OPTIMIZED BUNCHING IN THE SPIRAL INFLECTOR OF THE CYCLONE 44 INJECTION SYSTEM

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The University at Louvain-la-Neuve is building a compact cyclotron as a postaccelerator/separator for radioactive ion beams called CYCLONE-44[1]. To achieve a high overall injection efficiency it is necessary to ensure proper 6D beam matching of the injected beam to the cyclotron central region. The key component in the cyclotron injection system is a spiral inflector that will place the beam at correct position in the median plane. This paper deals with the optimized beam bunching in the inflector. Using transfer matrix techniques the conditions for obtaining minimized bunch lengths at the inflector exit were derived. Consistent with some previous publications, the time spread of the beam at the inflector exit for the large injected emittances required in this application is significant. It will be shown that these large pulse lengths are a direct consequence of the inflector optics.

1 Introduction

The overall goal of this study was to maximize the amount of beam that could be injected into the cyclotron. We began by optimizing the transverse phase space matching following the procedure developed by Baartman and Kleeven [2]. Then the first-order effect of a spiral inflector on the longitudinal phase space was studied using similar techniques.

The beam calculations were performed using a computer code based upon **TRANSOPTR** [3] in which the spiral inflectors are represented by transfer matrices that have been generated by the numerical orbit code **CASINO** [4]. In each case the **TRANSOPTR** fitting is used to adjust a number of beam parameters at the inflector entrance in order to minimize the size of the effective emittance at the entrance to central region, and/or to minimize the bunch length at this point.

Results of the simulations for the transverse matching calculations indicated that a rotation of the injection line (and/or skew element) could introduce the correct cross-coupling to compensate for the inflector induced correlation. It is also possible to cancel the contribution from the longitudinal phase plane, either assuming negligible momentum spread $\delta p(0)$ in the nearly bunched beam ($l(0) \approx 0$) at the inflector entrance (with the buncher located far upstream from the inflector) or by setting

$$l(0) = \sqrt{\sigma_{66}(0)}S_{56} = \delta p(0)S_{56} \quad , \tag{1}$$

where S_{56} is the element of the inflector transfer matrix and σ_{66} is the element of the beam σ-matrix giving the momentum spread in the beam. In the optimized beam line the minimized bunch length at the inflector exit can be calculated from the quadrature addition of the minimized contributions from the both of the transverse planes i.e.:

$$l(1)^2_{min} = l_x(1)^2_{min} + l_y(1)^2_{min} \tag{2}$$

Thus the problem is reduced to the calculation of the minimized bunch lengths in the separate planes.

In **TRANSOPTR** the beam was determined by three parameters in each of the transverse phase planes: (x_m, y_m) - beam half-sizes, (x', y') - half-divergences at the beam boundaries (x_m, y_m), and $(\varepsilon_x, \varepsilon_y)$ - projected emittances. In this notation each of the decoupled terms in Eqn. 2 (e.g. in the x-p_x plane) can be expressed as the function of beam parameters taken at the entrance to the inflector:

$$l_x(1)^2 = x_m^2(S_{51}^2) + x'x_m(2S_{51}S_{52}) + \left[\frac{\varepsilon_x^2 + (x'x_m)^2}{x_m^2}\right](S_{52}^2) \tag{3}$$

Here the transfer matrix elements $S_{51}, \ldots, S_{54}$, represent the longitudinal-transverse coupling in the inflector and are fixed for any given inflector. In our problem the beam sizes at the inflector entrance are comparable with the size of the inflector electrodes. Therefore, to simplify our analysis, we assumed (x_m, y_m) to be fixed and taken in some proportion (e.g. the aspect ratio of the inflector which is defined as the ratio of the electrodes width a over the electrode spacing b). In this study we have taken $a/b = 2$ with $b = 10$ mm, unless otherwise mentioned. Minimization of $l_x(1)^2$ w.r.t. (ε_x, x') gives the location of two minima:

$$\varepsilon_x = 0 \quad \text{and} \quad x' = -\left(\frac{S_{51}}{S_{52}}\right)x_m. \tag{4}$$

Similar expressions hold in the y-p_y plane.

2 Effect of inflector parameters

As a part of the overall study, we wished to determine how much of an effect the inflector parameters had on the ability to match large beams into the cyclotron. Thus inflector simulation studies were performed in (A, k') parameter space, where A is an electric radius (or height) and k' is a tilt parameter of the inflector. The work was done in two stages:

- To understand basic effects of inflector on beam bunching, we chose 28 points on the (A, k') plane, forming a rectangular mesh. At each of these points the corresponding inflector matrix was obtained using `CASINO` and then it was used to determine the beam longitudinal extension $l(1)$ at the inflector exit.
- Specific pairs (A, k') in inflector parameter space were chosen such that the resulting inflectors produce correct beam orbit radius ($\approx$1.8 cm) in the cyclotron median plane. Therefore A and k' are now correlated by some function. This function is shown in Fig. 1 as the curve in the (A, k') plane.

