PHASE SPACE STRETCHING AND DISTORTION ARISING FROM THE RF DEFLECTOR IN THE TRIUMF CYCLOTRON

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This report is devoted to the phase space aspects of the beam dynamics when particles cross auxiliary element of the TRIUMF extraction system — Radio Frequency Deflector (RFD). Special attention is given to analysing the nonlinear effects that the gradient of the RFD vertical component of the electric field produces on the vertical emittance. Computer simulations have concentrated on an improved RFD design which is primarily intended to significantly reduce vertical stretching while preserving the high extraction efficiency which the original RFD device provides. A few possible options are presented.

1 INTRODUCTION

During my work term at TRIUMF (1992-1993 years) I was asked to examine some aspects of beam dynamics when particles cross the Radio Frequency Device - the first element of the proposed H^- extraction system in the TRIUMF 500 MeV cyclotron [9]. The efficiency of the H^- extraction is improved by adding the precessional component to the radius gain per turn. RFD generates a large coherent oscillations by providing a radial electric field to the circulating beam when particles crossing ν_r=3/2 resonance at 428 $MeV(R \approx 292\ in.)$[1].

The cavity operates at a frequency of 11.5 MHz) and deflects adjacent buckets of beam in opposite direction on each turn [1,2]. At higher energies, as ν_r increases, the oscillations precessing generating modulations in beam density (fig.1). When phase of precession cycle coincides with radius gain per turn due to acceleration process an increase in the distance between two subsequent turns of a factor of four is produced. The total swing of a coherent radial oscillations would reach 2.4 $in.$ Beam is extracted later at 452 MeV ($\approx$ 305 $in.$) in the third precession minimum (fig.1).

Since TRIUMF cyclotron does not accelerate separate turns (beam is homogeneous), the extracted radial emittance is not a circulating one but depends on the radius gain per turn and the energy spread. The unavoidable side-effect of making beam more diffuse radially is an increase in extracted radial emittance [3].

A perturbation of the vertical phase space is another effect which accompanying improvement in extraction efficiency [4]. Deflection of the particles in vertical direction produced by the RFD fringe fields (which are present side by side with desired radial field) in conjunction with ν_z=0.25 resonance provoke a stretching of the vertical emittance [4].

L.Root [5] and R.Laxdal[3] have analyzed phase dependence of the RFD induced stretching and have shown that due to mixing of the perturbed beam stretching will lead to growth in vertical emittance of the extracted beam. Mismatched ellipses precess in vertical phase space. "If we start with a single phase ellipse, then the orientation of the stretched ellipses with different initial RF phases at extraction radius will depends upon the number of turns required for the ions to travel between ν_z=0.25 resonance point where the mismatch is generated and extraction point and upon the value of the ν_z"[5]. Single phase ellipses having different orientation and shape will be superimposed at extraction point. Total effective area in the vertical phase space filling in by the particles with different RF phases will be larger than initial one. Growth in vertical emittance as well as increase in radial beam size has been registered in experiments with TRIUMF beam.

2 NONLINEAR DISTORTION OF THE VERTICAL PHASE SPACE WHEN PARTICLES CROSSING ν_z=0.25 RESONANCE

2.1 Computer Code GOBLIN

All computer simulations have been done by using computer code GOBLIN which trucks each particle in a cyclotron by numerical integration of the Hamilton canonical equations of motion [6]. Program includes the extension of the equation of motion to second order in vertical direction. Effect of the RFD was simulated by analytical formula [7]. The RFD was assumed to have an uniform electric field azimuthally (since it is 0.5 m long) and no azimuthal gradient. A "snap-shop" of radial component of electric field for RFD with peak azimuthally integrated strength 110 $V/mm{\cdot}m$ is shown in fig.2a. Vertical component of electric field associated with gradient of the radial field is also presented. Contours of RFD electrodes are shown in fig.2b.

