## FRINGEFIELD EFFECTS IN SHORT QUADRUPOLES.

O.C. DERMOIS, A.M. VAN DEN BERG, H.K.T. VAN DER MOLEN Kernfysisch Versneller Instituut, Zernikelaan 25, 9747 AA Groningen, Netherlands<br>In quadrupoles with a small length to aperture ratio, roughly about 2 or less, fringefield effects are dominant in determining aberations. Due to the presence of a $B z$ component in the field a third order quadrupole and a seventh order twelvepole term comes in. Especially the quadrupole term can have easily the same or more influence then an octupole term one may like to introduce to compensate for aberration coefficients.<br>Field measurements done in the quadrupoles for the BBS spectrometer together with three dimensional calculations will show this effect.<br>Actually this contribution is a continuation of a preliminary one presented at the Berlin conference in 1989.

## 1. Introduction.

For our new Big Bite Spectrometer [1] we were forced to use rather short quadrupoles with a large aperture. The spectro meter is of the QQD type and the second quadrupole, called $Q 2$, has an iron length of 62 cm and an aperture of 35 cm . This means that the so called good field region is small, actually less then 20 cm , and the fringe field region needs special attention.

In addition the distance betweem Q1 and Q2 is only 30 cm and in one of the modes of operation the distance between Q2 and the dipole is so small that it modifies the fringe field of both. This forced us to get field maps for the elements separately and for the instrument as a whole. To use these field maps to calculate an ion-optical transfer map for the combination Q1-Q2 three dimensional calculations are done for comparison. As a first start an electrostatic program RELAX3D from TRIUMF is used. For another project the same kind of calculations where done on smaller quadrupoles [2], but now we had the possibility to compare the results with real field maps.

## 2. Theoretical represention of the field.

In the fringe field region of a quadrupole we get additional higher order terms next to the twelve pole and twenty pole we expect in a quadrupole with fourfold symmetry and a finite hyperbolic pole-
shape. This is due to the presence of a longitudinal field component $B_{z}$ and it is especially connected with the derivative of $B_{z}$ to $z$.

From a series expansion of the scalar potential with coefficients $a_{2}$ and $a_{6}$ as function of $z$ we get the formulas (1) to (3) for the components of $B$.
$a_{2}^{\prime} a_{2}^{\prime \prime}$ and $a_{2}^{\prime \prime}$ are the first, second and third derivatives to $z$.

$$
\begin{aligned}
& B_{z}=-a_{2}^{\prime} r^{2} \cos (2 \theta)+(1 / 12) a_{2}^{\prime \prime} r^{4} \cos (2 \theta)+ \\
& -a_{6}^{\prime} r^{6} \cos (6 \theta)+(1 / 28) a_{6}^{\prime \prime} r^{8} \cos (6 \theta)+\ldots \\
& \mathrm{B}_{\mathrm{r}}=-2 \mathrm{a}_{2} \mathrm{r} \cos (2 \theta)+(1 / 3) \mathrm{a}_{2}^{n} \mathrm{r}^{3} \cos (2 \theta)+ \\
& -6 a_{6} r^{5} \cos (6 \theta)+(2 / 7) a_{6}^{11} r^{7} \cos (6 \theta)+\ldots \\
& B_{\theta}=+2 a_{2} r \sin (2 \theta)-(1 / 6) a_{2}^{\prime \prime} r^{3} \sin (2 \theta)+ \\
& +6 a_{6} r^{5} \sin (6 \theta)-(3 / 14) a_{6}^{\prime \prime} r^{7} \sin (6 \theta)+\ldots \\
& B_{z} \cong-a_{2}^{\prime} r^{2} \cos (2 \theta) \text { thus for }(r \text { and } \theta)= \\
& \text { constant } \mathrm{a}_{2}^{\prime \prime}=\text { const* }\left(-\partial \mathrm{B}_{z} / \partial z\right) \text {. } \\
& \text { ( inside the quad } \partial B_{z} / \partial z \text { starts to be } \\
& \text { positive. ) }
\end{aligned}
$$

Next to the normal $r^{5}$ twelve pole we see a $r^{3}$ quadrupole and a $r^{7}$ twelve pole. Note that in regions were $B_{z}$ can be represented by the first term the third order quadrupole term is proportional to the derivative of $B_{z}$. Note also that the coefficients of the additional terms are

Q2, poleface: hyperbola with pole-end correction


Figure 1. Radial field distribution.
different in $B_{r}$ and $B_{\theta}$. This causes a difference in the effective length in the $y=0$ and $y=x$ plane (median plane and plane under 45 degree)
In addition all terms have equal sign inside the quad so the effective length increases with radius.

Unfortunately one cannot do much on the occurence of these terms but one can take measures to reduce the influence and especially the the integrated effect.

The presence of the third order quadrupole term becomes important in short quads with a length to aperture ratio of say less then 2. Especially when one has a strongly divergent beam in the fringe field region as e.g in a spectrometer and sometimes in a beamline, the local position of the field distribution is important in the total map from initial to final coordinates. And in a short quad there is no good field region anyhow.

## 3. Three dimensional calculations.

Figure 1 shows the calculated radial distribution of the field strength $B$ at different longitudinal positions.
The poleshape is a hyperbola in order not to disturb the picture with the presence of an octupole and sextupole.

