A FORMULATION OF SPIRAL INFLECTOR DESIGN AND ITS APPLICATION TO SF CYCLOTRON

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ABSTRACT

Ion trajectories through a spiral inflector are analyzed. In the case of a uniform magnetic field, it is shown that the central trajectory can be expressed as a simple analytical form, which includes all the inflector types already known. By using a curvelinear coordinate along the central trajectory, the equations of motion of non-central ions are modified to describe the ion motion in the transverse and longitudinal planes. A method is presented to expand the electric potential around the central trajectory to calculate the electric field to be used in the equations of motion. In the method, an electric potential thus obtained is used instead of the electric field, which guarantees the energy conservation of the ions through the inflector. Numerical studies have been carried out for design of an inflector for the SF cyclotron at the Institute for Nuclear Study, University of Tokyo.

1. INTRODUCTION

With advent of excellent external ion sources, such as ECR sources for heavy ions, more cyclotrons are equipped with them. Most cyclotrons are designed recently from the beginning to employ only them. It has been proved that the ions are most effectively injected along the main magnetic field of the cyclotron into its center. They must be deflected into the median plane in order for them to be accelerated successfully in the cyclotron.

In the early stage of the development of the modern cyclotron, many studies have been carried out for devices to inflect the ions injected along the main magnetic field into the median plane.[1-4] Nowadays, most cyclotrons employ a device called a spiral inflector [2,3,5,6] or an electrostatic mirror. [8] It is essential that a good optical device should be employed to accept ions effectively from the ion source.

A spiral inflector for ions may be defined as a device with a pair of electrodes which produce an electric field always perpendicular to the velocity of the ions moving in a given magnetic field. The ion velocity at its entrance is parallel to the magnetic field, while at its exit the velocity should have been turned into a direction perpendicular to the magnetic field. Although it is not evident whether such a device generally exists in any given shape of magnetic field, spiral inflectors can be actually manufactured for cyclotrons, in which the main magnetic field is along the initial ion velocity, whereas the final ion velocity should be in the median plane of the cyclotron.

In this paper, firstly, the central trajectory of an ion through the spiral inflector is reviewed by using a uniform magnetic field approximation. Secondly, the equations of motion for the non-central ions are modified in a curvelinear coordinate with the central trajectory as the reference curve to integrate them numerically. Thirdly, trajectory as the reference a method to expand the electric potential produced by the inflector electrodes around the central trajectory is presented. These formulae are applied to study the inflector design for the SF cyclotron at the Institute for Nuclear Study.

2. CENTRAL TRAJECTORY AND CENTERING OF THE CYCLOTRON ORBIT

The equations of motion of an ion with mass m and charge e, moving in a magnetic field $\boldsymbol{B}$ and electric field $\boldsymbol{E}$ with velocity v, can be written as

$$\boldsymbol{t}' = K\boldsymbol{e} - F\boldsymbol{n} + [\boldsymbol{t} \times \boldsymbol{b}], \tag{1}$$

where the prime designates differentiation with respect to the length s along the orbit, $\boldsymbol{b}$ is the magnetic curvature vector,

$$\boldsymbol{B} = \frac{mv}{e}\boldsymbol{b},$$

K and F are the electric curvatures,

$$\boldsymbol{E} = \frac{mv^2}{e}(K\boldsymbol{e} - F\boldsymbol{n}),$$

$\boldsymbol{t}$ is the tangent unit vector, $\boldsymbol{e}$ and $\boldsymbol{n}$ are unit vectors perpendicular to $\boldsymbol{t}$ and $\boldsymbol{n} = \boldsymbol{t} \times \boldsymbol{e}$.

