NUMERICAL STUDIES FOR SUPPRESSION OF VERTICAL OSCILLATIONS IN THE K1200 CYCLOTRON

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ABSTRACT

Calculations using vertical electric deflectors to eliminate the coherent vertical oscillations in the K1200 cyclotron are presented. A central region code is used to track individual particles through 3-dimensional electric fields of deflecting plates. The results are promising, indicating that this method could be used if simpler methods of reducing the vertical oscillations are unsuccessful.

1. INTRODUCTION

A new internal beam probe[1] in the K1200 cyclotron based on CCD TV camera is yielding information about the internal beam structure and dynamics in much greater detail than previously available.

Initial results show that the intrinsic beam height in the cyclotron is about 1 mm, but its effective size is dominated by vertical oscillation with a peak-to-peak amplitude of typically 4 mm. This oscillation increases the transverse beam emittance of the extracted beam in the axial direction.

Some experimental studies of correcting the oscillations have been done by moving the spiral inflector vertically and/or unbalancing trim coil 0. Preliminary measurements with trim coil 0 were promising, but it is not clear if this method will work for all beams. This simple method will be pursued further. The present paper presents an alternative technique.

Calculations have been carried out to show if some small electrostatic vertical deflection plates are added to the central region the vertical oscillations could be tuned. For these calculations the NSCL central region code CYCLONE[2] has been modified to include 3-dimensional electric fields of deflecting plates calculated with the code RELAX3D.[3]

2. SIMPLIFIED THEORY

The underlying idea to eliminate the coherent vertical oscillations is to use two electrostatic deflectors on successive hills near the central region. The 1st electric deflector applies an impulse to make the central ray in a beam pass through the magnetic median plane in the 2nd electric deflector. Then the 2nd deflector applies another impulse to keep the ray on the median plane. As a result the coherent oscillation of the beam would disappear if the phase space distribution is symmetric about the central ray in $z - p_z$ space. While in some cases one deflector is enough, two are needed to make the method general.

The impulse by a static electric field E_z to a particle of electric charge q, mass m, and orbital frequency ω is

$$\Delta p_z = \int \frac{qE_z}{\omega} d\theta \,, \tag{1}$$

where

$$\omega = \frac{d\theta}{dt} \,. \tag{2}$$

We define associated length units $\tilde{p}$ and a as

$$\tilde{p} \equiv \frac{p}{m\omega} \tag{3}$$

$$a \equiv \frac{c}{\omega} \,, \tag{4}$$

where c is the velocity of light.

If the electric field and orbital angular frequency are constant, the impulse to the particle of charge q and atomic mass A is given in the cyclotron units by:

$$\Delta \tilde{p}_z \quad \propto \quad \frac{q}{A} a^2 E_z \Delta\theta \,. \tag{5}$$

The vertical motion over a small radial range can be approximated by

$$\begin{aligned} z &= z_0 \cos(\nu\theta + \varphi) & (6)\\ \tilde{p} &= -z_0 \nu \sin(\nu\theta + \varphi) \,, & (7) \end{aligned}$$

where ν is the vertical tune and φ is initial phase. z_0 and ν are not constant in general but are nearly so for a range of several turns.

If we assume that the electric fields are localized in the centers of the two electric deflectors positioned at θ_1

and θ_2 respectively, then we can consider the impulse by the electric field as a momentum kick.

Now position and momentum in the 1st electric deflector are given by:

$$z_1(\theta_1) = z_{10}\cos(\nu\theta_1+\varphi) \quad (8)$$
$$\tilde{p}_1(\theta_1) = -z_{10}\nu\sin(\nu\theta_1+\varphi)+\Delta\tilde{p}_1\,, \quad (9)$$

where $\Delta\tilde{p}_1$ is the momentum kick by the 1st electric field.

Or with a redefined phase φ_1 and amplitude z^*_{10}:

$$z^*_1(\theta_1) = z^*_{10}\cos(\nu\theta_1+\varphi_1) \quad (10)$$
$$\tilde{p}^*_1(\theta_1) = -z^*_{10}\nu\sin(\nu\theta_1+\varphi_1)\,. \quad (11)$$

And position and momentum in the 2nd electric deflector are given by:

$$z_2(\theta_2) = z_{20}\cos(\nu\theta_2+\varphi_1) \quad (12)$$
$$\tilde{p}_2(\theta_2) = -z_{20}\nu\sin(\nu\theta_2+\varphi_1)+\Delta\tilde{p}_2\,, \quad (13)$$

where $\Delta\tilde{p}_2$ is the momentum kick by the 2nd electric field. (Note that z_{20} is equal to z^*_{10})