At $\mathrm{z}=21 \mathrm{~cm}$, so 10 cm inside the iron poles the influence of the third order term is already visible. It grows vastly towards the exit and gets an amplitude of several percent ( up to $8 \%$ ). It then changes sign and grows even further but here the influence is smaller because of the lower field strength.

In our case the sign of the third order quadrupole term is opposite to the wanted octupole term and the amplitude is much higher.

Table 1 shows a polynomial expansion of the field as function of $x$ in the median plane at different $z$ positions. The poleshape was a hyperbola in the first section, the real shape in the second and the third section gives an expansion of a
measured field at $70 \%$ of maximum excitation.

In case of a hyperbola the third order term at $z=0$ in section 1 should be zero. The rest value is due to the approximation of the poleshape in a square grid.

The poles of $Q 2$ are shaped such that there will be a sextupole term of $-1 \%$ and an octupole term of $-.8 \%$, but for the third order term this holds in the middle of the quad only as one notices in the first line of section 3 .

Table 1: polynomial expansion

| order 3 | 2 | 1 | $z$ position |
| :---: | :---: | :---: | :---: |
| hyperbola poleshape as |  |  | calculated |
| 0.0048 | 0.001 | 11.323 | 0 cm |
| 0.0300 | 0.001 | 10.967 | 21 |
| 0.0830 | 0.001 | 8.954 | 30 |
| 0.0740 | 0.001 | 8.574 | 31 |
| 0.0370 | 0.001 | 7.757 | 33 |
| -0.0390 | 0.001 | 6.461 | 36 |
| -0.1360 | 0.0005 | 4.139 | 42 |
| -0.1540 | -0.000 | 2.656 | 48 |
| real poleshape as |  |  | calculated |
| -0.0022 | -0.009 | 11.391 | 0 |
| 0.0230 | -0.009 | 11.031 | 21 |
| 0.0770 | -0.0084 | 8.995 | 30 |
| 0.0670 | -0.0081 | 8.610 | 31 |
| 0.0310 | -0.0071 | 7.784 | 33 |
| -0.0430 | -0.0055 | 6.479 | 36 |
| -0.1390 | -0.0034 | 4.144 | 42 |
| -0.1550 | -0.0023 | 2.656 | 48 |
| measured field 70\% |  |  | . excitat |
| -0.008 | -0.0115 | 0.3322 | 0 |
| -0.0058 | -0.0116 | 0.3316 | 10 |
| 0.0268 | -0.0121 | 0.3196 | 21 |
| 0.0763 | -0.0110 | 0.2564 | 30 |
| 0.0664 | -0.0104 | 0.2449 | 31 |
| 0.0321 | -0.0089 | 0.2203 | 33 |
| -0.0379 | -0.0066 | 0.1817 | 36 |
| -0.1439 | -0.0034 | 0.1128 | 42 |
| -0.1772 | -0.0017 | 0.0665 | 48 |
| -0.1789 | -0.0013 | 0.0556 | 50 |
| -0.1738 | -0.0006 | 0.0359 | 55 |

## 4. The shape of the pole ends.

One of the problems connected with the


Figure 2. View of Q2
additional terms is that the effective length becomes different in the median plane and the plane under 45 degree. In our case it also differs in the vertical plane but that is due to the pole shape cutoff.
A standard method to decrease e.g the fringe field twelve pole is to make a 45 degree chamfer at the pole end.
This however increases the influence of the third order quadrupole term because the region of positive and negative derivative of $\mathrm{B}_{\mathrm{z}}$ becomes more separated. As long as saturation is not a problem it is better to make a slot in the pole end in the $y=x$ direction. In our case this slot has a depth of 8 mm and a width of 8 cm and it has in cross section the shape of a hyperbola.

Figure 2 shows Q2. This quadrupole is open in the median plane. There is no return yoke. The slot in the end of the pole is not shown.

Figure 3 shows the effective length as function of radius in the $x=0, x=y, y=0$ plane.
Actually this shows that things are a bit overcompensated so the depth and width of the cut should have been a bit smaller.



Figure 3. Effective length.


Figure 4. Alternative pole end shape

We decided to leave it for the moment as it is. We still have reserve endpieces.

An even better compensation is shown in Figure 4. Here a small chamfer is covered with a thin slab having the same shape as the pole. It was calculated for a quad with an aperture of 13 cm , an iron length of 21 cm and a fuliy fourfold symmetry also in the yoke. The result is that the effective length in the $y=0$ and the $y=x$ plane is equal within a few tens of a mm and the change as function of radius is only 1 mm over $80 \%$ of the aperture. Unfortunately it works good for electrostatic quads but for magnetic quads at low fields only.

## Remarks

The multipole content of Q 2 has been measured with a rotating coil meter and those measurements show a sextupole of 1\% and an octupole of $.08 \%$. The wanted octupole is really there throughout the quadrupole but over-shadowed by the third order quadrupole term as shown in section 3 of table 1. Such a third order quadrupole term is not detected by a rotating coil meter. Table 1 also shows that an electrostatic model to calculate the field distribution works well, in this case up to $70 \%$ of maximum excitation. At full excitation there is a small difference.

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## References

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