# IMPROVING THE ENERGY RESOLUTION OF THE 30 MEV CYCLOTRON PROTON USING THE DESIGNED ACHROMATIC SYSTEM 

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#### Abstract

An achromatic system was designed to transport the proton beam emitted from the cyclotron $\mathrm{C}-30$, to the switching magnet while improving its energy resolution from $\Delta \mathrm{E}=600 \mathrm{KeV}$ to about $\Delta \mathrm{E}=30 \mathrm{KeV}$. This cyclotron is installed in NRCAM at Karadj city near Tehran. The specifications of the emitted proton beam from the cyclotron are; $\Delta \mathrm{X}=0.84 \mathrm{~cm}, \Delta \theta=6.38 \mathrm{mrad}, \Delta \mathrm{Y}=0.27 \mathrm{~cm}, \Delta \phi=6.25 \mathrm{mrad}$, with an energy spread of 600 KeV . The system designed includes a pair of quadruple doublets, bending magnets, and a water-cooled resolution slit. After calculations of the magnetic fields for each element of the system, the iteration method of the "TRANSPORT VARY CODE" was employed to optimise the beam energy resolution. The calculation shows that the specifications of the proton beam obtained at the switching magnet will be: $\Delta X=1.11 \mathrm{~cm}, \Delta \theta=15.88 \mathrm{mrad}$, $\Delta \mathrm{Y}=1.04 \mathrm{~cm}$, and $\Delta \phi=50.0 \mathrm{mrad}$, with an energy spread of 30 KeV .


## 1 Introduction

The Atomic Energy Organisation of Iran have recently purchased a cyclone-30 made by IBA in Belgium and installed it in NRCAM at Karadj city. This cyclotron made suitable for production of radio isotopes, provides proton and deuteron beams up to a maximum of 30 and 15 MeV in energy, with maximum intensities of 500 and $150 \mu \mathrm{~A}$, respectively. The energy resolution of the 30 MeV proton beam emitted from the cyclotron is about $\Delta \mathrm{E}=600 \mathrm{KeV}$ which is not suitable for the nuclear physics experiments. The major goal of this project was to design an achromatic system to transport the emitted proton beam onto the switching magnet while increasing the energy resolution to a maximum of 30 KeV . The achromatic system designed offers a 30 MeV proton beam with $\Delta \mathrm{E}=30 \mathrm{KeV}$ which opens a new era for research in the field of nuclear physics within our country for the first time.

## 2 Experimental Design

For the primary goal of improving the energy resolution, a particle spectrometer system was designed. This system contains two bending magnets with $\phi=70^{\circ}$ and $\phi=90^{\circ}$ which separates the particles according to their magnetic rigidities $^{3} \quad \chi=\chi_{0}(1+\Delta)=\mathrm{B} \rho=\mathrm{B} \rho_{0}(1+\Delta)$, where $\chi_{o}=\mathrm{P}_{\mathrm{o}} / \mathrm{Ze}=\mathrm{B} \rho_{\mathrm{o}}$ is the magnetic rigidity of the reference particle moving along the central trajectory of equal radius of curvature $\rho_{0}$, and $\Delta=\Delta \mathrm{P} / \mathrm{P}$. In the present case for the proton beam with $\mathrm{E}=30 \mathrm{MeV}, \Delta \mathrm{E}=600 \mathrm{KeV}$, and the magnetic field $B, \Delta$ stays constant and equal to 1 (percent).

The transform matrices that transform particles in the $X$ and Y-directions are obtained as;

$$
\begin{align*}
& {\left[\begin{array}{ccc}
\langle x \mid x\rangle & \langle x \mid a\rangle & \langle x \mid \Delta\rangle \\
\langle\mathrm{a} \mid \mathrm{x}\rangle & \langle\mathrm{a} \mid \mathrm{a}\rangle & \langle\mathrm{a} \mid \Delta\rangle \\
0 & 0 & 1
\end{array}\right]=} \\
& {\left[\begin{array}{ccc}
1 & 1_{2} & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
\operatorname{Cos} \phi_{0} & \rho_{0} \sin \phi_{0} & \rho_{0}\left(1-\operatorname{Cos} \phi_{0}\right) \\
-\operatorname{Sin} \phi_{0} / \rho_{0} & \operatorname{Cos} \phi_{0} & \operatorname{Sin} \phi_{0} \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
1 & 1_{1} & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right](\mathbf{1})} \\
& {\left[\begin{array}{cc}
\langle y \mid y\rangle & \langle y \mid b\rangle \\
\langle b \mid y\rangle & \langle b \mid b\rangle
\end{array}\right]=\left[\begin{array}{cc}
1 & 1_{2} \\
0 & 1
\end{array}\right]\left[\begin{array}{cc}
1 & \rho_{0} \phi_{0} \\
0 & 1
\end{array}\right]\left[\begin{array}{cc}
1 & 1 \\
0 & 1
\end{array}\right](2)} \tag{2}
\end{align*}
$$

