EXPERIMENTAL DETERMINATION OF THE EXTERNAL BEAM MATRIX ON 1 GEV SYNCHROCYCLOTRON.

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The critical question for beam optics calculations is a correct determination of the input beam parameters. Presented report is devoted to description the procedure of the experimental determination of the extracted beam parameters on I GeV synchrocyclotron. The beam ellipse parameters which are elements of σ-matrix can be determined by measuring the profiles for three profilometer position, but a procedure is very sensitive to the accuracy in determining the beam profile, especially for the comparatively low energy proton beams with a large emittans. The algorithm can be improved by measuring the profiles for more than three positions and applying a least-square fit (LSM). However, LSM-procedure under limited number of measurement points does not always eliminate of physically unrealizable results. The best results on determination σ-matrix were obtained at measurement of beam profiles in a drift space after lens doublet in two various operation modes. In each of operation modes in one plane the envelope had a waist. The series of measurements at different operation modes were processed together by using of least-square fit taking into account a conservation of a beam emittans as additional constraint.

1. Introduction.

The critical question for the beam optics calculations is a correct determination of the input beam parameters. Presented report is devoted to description the procedure of the experimental determination of the extracted beam parameters on 1 GeV synchrocyclotron.

According TRANSPORT [1] formalism the phase space occupied by the beam in characterized by two dimensional symmetrical sigma beam matrix $\boldsymbol{\sigma}$. The diagonal elements of matrix give the coordinate and angle envelopes $x_{max}=\sqrt{\sigma_{11}}$ and $\theta_{max}=\sqrt{\sigma_{22}}$ and matrix determinant is equal to beam emittance $\varepsilon^2=\det\boldsymbol{\sigma}$. The beam matrix is transformed along a beam line according the low

$$\boldsymbol{\sigma}(s)=\mathbf{R}(s)\,\boldsymbol{\sigma}_0(s)\,\mathbf{R}^T(s), \qquad (1)$$

where $\boldsymbol{\sigma}_0$ -is initial matrix, $\mathbf{R}(s)$-is optical matrix of transition. From (1) we receive expression

$$\sigma_{11}(s)=\sigma^0_{11}(s)R^2_{11}(s)+2\sigma^0_{12}(s)R_{11}(s)R_{12}(s)+\sigma^0_{22}R^2_{12}(s), \qquad (2)$$

which in a drift space is a parabola equation (since $R_{11}(s)=1$, $R_{12}(s)=s$).

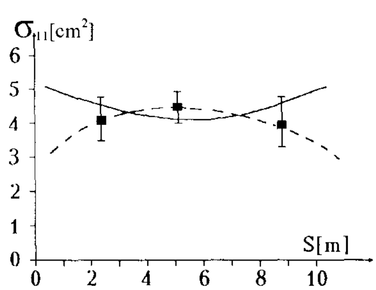

Fig.1 Beam envelope in drift space.

2. The method of measurements.

The beam profile was measured by using the profilometers, developed on PNPI synchrocyclotron [2]. The profilometer allows the distribution functions $f_1(x)$ and $f_2(z)$ of the beam density across two transversal coordinates x or z to be measured. In this experiment we were using two first moments of distribution function: the average coordinate or "centre of gravity" of the beam

$$\bar{x}=\int x\ f(x)dx$$

and mean-square deviation or mean-square beam size

$$<(x-\bar{x})^2>=\int(x-\bar{x})^2\ f(x)dx\,.$$

The beam ellipse parameters which are elements of matrix can be determined by measuring profiles for three profilometer positions. The most accurate measurements can be done in drift space as far as the transfer matrix can be determined accurately. The procedure is very sensitive to the accuracy in determining the beam profile. This situation is illustrated in fig. 1.

