BEAM DEPOLARIZATION IN THE NAC BEAM LINES AND CYCLOTRONS

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Since the completion of the second injector cyclotron in 1994, polarized proton beams have regularly been accelerated at NAC. Initially the polarization was erratic and generally low. We made a detailed theoretical and experimental study of causes of the depolarization in the cyclotrons and beam lines (consisting of solenoid lenses, dipoles and quadrupoles). We found that in the beam lines, the only significant depolarization which can occur is in the solenoids of the injection beam line to the second injector cyclotron. By proper adjustment of the solenoid lenses, the polarization could be improved significantly to values consistently in the 70% to 80% range. We present the results of measurements and calculations obtained with a computer program developed for this purpose. We also discuss a numerical study of a proton depolarizing resonance caused by first-harmonic field errors in the SSC.

1 Introduction

The first polarized beams were produced at NAC in August 1994. Improvements to the source and beam lines have been discussed elsewhere [1]. Until now, only vertical polarization has been used in the external beam lines and cyclotrons. The intensity available for injection into the separated-sector cyclotron SSC is between 700 nA and 1000 nA.

Initially the polarization was not reproducible and was generally low. This prompted us to make a detailed experimental and theoretical study of the possible causes of depolarization in the cyclotrons and beam lines (consisting of solenoid lenses, dipoles and quadrupoles).

The first part of this paper discusses the theoretical study of beam line depolarization and describes the program we have developed to study polarization properties of protons in beam lines. The second part discusses the result of calculations, made with the cyclotron orbit code COC [2], of the acceleration of polarized beams in the cyclotrons at NAC.

2 The Polarized Beam Injection Line

Beams are injected into the second injector cyclotron SPC2 through an axial injection system from a basement area, using either the ECR ion source or the polarized ion source. A schematic view of the polarized beam injection line is given in fig. 1.

The horizontal section of the injection line consists of the source P and solenoids L1 to L6. A Wien filter WF is situated between L3 and L4. Two 45° magnets bend the beam upwards with solenoid L7 placed between them. The vertical section consists of solenoid L8 followed by two quadrupole triplets and two small solenoids (in the yoke of SPC2).

The magnetic fields of L1 to L3 are symmetric about the vertical cross-section plane through their centres, the fields of L4 to L8 are antisymmetric (see fig. 2).

3 Equations of Motion

Polarized beams in beam lines and accelerators can be studied largely on the basis of classical equations of motion [3]. The spin motion $\mathbf{S}$ in an electromagnetic field is governed by the equation

$$\frac{d\mathbf{S}}{dt} = \mathbf{S} \times \boldsymbol{\Omega}_S \tag{1}$$

with the axial precession vector $\boldsymbol{\Omega}_S$ given by

$$\boldsymbol{\Omega}_S = \frac{q}{m_0\gamma}\left[(1+\gamma a)\mathbf{B} - (\gamma - 1)\, a\frac{\mathbf{p}(\mathbf{p}\cdot\mathbf{B})}{p^2}\right] \tag{2}$$

for a given magnetic field $\mathbf{B}$. The particle has momentum $\mathbf{p}$, charge q and rest mass m_0 ($\gamma = m/m_0$). The constant a is the gyromagnetic anomally (1.7928 for protons). Equation (2) ignores a term in the electric field which is significant only at very high velocity.

The particle motion is given by the usual expression for the Lorenz force

$$\frac{d\mathbf{p}}{dt} = \mathbf{p} \times \boldsymbol{\Omega}_p\,, \qquad \boldsymbol{\Omega}_p = \frac{q}{m_0\gamma}\mathbf{B} \tag{3}$$

These equations completely describe the motion and spin of a polarized particle in a magnetic field — the spin and momentum both precess about the magnetic field, but at different rates.