We will find solutions of the equations of motion by using $\boldsymbol{t}$, $\boldsymbol{e}$ and $\boldsymbol{n}$. The other two vectors should satisfy the following simultaneous equations

$$\boldsymbol{e}' = -K\boldsymbol{t} + G\boldsymbol{n} + [\boldsymbol{e} \times \boldsymbol{b}]$$

and

$$\boldsymbol{n}' = -G\boldsymbol{e} + F\boldsymbol{t} + [\boldsymbol{n} \times \boldsymbol{b}],$$

together with Eq.(1). The curvature G is also a function of s. K and F determine the bending electric field and G determines the rotation of the electric field along the trajectory.

By definition, the electric field acts just as the magnetic field or *vice versa* so long as the central trajectories are concerned, since it exerts force in the perpendicular direction to the ion velocity. This fact is incorporated into these equations. If the magnetic field distribution $\boldsymbol{b}$ and the functions K, F and G are given as functions of position of the ion, we can integrate above three equations to obtain the central orbit.

In the case of a uniform magnetic field B, it can be shown that the equations have simple solutions containing only trigonometric functions in the case

$$K = C_e, F = C_f \sin\alpha, G = -C_f \cos\alpha$$

and

$$\alpha = C_e s + \alpha_0,$$

where C_e and C_f are constants specifying the electric curvatures and α_0 is a constant determining the phase of the rotation of the electric field. Let us introduce the following notation: $R_0 = (1/B)\sqrt{2mV_d/e}$, $R_m = \sqrt{K_i}R_0$, $C_m = 1/R_m$, and $K_i = V_i/V_d$, where V_i and V_d are the injection voltage and dee voltage, respectively. The strength of the electric field is given by

$$E = 2V_i \left\{C_e^2 + C_f^2 \sin^2(C_e s + \alpha_0)\right\}^{1/2}.$$

This solution contains all of the types of the spiral inflectors so far studied in addition to new ones. With our notation, they are as follows:

normal [2] : $C_e \neq 0$, $C_f = 0$, $\alpha_0 = 0$
slanted [2] : $C_e \neq 0$, $C_f \neq 0$, $\alpha_0 = 0$
hyperboloid [3] : $C_e = C_m/\sqrt{6}$, $C_f = -C_m/2$, $\alpha_0 = 0$
paraboloid [3] : $C_e = C_m/2$, $C_f = -C_m/2$, $\alpha_0 = 0$.

It is known that the *hyperboloid* and *paraboloid* has no free parameters, while *normal* has one free parameter and the *slanted* has two free parameters. In the general case described above, we have three free parameters, C_e, C_f and α_0 in general. In the *normal* case, the strength of the electric field is constant along the trajectory and so is the gap between the electrodes. Among many possible combinations of parameters, we should note the case: $C_e = 0$, $C_f \neq 0$ (let's call this *parallel*, since the electric field is constant and the electrodes are parallel even if they are twisted). When $\alpha_0 = \pi/2$, this solution is the same as the *slanted*, except that the axis is rotated by $\pi/2$, but we can choose α_0 at will according to the requirement. This solution has also a constant gap and still has another free parameter.

After exit of the inflector, the ion orbit should be centered appropriately for further acceleration. In a simple approximation of a uniform magnetic field and the narrow-gap limit of acceleration, the tangent at the inflector exit should be the tangent of a circle with radius R_m. The center of this circle should be located properly for further acceleration in the cyclotron. This requirement determines one of the three parameters or the injection voltage.

3. OPTICAL PROPERTIES

For the non-central trajectories, we trace the ion orbits in a curvelinear coordinate system referring to the central trajectory as the reference curve. As in the theory of particle accelerator with large dimension, it is convenient to separate particle orbit in the transverse and longitudinal directions. For this purpose, let's introduce a vector in the transverse plane

$$\boldsymbol{q} = \boldsymbol{r} - \boldsymbol{r}_c$$

such that

$$\boldsymbol{q} \cdot \boldsymbol{t} = 0. \tag{2}$$

In order to trace the ion motion along the reference curve, let's introduce another vector

