COOLING SIMULATIONS WITH THE BETACOOL CODE

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Abstract

The BETACOOL program developed by the JINR electron cooling group is a kit of algorithms based on a common format of input and output files. The general goal of the program is to simulate long term processes (in comparison with the ion revolution period) leading to a variation of the ion distribution function in six dimensional phase space. The BETACOOL program includes three algorithms for the simulation of the beam dynamics and takes into account the following processes: electron cooling, intrabeam scattering, ion scattering on residual gas atoms, interaction of the ion beam with an internal target and some others.

INTRODUCTION

The goal of the first version of the Betacool program [1] was to investigate the electron cooling process using formulae for the friction force derived in [2]. Presently the program is a kit of algorithms allowing to simulate long term processes (in comparison with the ion revolution period) leading to the variation of the ion distribution function in six dimensional phase space.

Evolution of the second order momenta of the ion distribution function is realized in the so called "*rms dynamics*" algorithm based on the assumption of a Gaussian shape of the distribution. Here all heating and cooling effects are characterized by rates of variation of the emittances or of particle loss.

The investigation of the beam dynamics at arbitrary shape of the distribution is performed using multi-particle simulation in the frame of the Model Beam algorithm. In this algorithm the ion beam is represented by an array of model particles. The heating and cooling processes involved in the simulations lead to a change of the components of the particle momentum and of the particle number.

During the last years the program was used for simulations of ion beam dynamics in the following fields of a cooling application:

- luminosity preservation in ion-ion colliders:
  RHIC-II (BNL), PAX (FZJ), NICA (JINR),
- simulations of experiments with internal pellet target:
  PANDA (GSI, FZJ), WASA at COSY (FZJ),
- benchmarking of IBS and electron cooling models:
  CELSIUS (TSL), RHIC (BNL), Recycler (FNAL), Erlangen University, TechX,
- beam ordering investigations:
  S-LSR (Kyoto University), COSY (FZJ), NAP-M (BINP), ESR (GSI),
- simulations of cooling-stacking process:
  LEIR (CERN), HIRFL-CSR (Lanzhou).

In this report a brief description of a few basic Betacool algorithms is presented.

PHASE DIAGRAMS

Usually a design of a cooling system is started from an estimation of the cooling rate required for reaching equilibrium at the necessary value of the beam emittance. By definition, the cooling (heating) rate is equal to

$$\frac{1}{\tau}=\frac{1}{\varepsilon}\frac{d\varepsilon}{dt}, \qquad (1)$$

and in the general case it is a function of the beam phase volume and intensity. Here ε are the horizontal, vertical or longitudinal emittances. An equilibrium between heating and cooling processes corresponds to a vanishing sum of the rates:

$$\sum_j \frac{1}{\tau_j}=0\cdot \qquad (2)$$

The index *j* is the number of processes involved in the calculations. Equations (2), written for each degree of freedom, form a system of non-linear algebraic equations describing the equilibrium emittance of the beam. For the solution of such systems the phase diagram method was developed in the Betacool program. In a phase diagram the sum of the rates is plotted as a function of the beam emittance (assuming that the horizontal emittance is equal to vertical one) and of the momentum spread. The crossing of the lines of vanishing sum of the rates at the phase diagrams for all three degrees of freedom corresponds to an expected equilibrium beam parameters. An analysis of the phase diagrams permits to predict some peculiarities of the cooling process without simulation of its dynamics. For example, the efficiency of this method was demonstrated in the simulations of the beam ordering process [4].

Calculation of the characteristic times is also the basis of RMS dynamics algorithm.

RMS DYNAMICS

The physical model used in the *rms* dynamics simulations is based on the following general assumptions:

1) the ion beam has a Gaussian distribution over all degrees of freedom and does not change during the process.

2) the algorithm for the analysis of the problem is considered as a solution of the equations for the *rms* values of the beam phase space volumes of three degrees of freedom.

3) the maxima of all the distribution functions coincide with the equilibrium orbit.

The evolution of the ion beam parameters during the motion in the storage ring is described by the following system of four differential equations:

$$\begin{cases} \dfrac{dN}{dt} = N\sum_j \dfrac{1}{\tau_{life,j}}, \\ \dfrac{d\varepsilon_h}{dt} = \varepsilon_h \sum_j \dfrac{1}{\tau_{h,j}}, \\ \dfrac{d\varepsilon_v}{dt} = \varepsilon_v \sum_j \dfrac{1}{\tau_{v,j}}, \\ \dfrac{d\varepsilon_{lon}}{dt} = \varepsilon_{lon} \sum_j \dfrac{1}{\tau_{lon,j}}, \end{cases} \quad (3)$$

where N is the particle number, ε_h, ε_v, ε_{lon} are root mean square values of horizontal, vertical and longitudinal beam emittance, respectively. The characteristic times are functions of all three emittances and of the particle number and have positive sign for a heating process and negative for cooling. A negative sign of a lifetime corresponds to particle loss and the sign of the lifetime can be positive in the presence of particle injection, when the particle number increases. The structure of the algorithm is designed in such a way as to allow for including any process into calculation which can be described by cooling or heating rates. Numerical solution of the system (3) is performed using the Euler method with automatic step variation. Results of the simulation are the dependences on time of the emittance and particle number.