2.2 Vertical Phase Ellipses

Particles have been starting at energy 409 MeV which is beyond of extension of RFD electric field. Beam was assumed to be matched vertically to the cyclotron acceptance and centered in the radial direction. A set of

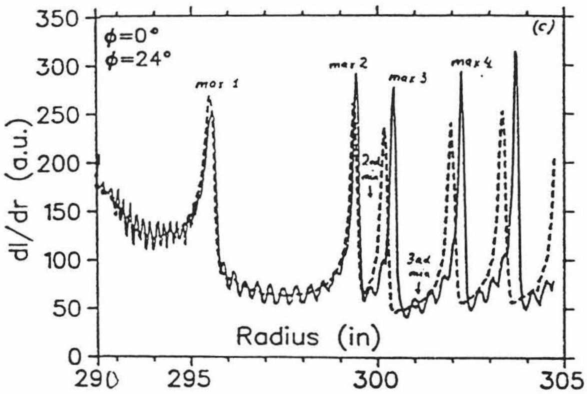

Figure 1: Radial scans using a differential probe showing the beam density pattern produced by the RFD for an optimized phase band and for a phase band off-set by 24°. Phase dependent amplitude growth at the ν_r=3/2 account for the difference.

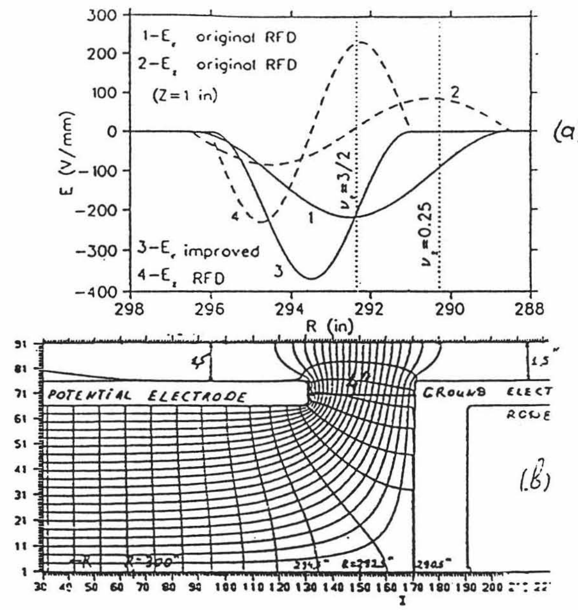

Figure 2: (a) A "snap-shot" of radial (E_r) and vertical (E_z) components of electric field for original RFD device (peak azimuthally integrated strength 110 $V{\cdot}m$) and proposed new RFD design with strength - 185 $V/mm{\cdot}m$. (b)Contours of lower part of ground and potential electrodes for original RFD device (radial gap 4 in, vertical gap 3 in.)Center of RFD (R = 292.5 in) coincides with ν_r = 3/2 resonance center line.

32 particles on the boundary of matched vertical ellipse were tracked through the acceleration process for RFD *OFF* case and RFD *ON* case. When RFD is turned *OFF* the static vertical phase ellipses are not distorted at least within dee aperture

For RFD *ON* case stretching is clearly observed for beam energy 420 MeV. Moreover, **NONLINEAR** effect — **DISTORTION** of vertical phase space for RFD strength 110 $V/mm{\cdot}m$ was apparent when circulating vertical emittance has exceed value $\approx 3\pi$ $mm{\cdot}mrad$ (fig.4a). Developing of stretching and following distortion of originally matched ellipses are shown in fig.3b, where *CENTER* of each vertical ellipse corresponds to the *RADIUS* of given turn.

In order to exclude from consideration coupling resonances $\nu_r + 2 \cdot \nu_z = 2$ (which is crossed by beam at 417 MeV) and $\nu_r - 2 \cdot \nu_z = 1$ (422 MeV) and taking into account that the swing of coherent radial oscillations produced by RFD is about 2 in the run has been computed for artificially *ZEROED* radial component of RFD field but with vertical component *ON*. Radial coherent oscillations disappeared but stretching of vertical phase space still present. So distortion of vertical phase space caused by RFD is NOT a coupling effect but just a result of beam crossing of an analog of parametric resonance ν_z = 0.5. Resonance ν_r = 3/2 here is analog of integer resonance ν_r = 1. Indeed coherent radial oscillations are excited by analog of the first harmonic of magnetic field and vertical stretching is excited by analog of gradient of first harmonic of magnetic field.