$$\boldsymbol{p} = \frac{d\boldsymbol{r}}{du} - \boldsymbol{t},$$

which describes the difference of the ion velocity with respect to the reference one. The quantity u has a dimension of length, $u = vt$, where v is the velocity of the reference ion and t is the time elapsed after entering the inflector. It is a function of s and corresponds to the flight time of the non-central ion in question. By differentiating Eq.(2), we obtain the equation for u:

$$u' = \frac{\lambda}{1 + \boldsymbol{p} \cdot \boldsymbol{t}}, \tag{3}$$

where

$$\lambda = 1 - (\boldsymbol{t}' \cdot \boldsymbol{e})(\boldsymbol{q} \cdot \boldsymbol{e}) + (\boldsymbol{n}' \cdot \boldsymbol{t})(\boldsymbol{q} \cdot \boldsymbol{n}).$$

With these vectors, we can rewrite the equations of motion in the following form:

$$\boldsymbol{q}' = u'\boldsymbol{p} + (u' - 1)\boldsymbol{t} \tag{4}$$

and

$$\begin{aligned} \boldsymbol{p}' = {} & (u' - 1)\{\boldsymbol{C}_c + [\boldsymbol{t} \times \boldsymbol{b}_c]\} \\ & + u'\{(\boldsymbol{C} - \boldsymbol{C}_c) + [\boldsymbol{t} \times (\boldsymbol{b} - \boldsymbol{b}_c)]\} + u'[\boldsymbol{p} \times \boldsymbol{b}], \end{aligned} \tag{5}$$

where the suffix c refers to the central trajectory and $\boldsymbol{C}$ is the electric curvature vector. It is easy to rewrite these vector equations into those for the components (q_e, q_n) of $\boldsymbol{q}$ and (p_e, p_n, p_t) of $\boldsymbol{p}$.

Up to present, we have not assumed that the magnetic field is uniform. If the magnetic and electric field distributions are given in the space and the central trajectory is given, we can integrate these equations to calculate the ion trajectories of non-central ions.

The energy conservation is easily verified in these equations if we use appropriate electric potential and electric field described in the next section. It should be noted that the longitudinal electric field appears for the non-central ions that are absent on the central trajectory. In the calculations using these equations, we can use the energy conservation as the checks for the calculations at each step of integration. When calculating transmission efficiency or acceptance of the inflector, we can check if an ion hit the inflector electrode or not by looking at the electric potential at each point in question.

4. ELECTRIC FIELD AND SHAPE OF ELECTRODE SURFACE

Since we are using a curvelinear coordinate moving with the central trajectory, the electric potential becomes somewhat complicated due to the twisting of the reference curve. In the transverse plane, the relation of the electric potential and the electric field is as usual:

$$\boldsymbol{E}\cdot\boldsymbol{e} = -\frac{\partial\phi}{\partial q_e}$$

and

$$\boldsymbol{E}\cdot\boldsymbol{n} = -\frac{\partial\phi}{\partial q_n}.$$

In the longitudinal direction, the electric field is given by

$$\begin{aligned}\boldsymbol{E}\cdot\boldsymbol{t} &= -\frac{1}{\lambda}\partial_s\phi \\ &\equiv -\frac{1}{\lambda}\left\{(\boldsymbol{e}'\cdot\boldsymbol{n})\left(q_n\frac{\partial\phi}{\partial q_e} - q_e\frac{\partial\phi}{\partial q_n}\right) + \frac{\partial\phi}{\partial s}\right\}.\end{aligned}$$

The Laplace equation then becomes

$$\begin{aligned}\Delta\phi &= \frac{1}{\lambda}\left\{\frac{\partial}{\partial q_e}\left(\lambda\frac{\partial\phi}{\partial q_e}\right) + \frac{\partial}{\partial q_n}\left(\lambda\frac{\partial\phi}{\partial q_n}\right)\right\} \\ &\quad + \frac{1}{\lambda}\partial_s\left(\frac{1}{\lambda}\partial_s\phi\right) \\ &= 0.\end{aligned}$$