When a beam envelope, shows up as a parabola with small curvature, small errors in profile can result in physically incorrect result. Parabola, passed through 3 measurement points, has curvature of other sign, that means, that $\sigma_{22}<0$ and $\det\boldsymbol{\sigma}=\varepsilon^2<0$. The algorithm can be improved by measuring profiles for more than three positions and applying a least-square fit (LSM) with minimizing functional

$$F=\sum_i\ [<(x-\bar{x})^2>_i-\sigma^0_{11}R^2_{11}(i)-2\sigma^0_{12}R_{11}(i)R_{12}(i)-\sigma^0_{22}R^2_{12}(i)]^2\,. \qquad (3)$$

However LSM-procedure under limited number of measurements points (≤10) does not always eliminate of physically unrealizable results. The attempts to modify functional by introduction of penalty functions, which "drive" parameters into the physically allowable region, were made. The penalty functions used here are as follows:

$$f_1 = D\cdot\exp(-\sigma^0_{22}/a) \text{ and}$$
$$f_2 = D\cdot\exp(-\det\sigma/\varepsilon^2_0), \qquad (4)$$

where a and ε_0 -are characteristic areas of changes σ_{22} and ε, D-is a parameter of the penalty. Functionals $\Phi_1 = F + f_1$ and $\Phi_2 = F + f_2$ were minimized, where F is defined by the formula (3). Fast rise of the exponent under $\det\sigma < 0$ and $\sigma_{22} < 0$ forces process of minimization to be shifted in the direction of physically allowable parameters. Under $\det\sigma > 0$ and $\sigma_{22} > 0$ the penalty function is being quickly tends to a zero and main role plays functional F. The matrix elements received as a result of this procedure have been in the physically allowable region, but beam envelopes, calculated with their use, do not sufficiently accurate describe experimental data.

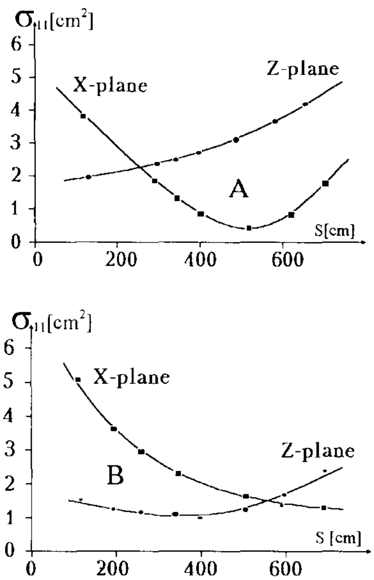

Fig.2 Beam envelope under two modes of the lense operation.

It is preferable to do profile measurements nearly of the waist position for both planes. But on the practice for large emittance beam it is very often impossible to produce the waists for the both planes simultaneously. The best results on determination σ-matrix were obtained at measurement of beam profiles in a drift space after doublet of lenses in two various modes of operation. In each of operation modes the envelope had a waist in one plane. The series of measurements at different modes operation were processed together taking into account a conservation of a beam emittans. The mathematical formulation of this problem is coming to functional minimization

$$F = \sum_i [\langle (x-\bar{x})^2\rangle_i - \sigma^A_{11} - 2\sigma^A_{12} s_i - \sigma^A_{22} s_i^2]^2 + \sum_j [\langle (x-\bar{x})^2\rangle_j - \sigma^B_{11} - 2\sigma^B_{12} s_j - \sigma^B_{22} s_j^2]^2 +$$
$$2\lambda\{[\sigma^A_{11}\sigma^A_{22} - (\sigma^A_{12})^2] - [\sigma^B_{11}\sigma^B_{22} - (\sigma^B_{12})^2]\} \qquad (5)$$

Indexes A and B concern to different mode operations of lenses, λ-is a Lagrange multiplier, enabling to take into account a condition

$$\varepsilon^A = \varepsilon^B \qquad (6)$$

Minimizing (5), we receive two systems of linear equations

$$C^K_1 = \sigma^K_{11} A^K_1 + 2\sigma^K_{12} A^K_2 + \sigma^K_{22}(A^K_3 - \lambda)$$
$$C^K_2 = \sigma^K_{11} A^K_2 + 2\sigma^K_{12}(A^K_3 + \lambda) + \sigma^K_{22} A^K_4$$
$$C^K_3 = \sigma^K_{11}(A^K_3 - \lambda) + 2\sigma^K_{12} A^K_4 + \sigma^K_{22} A^K_5 \qquad (7)$$

Here K-is a index of a lenses mode operation; the factors C and A are equal:

$$C_1 = \sum_i \langle (x-\bar{x})^2\rangle_i,$$
$$C_2 = \sum_i \langle (x-\bar{x})^2\rangle_i s_i,$$
$$C_3 = \sum_i \langle (x-\bar{x})^2\rangle_i s_i^2,$$
$$A_1 = \sum_i i = N, \qquad A_2 = \sum_i s_i,$$
$$A_3 = \sum_i s_i^2, \qquad A_4 = \sum_i s_i^3,$$
$$A_5 = \sum_i s_i^4, \qquad (8)$$

N-is a number of measurement points.