equation 1 describes the focusing and dispersing properties of a homogeneous bending magnet in the $x$ direction, whereas equation- 2 describes the corresponding properties in the y-direction. Note that equation 2 is nothing more than the transfer matrix of a drift distance of length $1_{1}+\rho_{o} \phi_{o}+1_{2}$, the path length of the optic axis between the profile planes. The matrix element $\langle x \mid a\rangle$ must vanish which characterises an object-image relation between the profile planes ${ }^{1}$. This implies a lateral $X$ magnification $\langle x \mid x\rangle=M_{x}$ and a corresponding angular magnification $\langle a \mid a\rangle=1 / M_{x}$. From Eq. -1 the relative object and image distances for a given magnification of $\mathrm{M}_{\mathrm{x}}$ are:
$\mathrm{l}_{2} / \rho_{\mathrm{o}}=\left(\operatorname{Cos} \phi_{\mathrm{o}}-\mathrm{M}_{\mathrm{x}}\right) / \operatorname{Sin} \phi_{\mathrm{o}}$
and
$1_{1} / \rho_{o}=\left(\operatorname{Cos} \phi_{o}-M_{x}\right) / \operatorname{Sin} \phi_{o}$
With $\quad \mathrm{M}_{\mathrm{x}}=-1$,
$1_{1} / \rho_{o}=1_{2} / \rho_{o}=\operatorname{Cotan}\left(\phi_{o} / 2\right)$
A particle with magnetic rigidity $\chi=\chi_{0}(1+\Delta)$ is separated from the optic axis in the image plane of a spectrometer by a distance ${ }^{4}\langle x \mid \Delta\rangle \Delta$. For $\Delta=1$, this distance $\langle x \mid \Delta\rangle \Delta$ is the rigidity dispersion, defined by ${ }^{2}$ $D=\langle x \mid \Delta\rangle=\rho_{0}\left(1-M_{x}\right)$. In order to separate particles of different magnetic rigidities $\chi$ and $\chi_{0}$ from each other, the distance $\Delta\langle\mathrm{x} \mid \Delta\rangle$ must be larger than the width of the image , $2 x_{0}\langle x \mid x\rangle$. Thus, the minimum resolvable $\Delta$, the rigidity resolution, is defined by :
$\Delta_{\text {min }}=2 x_{0}\langle x \mid x\rangle /\langle x \mid \Delta\rangle$.
The reciprocal of the smallest resolvable $\Delta$ is the rigidity resolving power, defined by ${ }^{2}$ :
$R=-1 / \Delta_{\min }=-\langle x \mid \Delta\rangle / 2 x_{0}\langle x \mid x\rangle=\rho_{0}\left[1-\left(1 / M_{x}\right)\right] / 2 x_{0}$.
To further improve the proton beam energy resolution, inhomogeneous wedge magnets, magnets with planar inclined pole faces were used instead of homogeneous magnets. For such magnets the field $B$ is described in the form $B=k / r^{n}$, where $k$ is a constant and $n$ is the radial gradient of the field. For a point-to-point focusing the matrix element $\langle x \mid a\rangle$ must vanish which characterises an object-image relation between the profile planes. This implies a lateral $x$ magnification $\langle x \mid x\rangle=M_{x}=\operatorname{Cosk}_{x} w$, and a corresponding angular magnification $\langle a \mid a\rangle=1 / M_{x}$. The object and image distances, $L_{1}$ and $L_{2}$, for $\langle x \mid a\rangle=0$ and $\langle x \mid x\rangle=M_{x}$ are :
$\mathrm{L}_{1}=\left(\operatorname{Cosk}_{\mathrm{x}} \mathrm{w}-1 / \mathrm{M}_{\mathrm{x}}\right) / \mathrm{k}_{\mathrm{x}} \operatorname{Sin}_{\mathrm{x}} \mathrm{w}$
$\mathrm{L}_{2}=\left(\operatorname{Cosk}_{\mathrm{x}} \mathrm{w}-\mathrm{M}_{\mathrm{x}}\right) / \mathrm{k}_{\mathrm{x}} \operatorname{Sink}_{\mathrm{x}} \mathrm{w}$
where; $\mathrm{w}=\rho_{\mathrm{o}} \phi_{\mathrm{o}}, \mathrm{k}_{\mathrm{x}}{ }^{2}=(\mathrm{l}-\mathrm{n}) / \rho_{0}{ }^{2}$, and $\mathrm{k}_{\mathrm{y}}{ }^{2}=\mathrm{w} / \rho_{0}{ }^{2}$.
For a point-to-point focusing in both X and Y directions, $k_{x}{ }^{2}=k_{y}{ }^{2}$. This happens only when $n=0.5$. Here, the dispersion :
$\mathrm{D}=\langle\mathrm{x} \mid \Delta\rangle=\left(1-\mathrm{M}_{\mathrm{x}}\right) / \rho_{\mathrm{o}} \mathrm{k}_{\mathrm{x}}^{2}=2 \rho_{\mathrm{o}}\left(1-\mathrm{M}_{\mathrm{x}}\right)$
and the resolving power :
$\mathrm{R}=-\langle\mathrm{x} \mid \Delta\rangle / 2 \mathrm{x}_{\mathrm{o}}\langle\mathrm{x} \mid \mathrm{x}\rangle=\rho\left(1-1 / \mathrm{M}_{\mathrm{x}}\right) / \mathrm{x}_{\mathrm{o}}$
are both improved by a factor of 2 . For the 30 MeV proton beam we like to improve the energy resolution from $\Delta_{\mathrm{E}}=1 / 100$ to $\Delta_{\mathrm{E}}=1 / 1000$, or to obtain a momentum resolution of $\Delta=\Delta \mathrm{p} / \mathrm{p}=1 / 2000$. Due to the geometry of the room $(12 * 13 \mathrm{~m})$ and the entrance angle of the proton beam ( $20^{\circ}$ ) with respect to the side wall, two bending magnets with $\phi=70^{\circ}$ and $\phi=90^{\circ}$ were used to guide the beam onto the switching magnet. Once again to further enhance the energy resolution, a pair of quadruples were placed before and after each bending magnet for higher dispersion and better convergence, respectively. A watercooled resolution slit was placed after the second bending magnet to let the desired portion of the proton beam through.