We can expand the potential ϕ around the central trajectory as a power series of $\boldsymbol{q}$ by using a polar coordinate (q, θ) in the transverse plane:

$$\phi = \sum_{n\geq 0}\phi_n,$$

where

$$\phi_n = Real\left\{a_n(s)q^n e^{in\theta} + \sum_{n<k, m\leq k-2} c_{nkm}(s)q^k e^{im\theta}\right\},$$

a_n and c_{nkm} are complex functions of s, and m is an odd or even non-negative integer as n is. Note that the factor $q^n e^{in\theta}$ is a solution of the Laplace equation when the reference curve is a straight line. The coefficient a_0 must be independent of s since the electric field is perpendicular to the reference curve and a_1 is determined by K and F. The other coefficients a_n introducing the $2n$-pole electric potential are free parameters for the shape of the inflector electrodes.

It should be noted that this expansion is only valid near the central trajectory: it is valid for small $C_m q$, $C_e q$ and $C_f q$.

5. DESIGN STUDY FOR THE INS SF CYCLOTRON

The SF cyclotron[9] at the Institute for Nuclear Study, University of Tokyo, has been using a mirror to deflect the ions both from a polarized-ion source and from an ECR ion source.[10] The cyclotron was designed and constructed to be used mainly with internal PIG sources. The transmission through the vertical injection line is not good(1-5%), which is the main motivation of the present study. It is an ordinary AVF cyclotron with an extraction radius of 730 mm. Usually it employs a constant orbit acceleration with 250 turns up to extraction: thus R_0 = 33 mm. The dee voltage used for usual operation lies in the range 20 - 60 kV. Since the magnetic field is uniform within 0.5% in the region of interest, the uniform field approximation may be adequate.

Numerical calculations have been carried out by using the polynomial expansion of the potentials up to 5th order in qC_m, qC_e and qC_f. Four types of potentials have been generated: two deflector types in $\boldsymbol{e}$ and $\boldsymbol{n}$ directions, two quadrupole types with hyperbolic shapes. The equations of motion, Eqs.(3-5), were integrated numerically for each ion starting at a point generated randomly in the phase space. The Monte Carlo calculations were performed for each combination of these potentials.

The numerical values of the potential thus obtained show distortion of the electrode surfaces due to the twist of the central trajectory. In the case of the *normal* type with a value of V_i = 0.3, radii of the magnetic and electric curvatures are 18 mm and 41 mm, respectively, the surfaces of the electrodes deviate from a straight horizontal line by about 0.5 mm at a point 5 mm apart from the central trajectory. Conversely, if we machine the electrode surface as a straight line in this case, considerable strength of the quadrupole and/or higher-order components of the electric potential is introduced inevitably at the edges of the electrode width, although it is not clear that these components improve or deteriorate the optical properties.

It was found that the *parallel* type has large freedom for choosing the injection voltage: V_i changes widely by changing α_0 for a given path length as shown in Fig.1. Note that $\alpha_0 = 90°$ is equal to the *normal* case. In our cyclotron, the injection energy of the ions must be varied widely, since the injection voltage is proportional to the dee voltage once the inflector structure is fixed. This point is an advantage of this type, because more ions with better emittance can be extracted usually from the ion sources and the ions with higher energy are less affected by disturbance such as stray field during the beam transport.

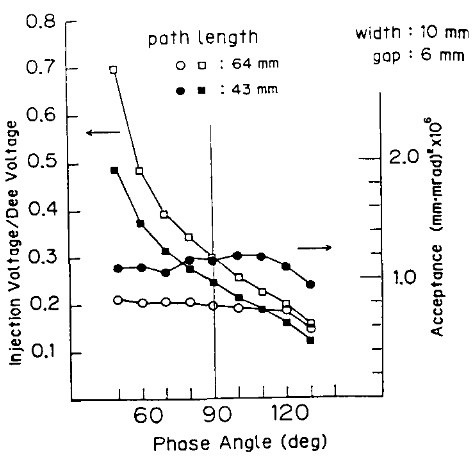

Fig. 1. Ratio of the injection voltage to the dee voltage and acceptance as a function of α_0 for the two different cases of the path length.