System (7) is a system of 6 linear equations for 7 variables: six σ^K_{ij} and λ. The idea of the solving (7) is basically the following. Assuming constant λ have same numerical value from 0 up to $5\cdot10^{-3}$ we can decide (7) and find σ^K_{ij} and $\varepsilon^2 = \det\sigma$. From this set of decisions we can select those σ^K_{ij} for which the conditions (5) and (6) are satisfacted.

The beam envelopes at two modes of the lenses operation are shown in fig. 2.

In fig. 3 the calculated emittanses $\varepsilon_X(\lambda)$ and $\varepsilon_Z(\lambda)$ are submitted as a function of the parameter λ. The point of crossing gives those meaning λ for plane x and z, which meet requirement (6).

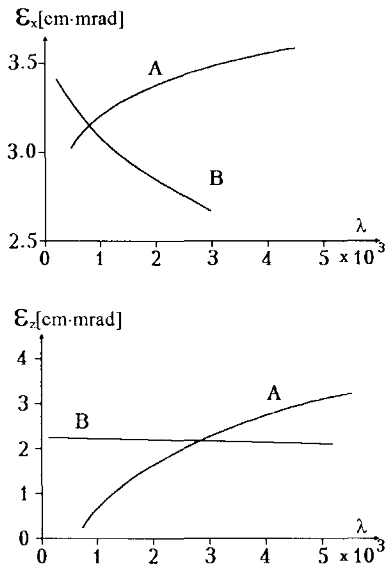

Fig.3 Dependence ε_X and ε_Z on λ

3. Results.

Two sets of matrix elements $\sigma_{ij}^{A,B}$, which answer condition (6) and correspond to A or B mode operation are received for each plane. The natural cross-checking method is the comparison the matrixes for modes A and B after its transition on the doublet input as far as on the doublet input the beam was the same for A and B operation modes. Results of the transformation according formula (1) can be presented as follows:

$$\sigma_x^A = \begin{bmatrix} 2.84 & 3.69\text{x}10^{-3} \\ 3.69\text{x}10^{-3} & 8.23\text{x}10^{-3} \end{bmatrix}$$

$$\sigma_x^B = \begin{bmatrix} 3.0 & 3.77\text{x}10^{-3} \\ 3.77\text{x}10^{-3} & 8.0\text{x}10^{-6} \end{bmatrix}$$

$$\sigma_z^A = \begin{bmatrix} 4.503 & -1.61\text{x}10^{-4} \\ -1.61\text{x}10^{-4} & 1.18\text{x}10^{-6} \end{bmatrix}$$

$$\sigma_z^B = \begin{bmatrix} 4.306 & 3.85\text{x}10^{-4} \\ 3.85\text{x}10^{-4} & 1.26\text{x}10^{-6} \end{bmatrix}$$

The emittance in X-plane is equal ε_x =3.13 cm·mrad and ε_z =2.3 cm·mrad for Z-plane respectively. Distinction in values of matrix elements is located within the limits of 5 %. The exception is made correlation element σ_{12}^{z}.

Having taken the σ-matrixes as an initial data for the code TRANSPORT /1/, we have been calculated beam transport line, for which we had the most complete information (a beam sizes and a beam envelopes at various modes operation of magnetic elements).

A qualitative agreement between calculation and experimental data were obtained.

The correctly determined σ-matrix of a beam provides a way of optimization existing and design new beam lines at the PNPI synchrocyclotron.

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References.

1. Karl L. Brown. SLAC-75, July 1967.
2. N.K. Abrossimov et al., Preprint LNPI 1487, L. 1989, 12 p.