Static runs (dee voltage V_{dee} = 0) at energy close to ν_z = 0.25 resonance have been completed in order to investigate nature of the effect. As a perturbation force is doubled the **TWISTING** of vertical phase space is apparent (fig.4b).

Comparison of the ellipse shapes for different values of initial vertical emittance and RFD strengths allows one come into conclusion:

1)for original RFD device and electric field strength 110 $V/mm{\cdot}m$ (which is desirable to achieve 90% H^- extraction efficiency) **distortion** of vertical space **noticeable** for beam emittance **greater** than 3π $mm{\cdot}mrad$ (likely emittance in the TRIUMF cyclotron is more close to value of 1π $mm{\cdot}mrad$ for selected tune);

2)increasing in RFD **strength** to improve extraction efficiency might lead to **distortion** of beam for selected tune and particular doubling of the RFD strength will cause a visible distortion of beam with $\approx 1\pi$ $mm{\cdot}mrad$ emittance beam (fig.4b).

To estimate the stop-band of resonance the vertical static phase space has been explored at different energies around resonance value. Phase space was uniformly filled in by grid of the representation points each of them belongs to particle with some initial condition (fig.5). At energy 409 MeV orbits are closed and oscillations are stable. The region of stability of betatron oscillations is stretched while energy is more close to resonance value and at E = 418.2 MeV is shrinked to zero. Two stable fixed points region appears at E = 418.2 MeV (we consider each *ODD* turn or each *EVEN* turn). Equilibrium orbit is unstable fixed point now. At energy 420.2 MeV the stable region around equilibrium orbit is restored.

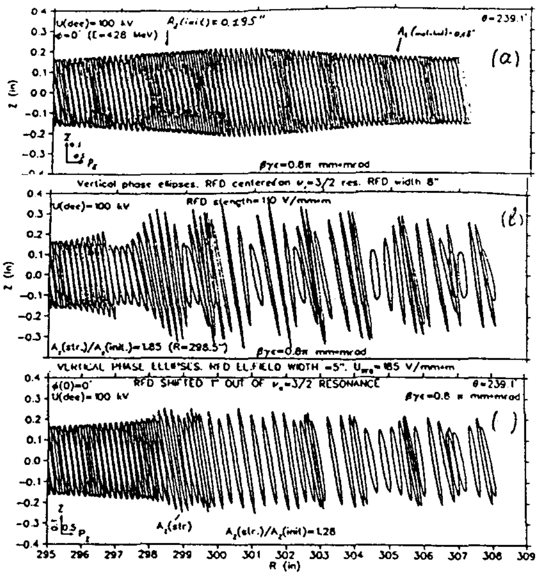

Figure 3: Vertical phase ellipses. Center of each ellipse corresponds to the radius of given turn: (a) RFD is OFF; (b) Original RFD design. Ratio of stretched vertical amplitude to initial one is $A_z(str)/A_z(init) = 1.85$; (c) Improved RFD design. $A_z(str)/A_z(init) = 1.28$. Ratio of stretching is reduced on 50%.

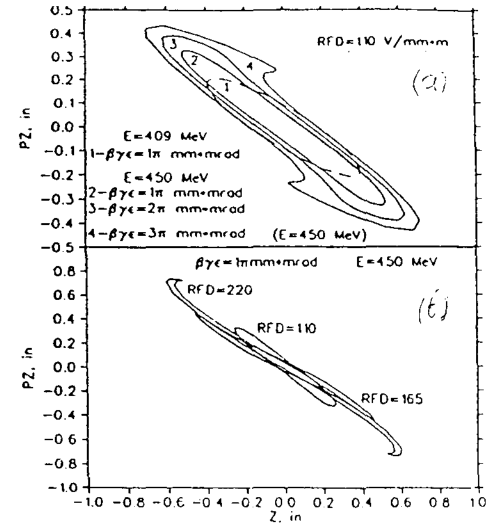

Figure 4: Comparison of vertical ellipse shapes: (a)for different values of initial emittance; (b) for different RFD strength.

