# ESTIMATION OF A BEAM CENTERING ERROR IN THE JAERI AVF CYCLOTRON 

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#### Abstract

A method for estimating a beam centering error from a beam density distribution obtained by a single radial probe has been developed. Estimation of the centering error is based on an analysis of radial beam positions in the direction of the radial probe. Radial motion of a particle is described as betatron oscillation around an accelerated equilibrium orbit. By fitting the radial beam positions of several consecutive turns to an equation of the radial motion, not only amplitude of the centering error but also frequency of the radial betatron oscillation and energy gain per turn can be evaluated simultaneously. The estimated centering error amplitude was consistent with a result of an orbit simulation. This method was exceedingly helpful for minimizing the centering error of a 10 MeV proton beam during the early stages of acceleration. A well-centered beam was obtained by correcting the magnetic field with a first harmonic produced by two pairs of harmonic coils. In order to push back an orbit center to a magnet center, currents of the harmonic coils were optimized on the basis of the estimated centering error amplitude.


## 1 Introduction

One of the reasons for loss of a beam during acceleration is displacement of an equilibrium orbit from a magnet center[1]. The displacement of an orbit center is due to a first harmonic in the magnetic field and/or inadequate adjustment of positions of an inflector and a puller. The displacement causes precession of the orbit. A large first harmonic drives a linear resonance at $\nu_{r}=1$ that makes the radial motion unstable. A crucial resonance will cause an increase in amplitude of radial oscillation that results in loss of the beam in a center region. The final beam quality may also become worse due to the resonance caused by the centering error.

On the other hand the resonance is helpful for increasing a turn separation just before extraction, what is called precessional extraction. The displacement of the orbit center may improve an extraction efficiency.

For the optimum tune-up of the cyclotron beam, the beam centering error has to be reduced during the early stages of acceleration. The beam centering error can be measured precisely with multiple radial probes [1, 2]. The JAERI AVF cyclotron[3] is equipped with three radial probes; a main probe, a deflector probe and a magnetic channel probe. The main probe covers a full radius, and the other two probes work only in an extraction region. Radial beam density distribution is obtained only by the main probe equipped with a differential head of threefinger type. Thus we devised a method for estimating the beam centering error from the single beam density distribution.

Estimation of the centering error is based on a concept that radial motion of a particle is expressed as the betatron oscillation around an accelerated equilibrium
orbit. A position of the accelerated equilibrium orbit is a function of particle energy and betatron oscillation frequency. The energy is calculated from the sum of an energy gain for each turn. The energy gain depends greatly on a phase of the particle. A beam spreads radially because of the different energy gain of particles. We can know an average position of orbits from the beam density distribution. The energy gain and the betatron oscillation frequency as well as the amplitude of the centering error can be determined by fitting the average beam positions to the equation of the radial particle motion. Here we assume that the variables obtained by the fitting are identical for several consecutive turns.

## 2 Method for Estimating the Centering Error

### 2.1 Measurement of beam density distribution

The radial main probe of the JAERI AVF cyclotron consists of two heads: an integral head and the differential head. The integral head is a copper block 20 mm thick cooled by water. The block has enough thickness to stop a 90 MeV proton fully. The differential head is divided into three tantalum blocks 8 mm thick aligned vertically. The three fingers are projected by 0.5 mm from the edge of the integral head. The beam density distribution of a 10 MeV proton beam obtained by the main probe is shown in Fig. 1. A radial position of each turn is determined by the position of a maximum current density.


Figure 1: Beam density distribution for the 10 MeV proton beam measured with the main probe.

### 2.2 Fitting of beam positions

A static equilibrium orbit is uniquely determined by the particle energy. The static equilibrium orbit changes outward due to an increase of the particle energy after acceleration. The radial position of the particle is expressed as

$$
\begin{equation*}
R(\theta)=A \cos \left(\nu_{r} \theta\right)+B \sin \left(\nu_{r} \theta\right)+R_{e q}(\Delta E, \theta) \tag{1}
\end{equation*}
$$

where $A$ and $B$ are amplitude of the radial betatron oscillation, that is the centering error, $\theta$ an azimuth integrated from the first acceleration, $\nu_{r}$ the radial betatron oscillation frequency, $\Delta E$ the energy gain per turn, $R_{e q}$ the radial position of the equilibrium orbit. The $A$ and $B$ represent amounts of the orbit center shift in the direction of the probe axis and in the perpendicular direction, respectively.

The radial position of the equilibrium orbit in the direction of the main probe axis is uniquely determined by the average radius of the equilibrium orbit. For the 10 MeV proton, correlation between the radial position of the equilibrium orbit and the average radius of the equilibrium orbit is shown in Fig.2. Using a second order polynomial of the average radius, the radial position of the equilibrium orbit is expressed as

$$
\begin{equation*}
R_{\epsilon q}=a_{1}+a_{2} \bar{R}+a_{3} \bar{R}^{2} \tag{2}
\end{equation*}
$$

where the constants $a_{1}, a_{2}$ and $a_{3}$ are determined by a fitting of the data in Fig. 2.