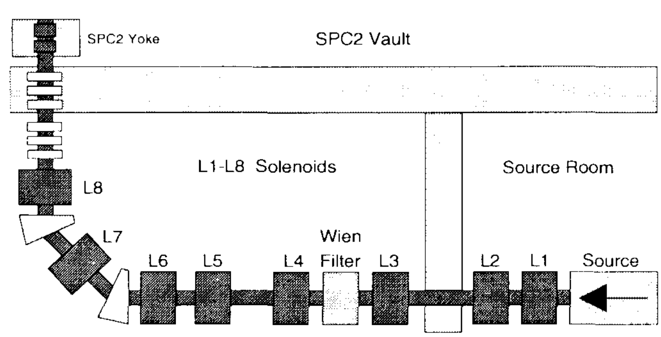

Figure 1: Schematic layout of the polarized beam injection line, showing the source and main beam line elements.

4 Polarization in Beam Lines

Protons from the source are horizontally polarized. The average spin precession in the two 45° bending magnets is 251.4°. The Wien filter is used to produce an additonal 18.6° to inject protons into SPC2 with average polarization vertically aligned (total average precession 270°).

The injection line has been designed [4] with a transport program to optimize beam properties. Unfortunately, information about spread in polarization was not available at that stage.

In a beam with very small emittance all particles experience approximately the same field components everywhere, but with increase in emittance different parts of the beam experience different fields, and precess at different rates and in different directions, with a resultant spread in polarization.

Since it is not possible to limit the emittance to very low values, and since our beam line has already been built up, the alternative is to achieve cancellation of, and/or minimize, the spread in precession through the tuning of beam line elements.

In order to study these effects, and to provide an accurate description of both beam and polarization transport in beam lines, we developed a computer program.

4.1 Beam Line Computer Program

To describe cancellation effects, which occur primarily at edge fields (at the entrance and exit of beam line elements), the program had to perform accurate numerical integration of the differential equations (1)–(3) in the magnetic fields of the beam line elements. The routines required to perform the integrations have been adapted from our orbit code [2].

The program, written in C^{++}, runs on the OS/2 operating system as a multi-threaded program with a graphical user interface. The user can edit and compose any beam line from its elements by specifying their types, positions, parameters and fields. The beam emittance, energy, polarization and other parameters are specified. Different data sets can be saved, to be recalled and modified, or a calculation can be launched. The results of any calculation are viewable as screen graphs. Coordinate values of polarization and beam envelopes can be read from the screen, and the graphs can be plotted directly from the program.

The coordinate system used by the program moves with the central particle, with its z-axis always in the direction of motion of the central particle (centre of beam line). Displacements of non-central particles are measured with respect to the central particle in two directions x and y perpendicular to the z-axis, as well as a longitudinal displacement along the z-axis. A beam of particles is simulated by integrating a central particle, plus two sets of 8 particles, distributed on horizontal and vertical emittance ellipses, respectively.

Solenoid fields are available as $B_z(r = 0, z)$ values measured along the axis of the solenoid. Examples of two fields are given in fig. 2. Off-axis values are obtained from the rotational symmetry by making a series expansion, which satisfies Laplace's equation, about $r = 0$

$$\begin{aligned} B_z(r,z) &= B_z(0,z) - \frac{r^2}{4}\frac{\partial^2 B_z(0,z)}{\partial^2 z^2} + \cdots \\ B_r(r,z) &= -\frac{r}{2}\frac{\partial B_z(0,z)}{\partial z} + \cdots \end{aligned} \tag{4}$$

The Wien filter field is assumed to be homogeneous, with its field distribution given by a parameterized form factor $f(z)$, normalized to a maximum value of 1. For a required spin precession angle θ_S, the fields are

$$\begin{aligned} B &= \frac{p\theta_S}{q(1+\gamma a)\int f(z)dz} \\ E &= \frac{p}{m_0\gamma} B \end{aligned} \tag{5}$$