And with another redefined phase φ_2 and amplitude z^*_{20}:

$$z^*_2(\theta_2) = z^*_{20}\cos(\nu\theta_2+\varphi_2) \quad (14)$$
$$\tilde{p}^*_2(\theta_2) = -z^*_{20}\nu\sin(\nu\theta_2+\varphi_2)\,. \quad (15)$$

Since $\tilde{p}^*_1=\tilde{p}_1$ and $z^*_1=z_1$ at θ_1, $\frac{\tilde{p}^*_1(\theta_1)}{z^*_1(\theta_1)} = \frac{\tilde{p}_1(\theta_1)}{z_1(\theta_1)}$ gives

$$-\nu\tan(\nu\theta_1+\varphi_1) = \frac{-z_{10}\nu\sin(\nu\theta_1+\varphi)+\Delta\tilde{p}_1}{z_{10}\cos(\nu\theta_1+\varphi)}\,. \quad (16)$$

If the ray passes through the magnetic median plane in the 2nd electric deflector i.e. $z_2(\theta_2)=0$, so it follows that from eq. 12 that

$$\nu\theta_2+\varphi_1 = (n+\frac{1}{2})\pi\,\cdots\,, \quad (17)$$

where $n = 0,1,2,\cdots$

Therefore, the required momentum kick in the 1st electric deflector to make the ray pass through the magnetic median plane in the 2nd electric deflector can be found from eq. 16 and eq. 17:

$$\Delta\tilde{p}_1 = \nu z_{10}\{\sin(\nu\theta_1+\varphi)-\cos(\nu\theta_1+\varphi)\cot(\nu(\theta_2-\theta_1))\}\,. \quad (18)$$

3. DESIGN CONSIDERATIONS

Eq. 18 has absolute values of minima(zeros) at $(\nu\theta_1+\varphi)+(\nu(\theta_2-\theta_1)) = \frac{1}{2}\pi,\ \frac{3}{2}\pi,\ \frac{5}{2}\pi,\ \cdots$. But in general the above conditions can not be satisfied since the locations of deflectors are not movable and tunes are also not fixed values for all beams. And from the characteristic of $\cot(\nu(\theta_1-\theta_2))$ the points of $\nu(\theta_2-\theta_1)=\pi, 2\pi, 3\pi,\cdots$ should be avoided. Therefore, it would be reasonable and safe to minimize only the second term of eq. 18 to include all beams of various tunes.

Then the appropriate positions of two deflectors would exist around

$$\nu(\theta_2-\theta_1) = \frac{1}{2}\pi,\ \frac{3}{2}\pi,\ \frac{5}{2}\pi,\ \cdots\,. \quad (19)$$

Let Δn be the difference of turns at two deflectors. For example $\Delta n = 1$ if $\theta_2=\theta_1+2\pi$. Then we have

$$(\theta_2-\theta_1) = 2\pi\Delta n\,. \quad (20)$$

From eqs.(19)-(20), the relative positions of two deflectors can be rewritten

$$\Delta n = \frac{(m+\frac{1}{2})\pi}{2\nu}\,, \quad (21)$$

where $m = 0,1,2,\cdots$

Since the locations of two deflectors are not movable, the values of tunes of various beams between them are important. Figure 2 shows the curve of tunes and relative positions of deflectors. If we go too far from the central region, the tune becomes larger and the gap between turns becomes smaller. Therefore, it might not be easy to find an appropriate solution, if the electric fields of the deflectors are not well confined to include just a few turns.

For example, in the K1200 cyclotron the vertical tunes of most of beams are $0.1\le\nu\le 0.14$ near the central region(about 2.5 $\sim$ 5 inches in radius from the center). So the oscillations of all beams could be suppressed with relatively well optimized conditions for $\Delta n\approx 2$ from Fig. 2.

Now let's consider the required magnitude of electric fields for the deflectors. If the relative positions of two deflectors are relatively well optimized, then from eq. 18 we have

$$\Delta\tilde{p}_1 \approx \nu z_0\sin(\nu\theta_1+\varphi)\,. \quad (22)$$

Therefore the required momentum kicks in both deflectors are proportional to the tune and the amplitude z_0 of z motion. For a given beam the angular frequency ω is constant, and $z_0^2\nu\approx$ constant the required electric fields for two deflectors are from eq. 5 in the nonrelativistic limit:

$$E_1,\ E_2 \propto \frac{(T/A)}{(q/A)\Delta\theta}\sqrt{\nu}\,, \quad (23)$$

where (T/A) is the final kinetic energy per nucleon of a beam.