The projected emittances at the inflector entrance were assumed to be equal in both planes and held (unless indicated otherwise) at $\varepsilon_{x,y} = 235$ mm-mrad, corresponding to the typical values obtained in the transverse matching calculations.

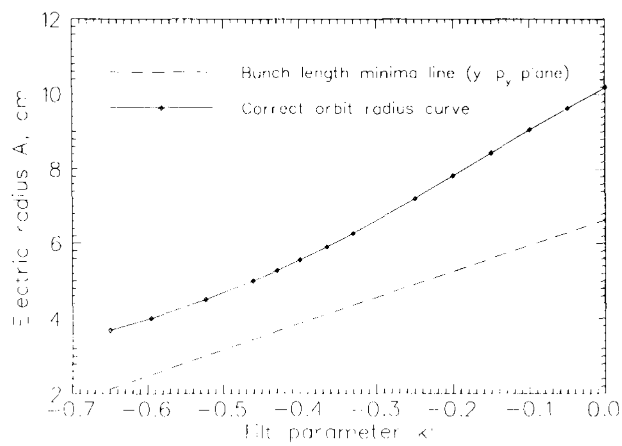

Figure 1: Inflectors with correct beam centering in the median plane

The results of bunch length minimization on the general $(A,\ k')$ plane are shown in Figs. 2 and 3. The first of these figures shows the overall effect of varying A and k' on the bunch length. Fig. 3 shows the contributions $l_x(1)$ and $l_y(1)$ from the individual phase planes. Over most of the k' range the x-plane produces most of the total bunch length with shorter inflectors providing narrower bunches at the exit. This effect is more pronounced as inflectors get more tilted. However, in general, the inflector parameters have a rather small effect on the total bunch length.

One can see that for any given inflector height A there is some tilt k' for which there is no bunch lengthening

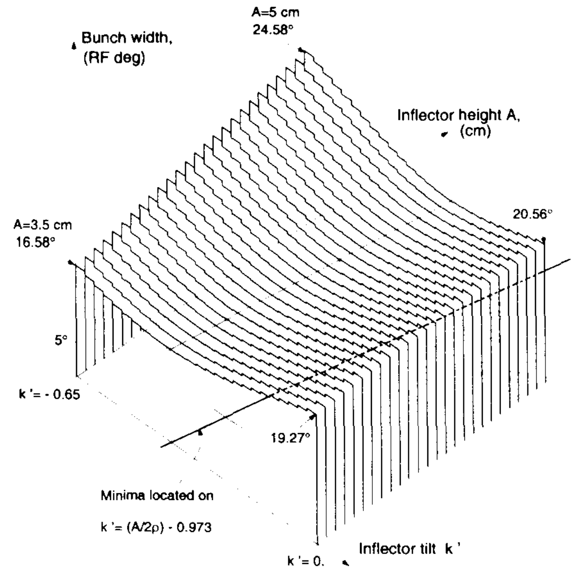

Figure 2: Minimized bunching in the $(A,\ k')$ plane

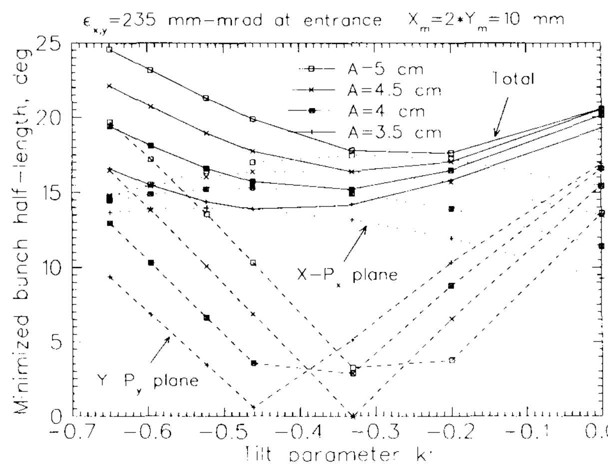

Figure 3: Minimized bunch length at inflector exit for various inflector configurations

in y-plane. These points define the minima in the total bunch length for each particular inflector and lie on the line in the $(A,\ k')$ plane given by $k' = (A/2\rho) - 0.973$. This minima line is shown in Figs. 1 and 2. This phenomenon can be understood if we recall that y-plane is the inflector bend plane and the value of $l_y(1)$ can be reduced to zero for some specific inflector setup. This can be further explained using the canonical F-matrix formalism to develop a differential increment of bunch length [5] as follows:

$$dl(s) = \left[\frac{2}{A}x(s) + \left(\frac{1}{\rho} + \frac{2k'}{A}\right)\sin\left(\frac{s}{A}\right)y(s) + \frac{\delta p}{p}\right]ds \tag{5}$$

where A is the electric radius of the inflector, s is the distance along the reference trajectory (s is zero at the inflector entrance and $\pi A/2$ at the exit), $x(s)$, $y(s)$ particle transverse coordinates relative to the central orbit, $k' = -|k'|$ is the tilt parameter and ρ is the magnetic radius of an ion in the centre of the cyclotron. With no momentum spread in the beam, one can decompose Eqn. 5 into two parts:

$$dl_x(s) = \left(\tfrac{2}{A}\right) x(s)ds \; ; \; dl_y(s) = \left(\tfrac{2}{A}\right) \Phi(s)y(s)ds,$$
$$\text{with} \quad \Phi(s) = \left(\tfrac{A}{2\rho} - |k'|\right) \sin \tfrac{s}{A} \qquad (6)$$

Here $\Phi(s)$ is the Larmour rotation angle. Then the contribution from the y-plane, in which the beam orbit was bent in the inflector, can be reduced o zero by setting $\Phi(s) = 0$. If the injected beam is monoenergetic, then the coupling in the inflector Hamiltonian between the spatial coordinates and the canonical momenta may be completely removed by a transformation to the 'Larmour frame' which is rotated around the reference trajectory over an angle $\Phi(s)$ [5]. Such a cancellation takes place whenever the Larmour rotation angle $\Phi(s)$ becomes zero regardless of the beam parameters at the inflector entrance. In fact, this corresponds to the minimized x-y coupling introduced in the beam by the spiral inflector [5], which is also beneficial for the beam transverse matching.

Fig. 4 presents the optimized bunching in the inflectors selected to provide the correct beam centering in the median plane. Here again the x-plane appears to be dominant in the bunch formation. In y-plane the bunches become smaller as inflectors get closer to the minima line in the $(A,\ k')$ plane. The change in the total length $l(1)$ over the entire range of k' is rather insignificant: $\pm 15\%$ ($\pm 3°$ about an average of $21°$) at $k' = -0.41$ with the shorter inflectors being somewhat more efficient.

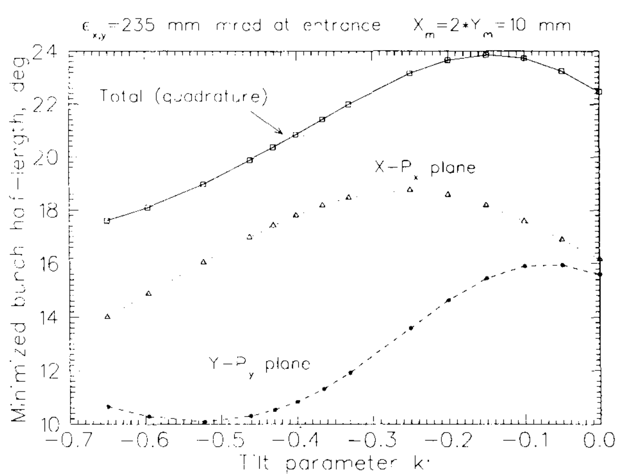

Figure 4: Minimized bunch length at the exits of inflectors providing the correct beam centering (x_m, y_m fixed)

3 Effect of beam parameters

Now consider the impact produced by the beam settings at the inflector entrance on the minimized bunch length at the exit. The inflector with A = 4 cm and $k' = -0.5957$ was chosen since it was used in our transverse matching studies. It has nearly the best bunching performance, (see Fig. 4). Results as a function of beam size and emittance for this specific inflector are shown in Fig. 5. The resulting hyperbolae level off after the beam becomes large (hence parallel) in both transverse directions ($x_m = 2y_m \geq 25$ mm). In the real device, however, the beam size is severely constrained by the aperture at the entrance ($x_m \leq 20$ mm).

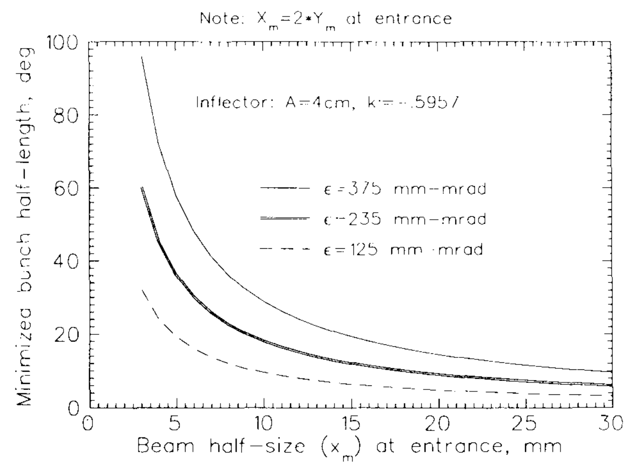

Figure 5: The effect of varying the beam size on bunch length for three different emittances

With the assumptions we have made, it can be shown that $l(1)$ decreases linearly as the projected emittance at the inflector entrance becomes smaller. However one loses in the mount of transverse emittance that can be transmitted to the central region, and vice versa. For example, with $x_m = 2y_m = 10$ mm at the entrance, a nearly twofold increase in the projected emittance from $\varepsilon_{x,y}$ = 125 to 235 mm-mrad leads to a doubling of the bunch length.