Field profiles for quadrupoles and bending magnets are flat inside the elements. The edge profiles are parameterized with suitable functions. The bending magnets at NAC have no field index. The dipole main field is selected to be in y-direction. The expansion about the median plane field B_{y0} is

$$\begin{aligned} B_x &= 0 \\ B_y &= B_{y0} - \frac{1}{2}\frac{\partial^2 B_{y0}}{\partial^2 z^2} y^2 + \cdots \\ B_z &= \frac{\partial B_{y0}}{\partial z} y + \cdots \end{aligned} \tag{6}$$

For quadrupoles the four-fold symmetry leads to the well-known expansion

$$\begin{aligned} B_r &= -B_1\frac{r}{R} sin(2\theta) + \cdots \\ B_\theta &= -B_1\frac{r}{R} cos(2\theta) + \cdots \end{aligned} \tag{7}$$

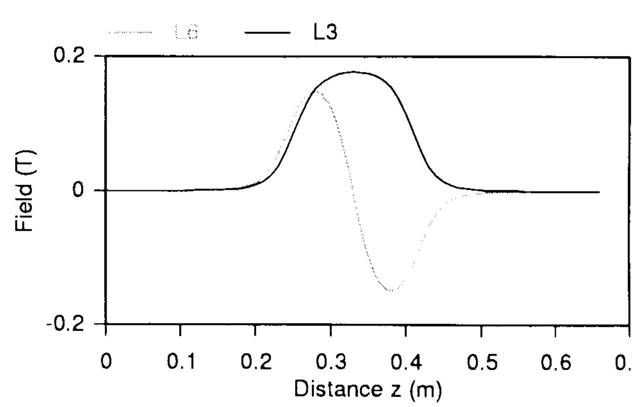

Figure 2: Fields measured along the axis, for two types of solenoids.

4.2 Calculations for Beam Lines

Calculations were performed for all the polarized proton beam lines at NAC. A comparison with our beam transport code shows that the program predicts exactly the same beam transport properties.

Calculations showed that depolarization is negligible in all beam lines, consisting of dipoles and quadrupoles only, after SPC2. However, in the injection beam line, considerable depolarization may occur as a result of improper settings of the solenoid fields. Consequently, we made a thorough study of this effect in order to determine suitable settings which still satisfy the beam transport requirements. In particular, we tried to identify the solenoids which are most sensitive to field changes that affect the polarization adversely. Starting with beam transport design values, the fields were set by trial and error and do not represent optimization, which would have been difficult and time consuming to implement.

As was to be expected, the results showed that the beam emittance is extremely important. Furthermore, for a given initial emittance, care should be taken that the beam dimensions stay as small as possible, particularly where it passes through solenoids. This means that pairs or sets of solenoids are used in the same way as quadrupole pairs or triplets. In our beam line good results are obtained by keeping equal the field values of both members of the respective pairs L2 and L3, L4 and L5, L6 and L8, and by varying the field settings to obtain the symmetrical beam envelope shown in the lower part of fig. 3. The field of solenoid L7 (between the 2 dipoles), is then adjusted to minimize the spread in polarization at the exit of L8.

