1 j_2 j_3}^j(t) x_0^{j_1} x_0^{j_2} x_0^{j_3} + \cdots \\ & + \beta_{j_1 j_2 \dots j_n}^j(t) x_0^{j_1} x_0^{j_2} \dots x_0^{j_n} + \cdots] \quad , \end{aligned} \tag{1}$$

then, the inverse transform of Eq.1 can be expressed as following :

$$\begin{aligned} x_0^j = \ & S_{i_1}^j(t) x_t^{i_1} + S_{i_1 i_2}^j(t) x_t^{i_1} x_t^{i_2} + \cdots \\ & + S_{i_1 i_2 i_3}^j(t) x_t^{i_1} x_t^{i_2} x_t^{i_3} + \cdots \\ & + S_{i_1 i_2 \dots i_n}^j(t) x_t^{i_1} x_t^{i_2} \dots x_t^{i_n} + \cdots \quad , \end{aligned} \tag{2}$$

here, $i, i_1, i_2, \dots, i_n; j, j_1, j_2, \dots, j_n = 1, 2, \dots, N$. The any order aberration coefficients $R_j^i(t)\beta_{j_1 j_2 \dots j_n}^j(t)$ had been obtained [1] from the motion differential equation. Here, deriving the any order transport coefficients $S^j_{i_1 i_2 \ldots i_n}(t)$ is the work of this paper by artificial intelligence. The following is the mathematical thought of deriving the transport coefficients by computers.

Rewrite Eq.1 :

$$\begin{aligned} x^j_0 = \; & [R^{-1}(t)]^j_{i_1} x^{i_1}_t - \beta^j_{j_1 j_2}(t) x^{j_1}_0 x^{j_2}_0 - \cdots \\ & -\beta^j_{j_1 j_2 j_3}(t) x^{j_1}_0 x^{j_2}_0 x^{j_3}_0 - \cdots \\ & -\beta^j_{j_1 j_2 \ldots j_n}(t) x^{j_1}_0 x^{j_2}_0 \ldots x^{j_n}_0 - \cdots \quad , \end{aligned} \tag{3}$$

where, the sum calculating of tensor is used:

$$\begin{cases} x^j_0 = & \delta^j_{j_1} x^{j_1}_0 \\ [R^{-1}(t)]^j_i x^i_t = & [R^{-1}(t)]^j_{i_1} x^{i_1}_t \\ = & [R^{-1}(t)]^j_i \delta^i_{i_1} x^{i_1}_t \quad . \end{cases} \tag{4}$$

Substituting Eq.2 in the left of Eq.3 and eliminating the first order terms in two sides of the equation, Eq.3 becomes:

$$\begin{aligned} & S^j_{i_1 i_2}(t) x^{i_1}_t x^{i_2}_t + S^j_{i_1 i_2 i_3}(t) x^{i_1}_t x^{i_2}_t x^{i_3}_t + \ldots \\ & \quad + S^j_{i_1 i_2 \ldots i_n}(t) x^{i_1}_t x^{i_2}_t \ldots x^{i_n}_t + \cdots \\ & = -\beta^j_{j_1 j_2}(t) x^{j_1}_0 x^{j_2}_0 - \beta^j_{j_1 j_2 j_3}(t) x^{j_1}_0 x^{j_2}_0 x^{j_3}_0 - \cdots \\ & \quad -\beta^j_{j_1 j_2 \ldots j_n}(t) x^{j_1}_0 x^{j_2}_0 \ldots x^{j_n}_0 - \cdots \quad , \end{aligned} \tag{5}$$

the coefficients of the first order term $x^{i_1}_t$ is

$$S^j_{i_1}(t) = [R^{-1}(t)]^j_{i_1} \quad . \tag{6}$$

With replacing the up-mark j by up-marks $j_1, j_2, \ldots, j_n$ respectively, substituting Eq.2 into the right of Eq.5; then arranging the right; finally, comparing the coefficient of every order term at left with that of the same order term at right, so, the expression can be obtained of any order transport coefficiects. The obtained expressions are recursion formulae, that is, if these coefficients from first-order $S^j_{i_1}(t)$ to the (n-1)th-order $S^j_{i_1 i_2 \ldots i_{n-1}}(t)$ are known, the (n)th-order coefficient $S^j_{i_1 i_2 \ldots i_n}(t)$ can be obtained. Above three procedures, namely, substituting; arranging and comparing are the logistic thought of deriving transport coefficients by artificial intelligence. Where, the arranging is the most difficult and important procedure of computer derivation. The mathematical methods of this procedure are as follows:

(a), making maltinomial multiplications in every order terms of the right of Eq.5 become the multiplication of two multinomials, namely:

$$x^{j_1}_0 x^{j_2}_0 \rightarrow x^{j_1}_0 (x^{j_2}_0) \quad ,$$

the 2th order term is the multiplying of two multinomials $x^{j_1}_0$ and $x^{j_2}_0$;

$$x^{j_1}_0 x^{j_2}_0 x^{j_3}_0 \rightarrow (x^{j_1}_0 x^{j_2}_0) x^{j_3}_0 \quad ,$$

the 3th order term is the multiplying of two multinomials $(x^{j_1}_0 x^{j_2}_0)$ and $x^{j_2}_0$, where, the first multinomial $(x^{j_1}_0 x^{j_2}_0)$ is the multiplying result of the 2th order term;

...... ;

$$x^{j_1}_0 x^{j_2}_0 \ldots x^{j_{n-1}}_0 x^{j_n}_0 \rightarrow (x^{j_1}_0 x^{j_2}_0 \ldots x^{j_{n-1}}_0) x^{j_n}_0 \quad ,$$

the (n)th order term is the multiplying of two multinomials $(x^{j_1}_0 x^{j_2}_0 \ldots x^{j_{n-1}}_0)$ and $x^{j_n}_0$, where, the first multinomial $(x^{j_1}_0 x^{j_2}_0 \ldots x^{j_{n-1}}_0)$ is the multiplying result of the (n-1)th order term;

....

(b), After every multiplying of two multinomials, combining the same terms and the symmetrical terms.

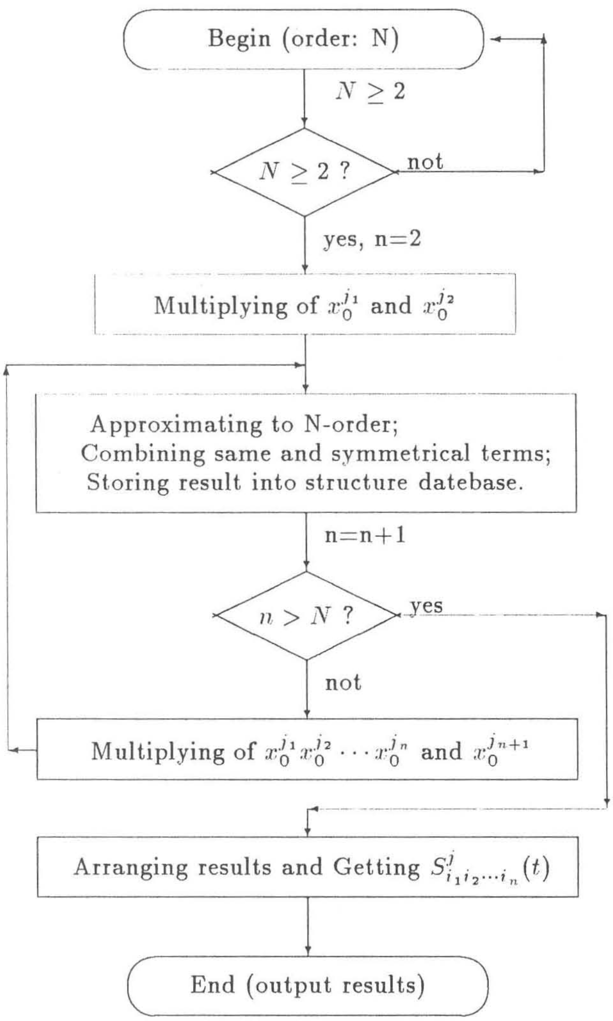
To summarize , above two procedures are such a process of processing every order terms of the right of Eq.5 one by one, and in every processing, the two mathematical procedures (a),(b) are done.

(c), After all multinomial multiplications in the right of Eq.5, combining same order terms.

In conclusion, the three procedures (a),(b),(c) are the main mathematical thoughts of deriving any order transport coefficients by artificial intelligence.

3. DERIVING ANY ORDER TRANSPORT COEFFICIENTS BY ARTIFICIAL INTELLIGENCE

Prolog is a logic programming language of intelligence. By using it, a practical artificial intelligence system can be established facilitately. In this paper, a computer program in Arity/Prolog was made which derive automatically the general expressions of any order phase space transport coefficients. With the supporting of Arity/Prolog interpreter or Arity/Prolog compiler, this program can be operated on IBM PC 286 or 386 machines. The following is the block diagram of the program structure.