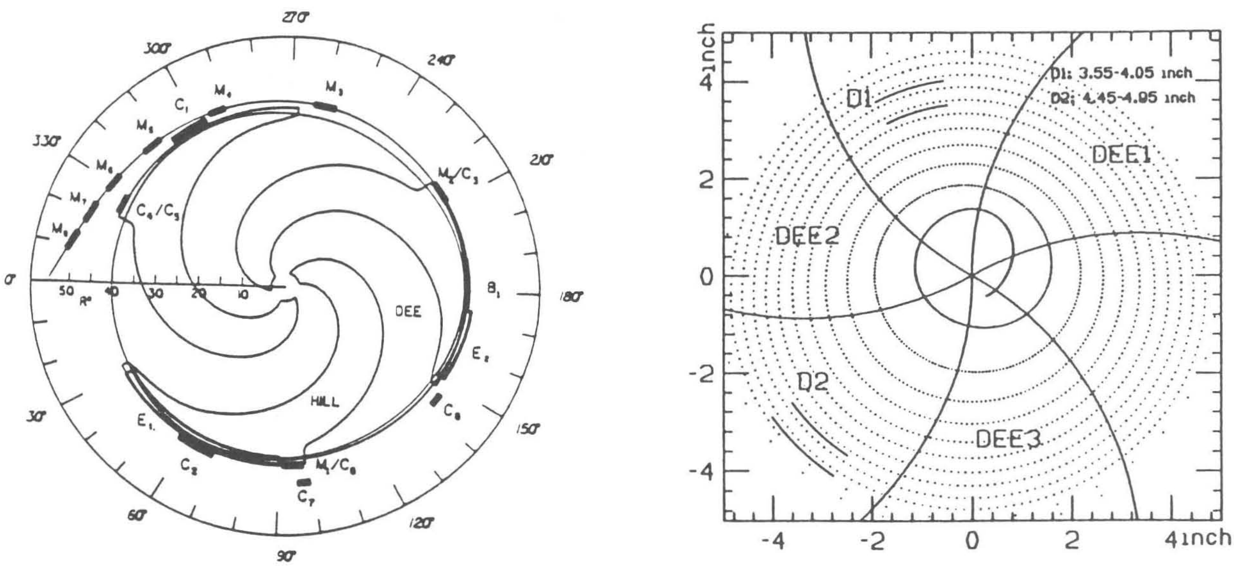

Fig. 1. Section view of the median plane of the K1200 cyclotron(left) and the enlarged top view of central region(right). The spiral dotted line shows a particle trajectory and D1, D2 are the electric deflectors locations.

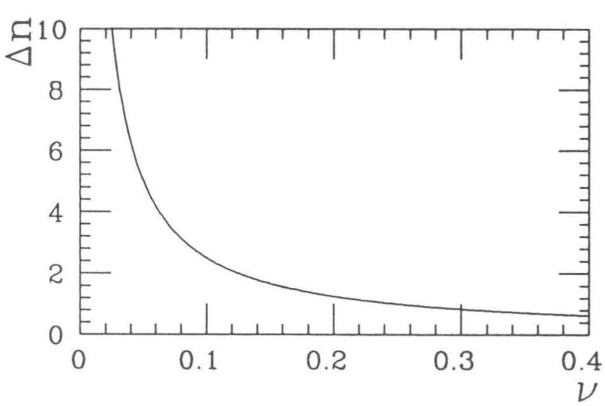

Fig. 2. The relative turns between two electric deflectors vs. the vertical tune with relatively well optimized conditions in eq. 21.

4. COMPUTER SIMULATION

The code RELAX3D[3] is a FORTRAN program to solve the three dimensional Helmholtz-Laplace equation. RELAX3D was used to calculate the 3-dimensional electric field distribution of a realistic electric deflector. The deflector has a sector shape with an angular width of 20 degrees, plate thickness of 0.225 inch, and a radial width of 1.5 inches. Edges of the deflector are rounded to reduce peak electric fields. (see Figure 3) The gap between the electric plates of the deflectors was taken to be 0.6 inch.

The raytracing was done with the NSCL central region code CYCLONE modified to include additional electric fields of two electric deflectors calculated from RELAX3D. One electric deflector was put on the hill between the 1st and 2nd dees at the radius of 3.80 inches from the center of the K1200 to the center of the deflector, while the other was put between the 2nd and 3rd dees at the radius of 4.70 inches.(see Figure 1)

The initial phase space for the beam in the vertical direction(see Figure 4) was guessed from the TV beam probe[1] data mentioned above. Figure 5 shows individual particle trajectories of nine rays with different initial phase space coordinates for $^4He^{1+}$ of 40 MeV/u. The coherent vertical oscillation(upper) was eliminated almost completely after passing the 2nd deflector(lower) to reduce the effective size of a beam by about two times. The required deflector voltages were -0.55 kV and -0.70 kV for the 1st and the 2nd deflectors, respectively.