By solving the equation of motion, the average radius of the equilibrium orbit is expressed as a function of the $\theta, \Delta E$ and $\nu_{r}$ :

$$
\begin{equation*}
\bar{R}=\overline{R_{0}}\left(\frac{E_{0}+\Delta E+\frac{\theta}{360} \Delta E}{E_{0}+\Delta E}\right)^{\frac{1}{2 \nu_{r}^{2}}} \tag{3}
\end{equation*}
$$



Figure 2: Correlation between the radial position of the equilibrium orbit in the direction of the main probe axis and the average radius for the 10 MeV proton beam.


Figure 3: The average radius of the equilibrium orbit as a function of the particle energy for a 10 MeV proton beam.
where $\overline{R_{0}}$ is the average radius of the first turn and $E_{0}$ is an injection energy.

A relation between the particle energy and an average radius of the static equilibrium orbit for a 10 MeV proton beam is shown in Fig. 3. By non-relativistic approximation, the particle energy can be expressed by a second order polynomial of the average radius. Thus we approximate the average radius of the first turn as

$$
\begin{equation*}
\overline{R_{0}}=b_{1} \sqrt{E_{0}+\Delta E+b_{2}}+b_{3} \tag{4}
\end{equation*}
$$

where the constants $b_{1}, b_{2}$ and $b_{3}$ are determined by a fitting of the data in Fig. 3.

The particle energy of the first turn is defined as $E_{0}$ $+\Delta E$, since particles are stopped by the main probe after having the first energy gain. The parameters of $A$, $B, \Delta E$ and $\nu_{r}$ are determined by a fitting of actual beam positions to the equation ( 1 )

## Results of the Centering Error Analysis

## 1 Estimation of the centering error

te centering error of the 10 MeV proton beam was timated from the beam positions of the first twenty© turns shown in Fig. 1. The fitting for estimating - centering error was applied to consecutive five or ten rus on the assumption that the parameters $A, B, \Delta E$ d $\nu_{r}$ don't change drastically during ten revolutions the center region. The positions of the first twenty© revolutions are plotted by dots in Fig. 4-7. Radial sitions of the accelerated equilibrium orbit obtained by $c$ fitting is also drawn in the figures. The parameters termined by the fitting are summarized in Table 1.
able 1:Amplitude of the Centering error $A$ and $B$, frequency ithe radial betatron oscillation $\nu_{r}$ and a energy gain per turn $\Delta E$ of the 10 MeV proton beam.

| Turn <br> number | $A$ <br> $(\mathrm{~mm})$ | $B$ <br> $(\mathrm{~mm})$ | $\nu_{r}$ | $\Delta E$ <br> $(\mathrm{MeV} / \mathrm{turn})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 to 5 | 2.6 | -50 | 0.99 | 0.037 |
| 6 to 15 | 0.3 | -2.2 | 0.98 | 0.042 |
| 11 to 20 | 0.7 | 1.4 | 0.98 | 0.043 |
| 16 to 25 | -0.1 | -0.9 | 0.97 | 0.040 |

The centering error amplitude of the first five turns is quite large and the energy gain was smaller than $\therefore$ others because of a phase slip caused by a center mp of the magnetic field. After the sixth turn the atering error was minimized and the beam was almost ll-centered. A correction of the centering error of the st five turns was made by the first harmonic in the id produced by two sets of harmonic coils.
In order to evaluate the results of the estimation, an :it simulation was carried out using the operation paneters of the 10 MeV proton beam. The amplitudes and $B$, the frequency $\nu_{r}$ and the energy gain $\Delta E$ obned by the simulation were $-1.1 \mathrm{~mm},-0.2 \mathrm{~mm}, 0.99$ d $0.042 \mathrm{MeV} /$ turn, respectively. The result of the esration from the real beam density distribution was asistent with the computer simulation.

## $\therefore$ Energy gain

order to compare the estimated energy gain to a noma value deduced from a Dee voltage of the practical ration, the beam density distribution measured at Terent Dee voltages was analyzed. The estimated val-- are shown in Table 2.


Figure 4: Radial positions(dots) from the first to the fifth turn of the 10 MeV proton beam. The curve accounts for the accelerated equilibrium orbit determined by the fitting.


Figure 5: Radial positions(dots) from the sixth to the fifteenth turn of the 10 MeV proton beam.

Table 2 : The energy gain obtained at different dee voltages

| Nominal <br> Dee voltage <br> $(\mathrm{kV})$ | Nominal <br> maximum $\Delta E$ <br> $(\mathrm{MeV} /$ turn $)$ | Estimated <br> $(\mathrm{MeV} /$ turn $)$ |
| :---: | :---: | :---: |
| 11.45 | 0.046 | 0.042 |
| 19.69 | 0.079 | 0.070 |
| 29.69 | 0.118 | 0.128 |

The estimated energy gain for the nominal Dee voltages of 11.45 kV and 19.69 kV was smaller by $10 \%$ than the nominal value. A center of the acceleration phase might shift from the phase of the maximum gain. On the other hand, the estimated value for the nominal Dee voltage of 29.69 kV was larger than the nominal one. In this case an error of the estimation might be larger than the others partly because the beam was extremely offcentered. The centering error amplitude for the nominal

Dee voltage of 29.69 kV was $A=-21 \mathrm{~mm}$ and $B=-6.4$ mm . Evaluation of the errors in the estimation is in progress.


Figure 6: Radial positions(dots) from the eleventh to the twentieth turn of the 10 MeV proton beam.


Figure 7: Radial positions(dots) from the sixteenth to the twenty-fifth turn of the 10 MeV proton beam.

## References

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