When the beam size aspect ratio $AR = y_m/x_m$ is varied at the inflector entrance, the minimized bunch length at the exit $l(1)_{min}$ changes as shown in Fig. 6. As y_m exceeds x_m (i.e. $AR > 1$), $l(1)_{min}$ shortens and, as can be shown, approaches the minimized bunch width in the x-plane $l_x(1)_{min}$. On the other hand, one can observe increasingly poor bunching when $AR < 1$, i.e. $y_m < x_m$. For instance, if $x_m = 10$ mm is fixed, then as $y_m = 5$ mm shrinks to $y_m = 3$ mm the bunch extension $l(1)_{min}$ grows by $\approx 28\%$ from $18°$ to $23°$.

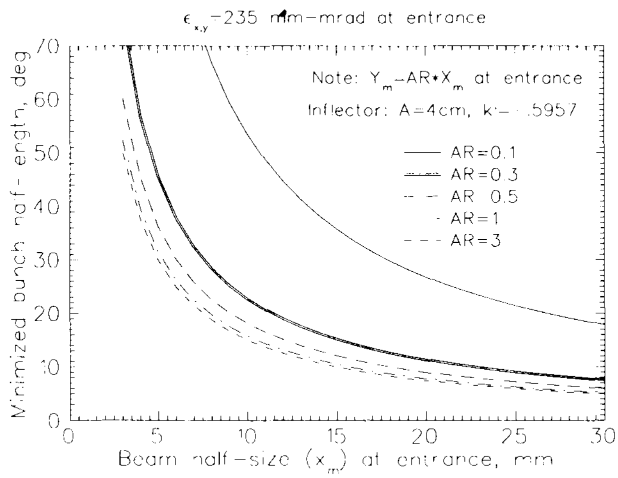

Figure 6: The minimized bunch length $l(1)_{min}$ as a function of the beam aspect ratio at the inflector entrance

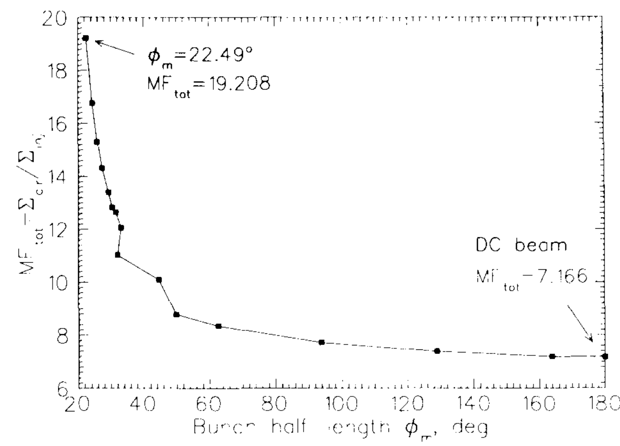

Figure 7: Trade-off in the six dimensional beam matching

4 Conclusions

Inflector parameters appeared to have relatively small effect on the minimized bunch length. On the other hand, the size of the beam at the inflector entrance has a large influence on the bunching performance, as was also found in [6], where the beam size was comparable to the aperture of the inflector. Thus optimizations of the longitudinal and transverse matching are closely coupled and require thorough investigation of possible trade-offs.

In our studies transverse matching was characterized by the merit factor MF_{tot} defined as the ratio of the sum of the circulating emittances in the cyclotron median plane and the sum of the two injected emmittances taken in both of the transverse phase planes [2]. Perfect matching of the beam to the cyclotron acceptances in the 4D phase space is achieved when $MF_{tot} = 1$. The results shown in Fig. 7 reveal the trade-off between the optimum matching in the longitudinal and transverse domains. In most of the cases shown in this figure, the beam was found to be circular at the inflector entrance ($AR \approx 1$).

When the goal of a matching system is to get the maximum amount of beam injected into a cyclotron, then a trade-off between transverse matching and longitudinal matching must be made. As the beam sizes in the inflector increase, so do the non-linear effects, and generally it becomes harder to get a clean transverse phase space at the entrance to the central region. In this case it seems prudent to put more emphasis on obtaining good longitudinal capture into the cyclotron, and limiting the transverse size to about half of the inflector aperture.

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