In the figure, the values of C_f were so changed that the path length has two different values of 64 mm and 43 mm for each α_0. The former is a realistic value having a height of about 40 mm and the latter, having a height of 30 mm, seems too small in our case.

Acceptance is also shown in Fig.1, calculated by the Monte Carlo method mentioned above. A waist with a radius covering the physical aperture and with an angular spread of 100 mrad was assumed for the beam at the entrance. It can be seen that the acceptance depends little on α_0, which allows us to select an adequate injection voltage without affecting the acceptance.

As expected, calculations show that acceptance is primarily determined by the path length of the central trajectory. The values of the acceptance shown in Fig.1 appear to be large enough, but calculations show that the two transverse motions and the longitudinal motion are highly coupled in the inflector that the transmission should be calculated for individual design of the inflector.

For simplicity, we have determined to start from the simplest type of the inflector, the *normal* type.

6. SUMMARY

The formulation of the central trajectory described in this paper can classify and compare several kinds of inflector types already known in the uniform field approximation. It can be used for compact-type cyclotrons in which the magnetic field is almost uniform in the central region of the cyclotron. It can be modified or used as a starting point of estimation also for separated-sector type cyclotrons in which the field changes considerably. It should be noted that the three free parameters allows us to choose a best central orbit for a specific requirement at a cost of difficulty of machining.

Our formulation of the non-central orbit is more transparent than those of other works by guaranteeing the energy conservation, when used with appropriate electric potentials. However, we don't believe that we have clarified the optical properties sufficiently by present numerical calculations because of the inherent complexity of the problem. Further numerical studies seem to be necessary.

7. REFERENCES

1) Powell, W. B. and Reece, B., L., "Injection of Ions into a Cyclotron from an External Source," Nucl.Instrum. Meth., **32**, 325(1965).
2) Belmont, J. L. and Pabot, J. L., "Study of Axial Injection for the Grenoble Cyclotron," IEEE Trans.Nucl.Sci., **NS-13**, 191(1966).
3) Müller, R. W., "Novel Inflectors for Cyclic Accelerators," Nucl.Instrum.Meth., **54**, 29(1967).
4) Hazewindus, N., "Calculation of Particle Trajectories in a Cyclotron Axial Injection System with an Electrostatic Deflector," Nucl.Instrum.Meth.,**76**, 273(1969).
5) Root, L. W., "Design of an Inflector for the TRIUMF Cyclotron," M. Sc. Thesis, Univ. of British Columbia, Vancouver, 1972.
6) Root, . W., "Experimental and Theoretical Studies of the Behaviour of an H^- Ion Beam during Injection and Acceleration in the TRIUMF Central Region Model Cyclotron," Ph.D. Thesis, Univ. of British Columbia, Vancouver, 1974.
7) Balden, R. J. *et al.*, "Aspects of Phase Space Dynamics in Spiral Inflectors," in **Proc. 12th Int. Conf. on Cyclotrons and their Applications** (World Scientific, Singapore, 1989) pp.435–438.
8) Bellomo, G. *et al.*, "On the Feasibility of Axial Injection in Superconducting Cyclotrons," Nucl.Instrum. Meth.,**83**, 19(1983).
9) Hirao, Y. *et al.*, "The INS 176cm Sector Focusing cyclotron," in **Proc. 7th Int. Conf. on Cyclotrons and their Applications**, (Birkhauser, Basel, 1975), pp103-106.
10) Ueda, N. *et al.*, "Acceleration of the Polarized Proton and Deuteron at the INS SF Cyclotron," in **Proc. 5th Int. Symp. on Polarization Phenomena in Nuclear Physics**, (American Institute of Physics, New York, 1981), pp.901-903.