5. DISCUSSION

The electric deflector method to eliminate undesirable coherent vertical oscillations is promising in these calculations. Alternative methods of correcting the oscillations, such as moving the spiral inflector vertically, with or without unbalancing trim coil 0, would not require new hardware, but needs further investigation to determine their range of usefulness in the K1200 cyclotron.

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6. REFERENCES

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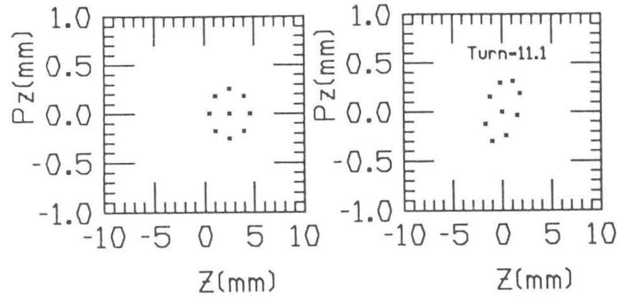

Fig. 4. Phase space coordinates of 9 rays at the beginning (left) and at turn=11.1 after passing the 2nd electric deflector(D2). The central ray has moved to the origin.

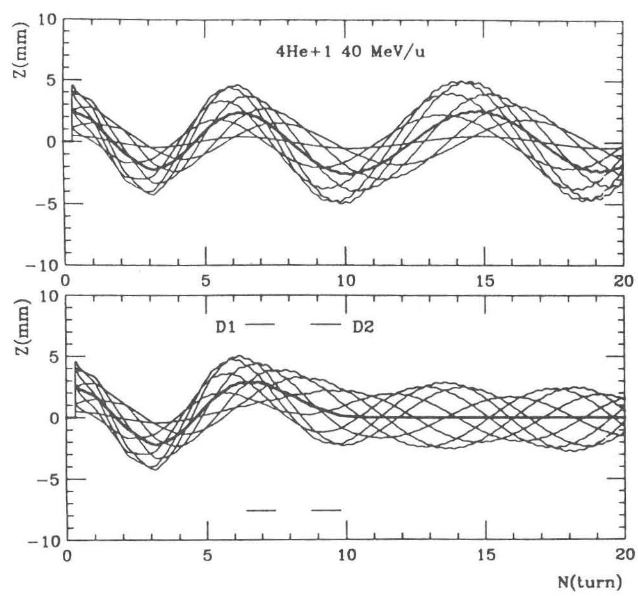

Fig. 5. Individual particle trajectories with different initial conditions as a function of turn number. A coherent vertical motion is simulated in the upper part. The lower part illustrates the calculated effect of the two vertical deflection plates. Also note that the central ray(the thick solid line) is on the median plane after passing the 2nd plate.

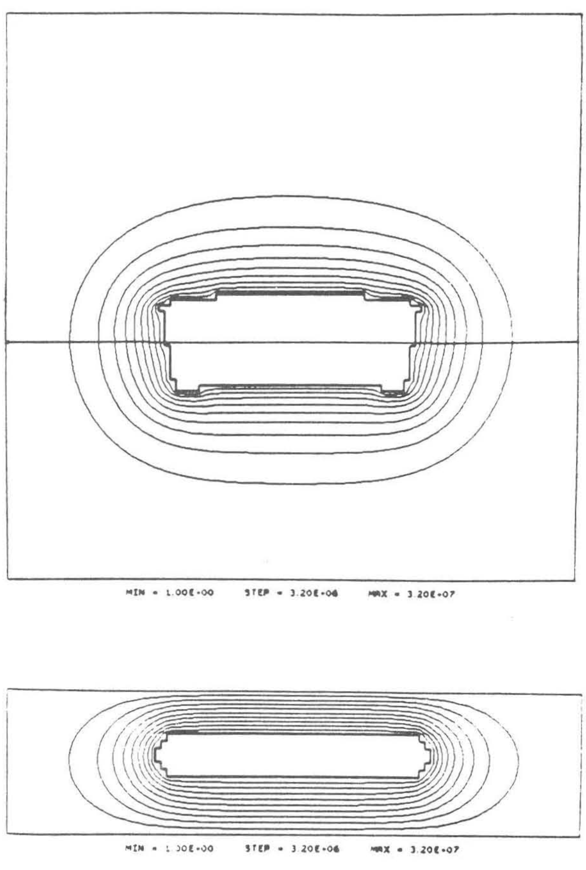

Fig. 3. A RELAX3D field calculation of a deflector plate showing equipotential lines both for top view(upper) and for cross sectional view(lower). The top solid line in the sectional view(lower) is the copper hill cover and the bottom solid line is the magnetic median plane.