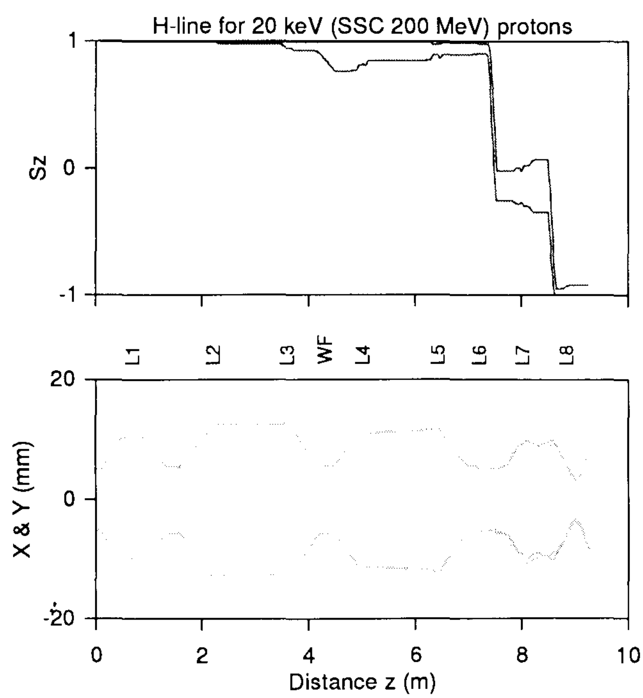

Figure 3: Calculation for polarized beam in the injection line. The beam emittance is 100 mm.mrad and the initial polarization is in the direction of the beam line. The upper plot shows envelopes of minimum and maximum spin projection along direction of travel, the difference between the two lines is the spread in polarization. The lower plot shows horizontal and vertical beam envelopes.

The result of a calculation is shown in fig. 3. In the upper plot the two lines give the minimum and maximum polarization of the simulated beam. Due to the emittance of the beam, there is a spread in polarization. This spread can be minimized at the exit of L8, but for beams with emittance 100 mm.mrad, the polarization can never be made better than about 95%. The calculations also show that the entrance and exit fields of solenoids do not always provide cancellation, which is contrary to what we had previously assumed.

4.3 Measurement of Beam Line Polarization

We have made measurements with a polarimeter in the beam line between SPC2 and SSC, to compare predictions of the program with the actual behaviour of polarization in the beam. We discuss here only some of the typical measurements which were made.

The field setting of solenoid L7 is not critical — a field change of $\pm30\%$ produces a small change in polarization. Changing the field of a solenoid early in the line has a bigger effect, e.g. changing solenoid L2 or L3 by $\pm20\%$ reduces the polarization by approximately 25%. This

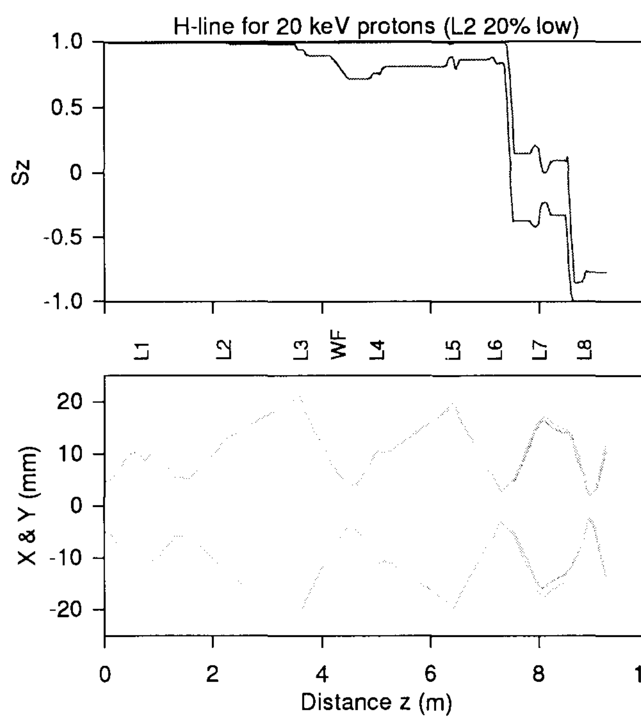

Figure 4: Calculation for the injection line with field setting of solenoid L2 20% too low. Polarization decreases to 75%.

behaviour corresponds well with calculations (see fig 4).

These measurements confirm qualitatively that the program correctly predicts the relation between polarization and field settings of the solenoids. It is reassuring that both measurements and calculations show that a well-tuned beam is tolerant to moderate variations in field settings. The results show that we can rely on simulations with the beam line program, and that such a tool is of great value, not only during the design of new polarized beam lines, but also in simulating the behaviour of existing lines.