Block diagram of the program structure

As the block diagram shows, the mathematical thought of deriving any order tramsport coefficients is comprised in the infereme procedure of this program.

With the using of many programming techniques[2], such as backtracking, structure database, etc., this deriving program with a tight structure can be read easily. The key technique of replacing recursion with interation broke through the 640k RAM limit and realized the practical operating of the program on IBM-PC 286 or 386 computer. By using this intelligent system, any order transport coefficients can be derived out automatically and fastly. The following are 2th,3th,4th,5th and 9th order transport coefficints output directly from a computer, and where, the results of 2th,3th,4th and 5th order are showing no difference with those by manual doing. The symbol corresponding relations between computer output and mathematical formulae are:

$$S\langle j; i(1), i(2), \cdots, i(n)\rangle \Longleftrightarrow S^j_{i_1 i_2 \dots i_n}(t)$$
$$beta\langle j; j(1), j(2), \cdots, j(n)\rangle \Longleftrightarrow \beta^j_{j_1 j_2 \dots j_n}(t).$$

```
N = 2
S<j;i(1),i(2)>=
  -beta<j;j(1),j(2)>
      *[S<j(1);i(1)>*S<j(2);i(2)>]
N = 3
S<j;i(1),i(2),i(3)> =
  -beta<j;j(1),j(2)>*[2*S<j(1);i(1)>
      *S<j(2);i(2),i(3)>]
  -beta<j;j(1),j(2),j(3)>*[S<j(1);i(1)>
      *S<j(2);i(2)>*S<j(3);i(3)>]
N = 4
S<j;i(1),i(2),i(3),i(4)> =
  -beta<j;j(1),j(2)>*[S<j(1);i(1),i(2)>
      *S<j(2);i(3),i(4)>+2*S<j(1);i(1)>
      *S<j(2);i(2),i(3),i(4)>]
  -beta<j;j(1),j(2),j(3)>*[3*S<j(1);i(1)>
      *S<j(2);i(2)>*S<j(3);i(3),i(4)>]
  -beta<j;j(1),j(2),j(3),j(4)>*[S<j(1);i(1)>
      *S<j(2);i(2)>*S<j(3);i(3)>*S<j(4);i(4)>]
N = 5
S<j;i(1),i(2),i(3),i(4),i(5)> =
  -beta<j;j(1),j(2)>*[2*S<j(1);i(1),i(2)>
      *S<j(2);i(3),i(4),i(5)>+2*S<j(1);i(1)>
      *S<j(2);i(2),i(3),i(4),i(5)>]
  -beta<j;j(1),j(2),j(3)>*[3*S<j(1);i(1)>
      *S<j(2);i(2),i(3)>*S<j(3);i(4),i(5)>
      +3*S<j(1);i(1)>*S<j(2);i(2)>
      *S<j(3);i(3),i(4),i(5)>]
  -beta<j;j(1),j(2),j(3),j(4)>*[4*S<j(1);i(1)>
      *S<j(2);i(2)>*S<j(3);i(3)>
      *S<j(4);i(4),i(5)>]
  -beta<j;j(1),j(2),j(3),j(4),j(5)>
      *[S<j(1);i(1)>*S<j(2);i(2)>*S<j(3);i(3)>
      *S<j(4);i(4)>*S<j(5);i(5)>]

N = 9
S<j;i(1),i(2),i(3),i(4),
                   i(5),i(6),i(7),i(8),i(9)> =
  -beta<j;j(1),j(2)>
      *[2*S<j(1);i(1),i(2),i(3),i(4)>
      *S<j(2);i(5),i(6),i(7),i(8),i(9)>
      +2*S<j(1);i(1),i(2),i(3)>
      *S<j(2);i(4),i(5),i(6),i(7),i(8),i(9)>
      +2*S<j(1);i(1),i(2)>
      *S<j(2);i(3),i(4),i(5),
                   i(6),i(7),i(8),i(9)>
      +2*S<j(1);i(1)>
      *S<j(2);i(2),i(3),i(4),i(5),
                   i(6),i(7),i(8),i(9)>]
  -beta<j;j(1),j(2),j(3)>
```

```
    *[S<j(1);i(1),i(2),i(3)>
    *S<j(2);i(4),i(5),i(6)>
    *S<j(3);i(7),i(8),i(9)>
    +6*S<j(1);i(1),i(2)>
    *S<j(2);i(3),i(4),i(5)>
    *S<j(3);i(6),i(7),i(8),i(9)>
    +3*S<j(1);i(1)>
    *S<j(2);i(2),i(3),i(4),i(5)>
    *S<j(3);i(6),i(7),i(8),i(9)>
    +3*S<j(1);i(1),i(2)>*S<j(2);i(3),i(4)>