STOP-BAND of $\nu_z = 0.25$ resonance for present RFD design is about 2 MeV or 0.5 in in radius. For energy gain per turn 360 $keV/turn$ and radius gain per turn 0.06 $inch/turn$ particles will spend about 8 turns in the resonance region. For such a narrow stop-band adiabatic growth of the amplitude of the vertical betatron oscillations should be just as 20% of initial amplitude while from GOBLIN calculations and from experimental results followed that the maximum value of stretching is about of 80÷100%. The reason might be that most of the stretching due to lack of adiabaticity take place inside of band width

$$\delta\nu_z = \sqrt{\partial\nu_z/\partial N} = 0.04$$

which is four times larger than static stopband of resonance [8]

$$\delta\nu_z = N \cdot (\partial\nu_z/\partial N) = 0.01 \ .$$

3 REDUCING PHASE-DEPENDENT EMITTANCE GROWTH WITH LOCAL FLATTOPPING

3.1 Vertical Emittance

GOBLIN simulations were used to explore phase dependence of emittance growth. Particles were tracked through the RFD for various initial phases of main RF.

Center of each vertical ellipse in fig.6 corresponds to radial coordinate of the beam centroid at given turn. Ellipses stretched when particles crossing resonance and rotate in phase space since ν_z is not fixed on 0.25. In most cases the extraction will take place in a few turns and extracted vertical emittance for narrow phase band will be larger than circulating one because the stretched ellipses from few turns with different orientation will be overlapped on deflector entrance.

Further increase in the emittance will be observed for particles with different RF phases because at any one radial position vertical ellipses for these particles will have different orientations since particles are being accelerated from resonance region to extraction radius for the different number of turns (fig.6a).

Vertical phase ellipses for different initial RF phases superimposed on each other are shown in fig.7. For a wide phase band ellipses with various orientation will be superimposed in the deflector and the extracted vertical emittance should exceed circulating one in about 3 times (fig.7b).

Phase dependent emittance growth can be reduced by employing the 4^{th} harmonic auxiliary accelerating cavity (AAC), which was primary installed in the TRIUMF cyclotron for another purposes, to flattop the local energy gain per turn. In our simulations small fourth harmonic is added in opposition to the main accelerating field. The stability of 4^{th} maximum radial position for a finite 40^o phase band was chosen as a criterion for flattopping. Simulations have shown that a AAC voltage of 20 kV is sufficient for this purposes [2]. Orientation of whole ensemble for **FLATTOP ON** case is the same over RF phase range from -20^o to $+20^o$ (fig.6b) . Area in the vertical phase space occupied by 40^o phase band beam is 50% less than for case Flattop *OFF* but still exceed circulating in a 2 times.

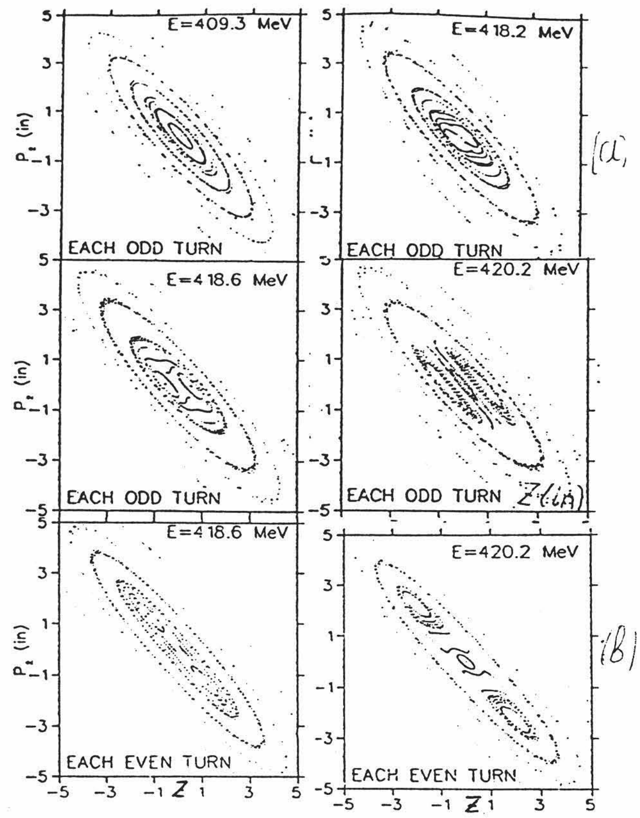

Figure 5: Vertical Phase Space diagrams. Original RFD device is ON: (a)- each odd turn, (b) - each even turn.