## 3 Dispersed or Achromatic Image with Two Magnets

Two magnets, deflecting in opposite directions, are often used to produce a highly dispersed image. Yet one might desire at will to have the maximum beam intensity at the final slit without regard to homogeneity of energy. One way to accomplish this is to open the slit wide, but the resulting dispersed image might be too wide for the purpose intended. A better solution is to produce an achromatic final image. A happy solution exists when $\mathbf{n}=0.5$ for both magnets. If the object and image distances are equal, $\mathrm{M}_{\mathrm{x}}=-1$, then :
$\mathrm{L}_{1}=\mathrm{L}_{2}=\sqrt{2} \rho_{\mathrm{o}} \operatorname{Cotan}(\phi / 2 \sqrt{2})$

If the magnets are separated by $\mathrm{S}=\mathrm{L}_{1}+\mathrm{L}_{2}$, the dispersion power of each becomes $\langle x \mid \Delta\rangle=4$, and a highly dispersed and double-focused image (doubly achromatic) is produced. A pair of magnets as such will give an achromatic double-focused image, irrespective of their separation.

To switch the system from "dispersive" to the "achromatic" mode, it suffice to relocate the object from the distance $\mathrm{L}_{1}$ to the $\mathrm{L}_{\mathrm{f} 2}$ while $\langle\mathrm{x} \mid \Delta\rangle=0 \&\langle\mathrm{a} \mid \Delta\rangle=0$. Under such circumstances
$\mathrm{L}_{\mathrm{f}_{1}}=\mathrm{L}_{\mathrm{f}_{2}}=\sqrt{2} \rho_{0} \operatorname{Cotan}(\phi / 2)$.
Since it is impractical to move the whole apparatus, one can merely adjust the magnetic fields of the quadruples. For the magnet with $\phi=70^{\circ}, \mathrm{L}_{1}=\mathrm{L}_{2}=3.067 \mathrm{~m}$ and $\mathrm{L}_{\mathbf{f}_{1}}=\mathrm{L}_{\mathrm{f}_{2}}=1.207 \mathrm{~m}$. For the magnet with $\phi=90^{\circ}$,
$\mathrm{L}_{1}=\mathrm{L}_{2}=2.279 \mathrm{~m}$, and $\mathrm{L}_{\mathrm{f}_{1}}=\mathrm{L}_{\mathrm{f}_{2}}=0.7008 \mathrm{~m}$. The separation S for both cases of dispersive and achromatic becomes, $S=3.0678+2.2279=5.3469 \mathrm{~m}$. In order to operate the system in both achromatic and dispersive modes, a DF and FD double quadruples were placed before and after each bending magnets, respectively. By using the transport code progranme, the magnification of the first quadruple is calculated to be $M_{x}=\langle x \mid D\rangle=0.3$. Having $M_{x}=0.3, d / L_{1}=0.15$, and $L_{2} / L_{1}=0.5$, with $d$ being the distance which separates the D and F lenses within cach double quadruple. Choosing $\mathrm{L}_{1}=1.5 \mathrm{~m}, \mathrm{~L}_{2}=0.75$ m and $\mathrm{d}=0.225 \mathrm{~m}$ are obtained. For the point-to-point
focusing, the $f_{1}$ for the $D$ lens, and $f_{2}$ for the $F$ lens of the quadruples with $a_{1}=5.3 \mathrm{~cm}, a_{2}=12 \mathrm{~cm}$ and $\mathrm{W}_{1}=\mathrm{W}_{2}=0.212 \mathrm{~m}$, were calculated as follows ${ }^{3}$ :