    *S<j(3);i(5),i(6),i(7),i(8),i(9)>
    +6*S<j(1);i(1)>*S<j(2);i(2),i(3),i(4)>
    *S<j(3);i(5),i(6),i(7),i(8),i(9)>
    +6*S<j(1);i(1)>*S<j(2);i(2),i(3)>
    *S<j(3);i(4),i(5),i(6),i(7),i(8),i(9)>
    +3*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3),i(4),i(5),
                 i(6),i(7),i(8),i(9)>]
-beta<j;j(1),j(2),j(3),j(4)>
    *[4*S<j(1);i(1),i(2)>
    *S<j(2);i(3),i(4)>*S<j(3);i(5),i(6)>
    *S<j(4);i(7),i(8),i(9)>+12*S<j(1);i(1)>
    *S<j(2);i(2),i(3)>
    *S<j(3);i(4),i(5),i(6)>
    *S<j(4);i(7),i(8),i(9)>+12*S<j(1);i(1)>
    *S<j(2);i(2),i(3)>*S<j(3);i(4),i(5)>
    *S<j(4);i(6),i(7),i(8),i(9)>
    +12*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3),i(4),i(5)>
    *S<j(4);i(6),i(7),i(8),i(9)>
    +12*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3),i(4)>
    *S<j(4);i(5),i(6),i(7),i(8),i(9)>
    +4*S<j(1);i(1)>
    *S<j(2);i(2)>*S<j(3);i(3)>
    *S<j(4);i(4),i(5),i(6),i(7),i(8),i(9)>]
-beta<j;j(1),j(2),j(3),j(4),j(5)>
    *[5*S<j(1);i(1)>*S<j(2);i(2),i(3)>
    *S<j(3);i(4),i(5)>*S<j(4);i(6),i(7)>
    *S<j(5);i(8),i(9)>+30*S<j(1);i(1)>
    *S<j(2);i(2)>*S<j(3);i(3),i(4)>
    *S<j(4);i(5),i(6)>
    *S<j(5);i(7),i(8),i(9)>
    +10*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4),i(5),i(6)>
    *S<j(5);i(7),i(8),i(9)>+20*S<j(1);i(1)>
    *S<j(2);i(2)>*S<j(3);i(3)>
    *S<j(4);i(4),i(5)>
    *S<j(5);i(6),i(7),i(8),i(9)>
    +5*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4)>
    *S<j(5);i(5),i(6),i(7),i(8),i(9)>]
-beta<j;j(1),j(2),j(3),j(4),j(5),j(6)>
    *[20*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4),i(5)>
    *S<j(5);i(6),i(7)>*S<j(6);i(8),i(9)>
    +30*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4)>
    *S<j(5);i(5),i(6)
    *S<j(6);i(7),i(8),i(9)>
    +6*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4)>*S<j(5);i(5)>
    *S<j(6);i(6),i(7),i(8),i(9)>]
-beta<j;j(1),j(2),j(3),j(4),j(5),j(6),j(7)>
    *[21*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4)>*S<j(5);i(5)>
    *S<j(6);i(6),i(7)>*S<j(7);i(8),i(9)>
    +7*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4)>*S<j(5);i(5)>
    *S<j(6);i(6)>*S<j(7);i(7),i(8),i(9)>]
-beta<j;j(1),j(2),j(3),j(4),j(5),
                 j(6),j(7),j(8)>
    *[8*S<j(1);i(1)>*S<j(2);i(2)>
    *S<j(3);i(3)>*S<j(4);i(4)>*S<j(5);i(5)>
    *S<j(6);i(6)>*S<j(7);i(7)>
    *S<j(8);i(8),i(9)>]
-beta<j;j(1),j(2),j(3),j(4),j(5),
                 j(6),j(7),j(8),j(9)>
    *[S<j(1);i(1)>*S<j(2);i(2)>*S<j(3);i(3)>
    *S<j(4);i(4)>*S<j(5);i(5)>*S<j(6);i(6)>
    *S<j(7);i(7)>*S<j(8);i(8)>*S<j(9);i(9)>]
```

4. DISCUSSION

By using the artificial intelligence, any high order transport coefficients can be obtained fastly and accurately. Obviously, with this new method man's brain will be extended and more time can be saved for many physicists.

Based on the transport coefficient expressions obtained from machines, a numerical calculating program can be generated and will be applied in the practical beam transport.

5. REFERENCES

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