5 Calculations for Cyclotrons

The depolarization of a polarized beam of charged particles during acceleration in a cyclotron is discussed in reference [5]. Equations (1) and (2) are rewritten in the form

$$\frac{d\mathbf{S}}{dt} = \mathbf{S} \times \frac{q}{m_0\gamma} \left[(1+\gamma a)\mathbf{B}_t + (1+a)\mathbf{B}_p\right] \qquad (8)$$

where $\mathbf{B}_t$ and $\mathbf{B}_p$ are respectively the components of the magnetic field perpendicular, and parallel to the momentum $\mathbf{p}$. Equation (8) predicts different rates of spin precession about the two components of the magnetic field. The appearance of γ in the coefficient of the transverse field $\mathbf{B}_t$ increases the possibility that at some energy the precession frequency may equal the frequency of an oscillatory component of the field experienced by the particle, thus producing a condition for resonant depolarization. This condition is:

$$a\gamma = N \pm l_z\nu_z \pm l_x\nu_x\,, \qquad (9)$$

where l_z and l_x are zero or positive integers and N the harmonic of the field causing the resonance. In a field with no mid-plane errors only odd values of l_z are allowed. A field error caused by mid-plane errors may have an even value of l_z.

Equation (8) has been programmed into a special version of the cyclotron orbit code. We have also included the ability to do orbit calculations for sector magnets which are displaced and/or rotated from the ideal position.

5.1 Polarization in Injector Cyclotrons

Calculations with the orbit code showed that there are no depolarization resonances at any of the proton energies in the injector cyclotron SPC2.

5.2 Polarization in the SSC

No depolarization resonances have been detected during extensive orbit code calculations in ideal magnet fields of the SSC. Since non-ideal fields may introduce new oscillatory field components, we also investigated the effect of field components which may be caused by small known positional errors of the SSC sector magnets.

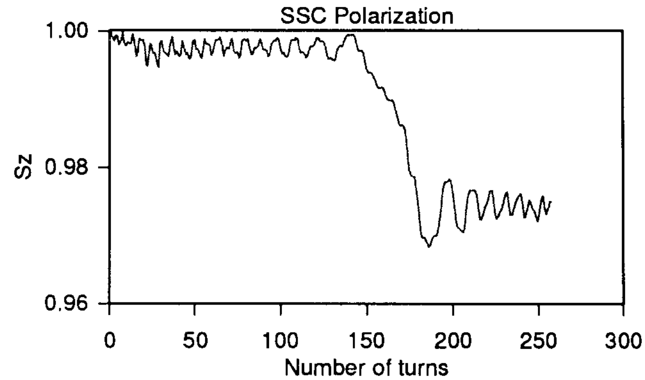

Figure 5: Calculation for 200 MeV protons in a field with mid-plane errors, obtained by displacing a sector magnet 1 mm vertically. The beam amplitude is 1 mm, and depolarization of a few percent is predicted at an energy near 114 MeV.

We considered first-harmonic field errors ($N = 1$), obtained by a vertical displacement of a single sector magnet, and integrated the spin precession equation (8) in the resultant magnetic field. In the SSC conditions for a resonance are favourable at an energy between 110 and 115 MeV, where $\gamma \approx 1.12$. In this energy range the focussing frequencies are $\nu_x \approx 1.37$, and $\nu_z \approx 1.16$, respectively. For these values it is seen that the resonance condition, eq. (9), is satisfied by $l_z = 2$ and $l_x = -1$

$$a\gamma = 1 + 2\nu_z - \nu_x \qquad (10)$$

Calculations (fig. 5) have shown that the depolarization is quadratic in the vertical beam amplitude and also quadratic in the vertical displacement of the sector magnet. However, for a vertical magnet displacement of 1 mm, the amount of depolarization should not exceed 10%. The last survey of SSC magnets indicates that none of the sectors are displaced vertically by more than 0.35 mm.

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