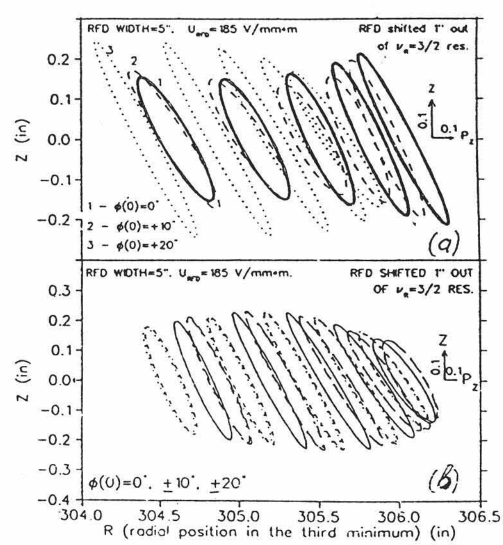

Figure 6: Vertical ellipses for turns in the extraction region (third precession minimum). $\delta\phi = 40^o$. The phase dependence in the orientation of the stretched ellipses evident in (a) is absent in (b) by the addition of a flattopping voltage.(Improved RFD design)

3.2 *Radial Emittance*

Same method was applied to find acceptance in radial phase space. Beam arriving to RFD was assumed to be continuous and matched to cyclotron acceptance. A set of 12 particles on the boundary of a matched radial phase ellipse were tracked using GOBLIN through the RFD for various initial RF phases.

After certain number of turns particles have being spread out in energy and time due to phase-dependent coupling between longitudinal and transverse motion. To reconstruct parameters of particles to one energy while from GOBLIN output one could find parameters of particles at given turn the interpolation procedure have been applied. Without this procedure area occupied by beam in radial phase space does not conserved.

Radial ellipses are stretched when crossing radial parametric resonance $\nu_r = 3/2$, became mismatched and rotate in radial phase space as ν_r increases with energy. Presence of RFD doesn't affect stretching process.

Radial phase space ellipses in extraction region for two different initial RF phases, 0^o and $+20^o$ are presented in fig.8. Part of the previous precession cycle is also shown, as is an extraction septum. The hatched region represent the area in radial phase space occupied by the extracted beam of $\Delta\phi = 40^o$ phase band. Since beam is homogeneous and extraction take place over more than one turn the radial extracted emittance over narrow phase band is determined by full radius gain per turn and will be few times larger than circulating one. Some phases will be extracted in one turn but most will not be extracted cleanly.

The $+20^o$ and particles follows a slightly different precession path in phase space than the 0^o particles.. This will lead to further increases in the extracted radial emittance (fig.8a).

When proper flattop voltage is added to the main accelerating field the ellipses with different initial phases will be followed more close the same precession trajectory and hence occupy a smaller phase space area once extracted (fig.8b).

From Monte-Carlo simulations presented in [2] follows that for an RFD voltage of 110 $V/mm{\cdot}m$, a circulating radial emittance of 1π $mm{\cdot}mrad$ and a phase band of 40^o, the extracted emittance is 4π $mm{\cdot}mrad$ while addition of flattopping voltage will **REDUCE** extracted emittance to the value of 3π $mm{\cdot}mrad$.

4 CHANGING IN RFD IN ORDER TO REDUCE VERTICAL STRETCHING

Some additional computer simulations have been done in order to diminish vertical stretching arising from the RFD while preserving extraction efficiency.