$f_{1}=-L_{1} /\left[\left(L_{1}+d+L_{2}\right)\left(L_{1}+d\right) / d\left(L_{2}+d\right)\right]=-0.34 m$
$f_{2}=L_{2} /\left[\left(L_{1}+d+L_{2}\right)\left(L_{2}+d\right) / d\left(L_{1}+d\right)\right]=0.30 m$.
As shown in Fig.-1 the resolution slit was placed at $L_{2}=2.279 \mathrm{~m}$. From $\mathrm{f}_{1}=(\mathrm{l} / \mathrm{k})($ Sinlıkw), $k=3.555$, and $\mathrm{B}_{1 \mathrm{D}}=-\mathrm{kG} \chi=-5.30 \mathrm{kG}$. Likewise, $\mathrm{B}_{\mathrm{IF}}=+7.65 \mathrm{kG}$, $\mathrm{B}_{2 \mathrm{~F}}=17.1 \mathrm{kG}$, and $\mathrm{B}_{2 \mathrm{D}}=-12.0 \mathrm{kG}$ had been calculated.


Figure 1: (A)-Dispersive, (B) - Achromatic

By using the transport programme, the transform matrix for the dispersive system calculated showed that $\langle x \mid x\rangle=0.276$ and $\langle x \mid \Delta\rangle=8$. The resolution for the designed system found to be $\mathrm{R}=1722$. This can further be improved if $M_{x}<0.3$, where $M_{x}$ is the magnification of the first double quadruple as before. The schematic diagram for the dispersive and achromatic system is presented in Fig.-1.A and Fig.-1.B, respectively. To change the dispersive to the achromatic mode, $L_{2}$ should be changed from 0.75 m to
$\mathrm{L}_{2}=0.75+(3.0678-1.2079)=2.609 \mathrm{~m}$
but $L_{1}$ and d stay unchanged. With the new value for $L_{2}$, the new parameters for the first quadruple were calculated as; $f_{1}=-0.438 \mathrm{~m}, \mathrm{~B}_{1 \mathrm{D}}=-4.195 \mathrm{kG}, \mathrm{f}_{2}=0.4638 \mathrm{~m}$, and $\mathrm{B}_{1 \mathrm{~F}}=4.6319 \mathrm{kG}$. Likewise, $\mathrm{L}_{1}$ for the second quadruple should be changed from 0.75 to
$L_{1}=0.75+(2.2791-0.7008)=2.328 \mathrm{~m}$
but $L_{2}$ and $d$ stay unchanged. The new parameters for the second quadruple were calculated as; $f_{1}=0.451 \mathrm{~m}$, $B_{2 F}=-10.58 \mathrm{kG}, \mathrm{f}_{2}=-0.430 \mathrm{~m}$, and $\mathrm{B}_{2 \mathrm{D}}=-9.667 \mathrm{kG}$.

By inserting the above values to the transport matrix, the values of ${ }^{4}\langle x \mid x\rangle=0.956$ and $\langle x \mid \Delta\rangle=0.3$ were obtained. The specifications of the beam transported through the system, onto the switching magnet were; $\Delta \mathrm{X}=1 \mathrm{~cm}$, $\Delta \theta=41.2 \mathrm{mrad}, \Delta \mathrm{Y}=1 \mathrm{~cm}$, and $\Delta \phi=8.79 \mathrm{mrad}$, which served the purpose intended quite well.

## References

1. H. Wollnik, Optics of charged particles, ( Academic Press, Inc., 1987).
2. H. Wollnik, Applied charged particles optics, (Academic Press, New York, 1980).
3. John J. Livingood, The Optics of Dipole Magnets, (Academic Press, Inc New York and London, 1969).
4. K. L. Brown and F. Rothacker, Transport Appendix, Linear Accelerator Laboratory, Stanford, California.(1977).