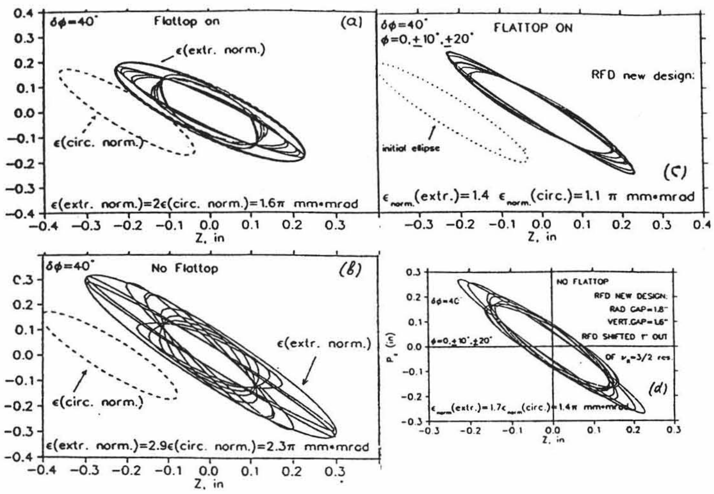

Figure 7: Extracted vertical emittance as a superposition of vertical phase ellipses for different initial RF phases ($\Delta\phi = 40^o$). Original RFD design: (a)FLATTOP ON; (b)FLATTOP OFF. Improved RFD design: (c)FLATTOP ON; (d)FLATTOP OFF

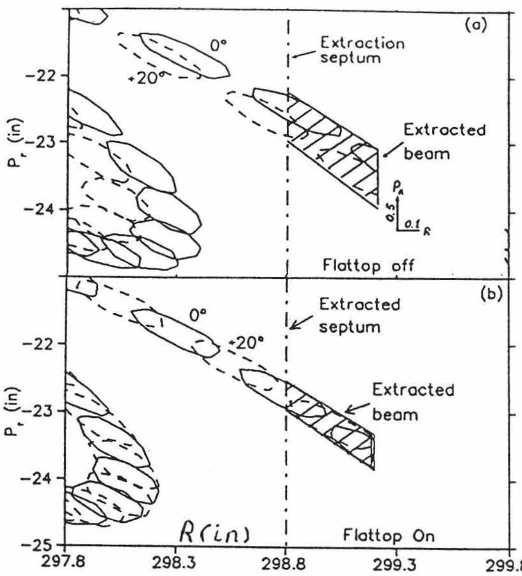

Figure 8: Position in radial phase space of a matched beam ellipse undergoing precession extraction for two different starting phases, 0^o and $+20^o$. (a) NO FLATTOPP. In (b) a 4^th harmonic FLATTOPPING voltage has been added to the accelerating field reducing the variation in precession between different RF phases. The hatched region represent the area in radial phase space occupied by the extracted, 40^o wide, beam.

Two resonances $\nu_z = 0.25$ and $\nu_r = 1.5$ are separated by 2 *in* radially (fig.2a). Maximum of the vertical electric field gradient coincides in present RFD design with center line of vertical resonance. It was proposed to minimize RFD radial gap and to displace device in order to reduce as much as possible vertical field at the radius of vertical resonance. RFD strength was varied for each new setting until swing of precession loop was adjusted to the original design in order to avoid decreasing in the radial oscillation amplitude.

Vertical Phase ellipses have been tracked for different RFD configurations and positions. For some inappropriate RFD settings and strength the typical result was distortion or even "blow up" of vertical phase space The radial range of RFD repositioning is restricted by ± 2 *in*.

Few variants with asymmetric electrodes have been calculated. It was found no noticeable improvement for asymmetric RFD electrodes in comparison with symmetric design.

Minimum of vertical stretching — 50% less in amplitude from the original RFD design — occurs when device is shifted radially 1 *in* out of ν_r=3/2 resonance and consists of two pairs of electrodes separated by 1.8 *in* radial gap and symmetric around median plane (vertical gap — 1.6 *in*). Desired RFD strength is 185 $V/mm \cdot m$ which is acceptable from the technical point of view.

For improved RFD design the extracted vertical emittance enclosed single phase ellipses with different orientation and shape will be still in 1.7 times *larger* than circulating one (fig.7d).

Proper flattopping voltage should further reduce phase dependent emittance growth and extracted vertical emittance for new RFD design should be just in ≈ 1.4 times exceed circulating emittance (fig.7c).

5 ACKNOWLEDGMENTS

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