The time step in the integration of the system (3) is determined by the characteristic times of the investigated effects and the speed of the calculation can be very fast. However, in some cases the basic physical model cannot provide a realistic simulation mainly due to the basic assumption of a Gaussian shape of the ion distribution function. This assumption is more or less realistic in an equilibrium state of the ion beam when the equilibrium is determined by many processes of stochastic nature. If there does not exist an equilibrium due to fast particle loss or in the initial stage of the beam cooling, the ion distribution function can be far from Gaussian. The same situation takes place in an experiment with internal targets which dimensions are not comparable with the ion beam dimensions. Neither can be calculated in the framework of this model correctly ionization energy losses of the ion beam in the target.

MODEL BEAM ALGORITHM

An investigation of the ion beam dynamics with an arbitrary shape of the distribution function is performed using multi-particle simulation in the frame of the Model Beam algorithm. In this algorithm the ion beam is presented by an array of modeling particles. Heating and cooling processes involved in the simulations lead to a change of the particle momentum components and of the particle number, which is calculated in accordance with the time step of the dynamical simulation. Each effect is located at some position of the ring characterized by the ring lattice functions. Transformation of the beam inside the ring is provided using a linear matrix with a random phase advance between the different objects. The results of the simulations can be presented both in form of a beam profile evolution in time or as time dependencies of the beam emittance and the particle number.

The Model Beam algorithm is based on the solution of the Langevin equation in momentum space that is realized in the Betacool program using the Euler method at fixed integration step. Action of some physical processes (IBS, scattering on gas etc.) results in a regular and (or) stochastic variation of the model particle momentum components. The momentum variation after the integration step of Δt is calculated in accordance with the following equation,

$$\Delta P_i = F_i \Delta t + \sqrt{\Delta t}\sum_{j=1}^{3} C_{i,j}\xi_j , \quad (4)$$

where $i = x, y, s$ are the horizontal, vertical and longitudinal co-ordinates, F_i are the components of the friction (or leading) term, ξ_j are three Gaussian random numbers with unit dispersion. The coefficients $C_{i,j}$ have to be calculated from the components of the diffusion tensor. In the Betacool program the component of the particle momentum are chosen to be $\left(p_x / p, p_y / p, \Delta p / p\right)$.

In the general case that the components of the diffusion tensor form a diagonal symmetric matrix,

$$\begin{pmatrix} D_{x,x} & D_{x,y} & D_{x,z} \\ D_{x,y} & D_{y,y} & D_{y,z} \\ D_{x,z} & D_{y,z} & D_{z,z} \end{pmatrix}, \quad (5)$$

and depending on the process some of them can be equal to zero. In the presence of diffusion the mean values of the variation of the components of the momentum can be expressed via components of the diffusion tensor in accordance with the definition,

$$\left\langle \frac{d\left(P_i P_j\right)}{dt} \right\rangle = D_{i,j} , \quad (6)$$

where triangular brackets mean averaging over the particles. From these expressions one can deduce the values of the coefficients $C_{i,j}$. They have to satisfy the following system of equations:

$$\sum_{k=1}^{3} C_{i,k} C_{j,k} = D_{i,j} . \quad (7)$$

This system has an infinite number of solutions but it can be simplified when the diffusion tensor has zero components. For example for a diagonal diffusion tensor (this corresponds to the case when the variations of momentum components do not correlate with each other) the simplest solution is:

$$C_{x,1} = \sqrt{D_{x,x}} , \; C_{y,2} = \sqrt{D_{y,y}} , C_{z,3} = \sqrt{D_{z,z}} ,$$

all other coefficients are equal to zero.

The Fokker-Plank approach (which is the physical basis of the Model Beam algorithm) allows for providing a uniform treatment of most of the heating and cooling effects, such as electron and stochastic cooling, interaction with residual gas and internal gas target,

intrabeam scattering process, heating due to noise of magnetic system power supply and others. However, when an accurate calculation of the distribution tail width and intensity is necessary one needs to provide direct simulations of the scattering processes. A more important example of such a situation is the simulation of an experiment with an internal pellet target.

SIMULATION OF PELLET TARGETS

When a beam interacts with an internal pellet target each particle crosses the pellet once during a few thousands of revolutions in the ring. In this case each crossing can be simulated directly. In order to simulate the variation of the components of the ion momentum the program calculates expectation values of the numbers of elementary events after a single crossing of the target, i.e. the numbers of ionizations of the target atoms and number of scattering on the nuclei. The actual numbers of the events is assumed to be distributed around its expectation values in accordance with Poisson law. In each elementary event the momentum variation is calculated as a random number distributed in accordance with corresponding law.

This algorithm can be illustrated on the example of the simulation of the ionization energy loss The variation of the longitudinal momentum component is caused mainly by the ionization energy loss distributed according to the function:

$$g(E) = \frac{(E_{\max} + I)I}{E_{\max}} \frac{1}{E^2} . \quad (8)$$

The macroscopic cross-section for ionization is

$$\Sigma = \frac{\Delta E_{BB}}{\Delta x} \frac{E_{\max}}{I(E_{\max} + I)\ln\left(\frac{E_{\max} + I}{I}\right)} . \quad (9)$$

$E_{\max}$ is the maximum transferable energy:

$$E_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma\frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} , \quad (10)$$

with m_e being the electron mass and M the projectile mass. I is the mean excitation energy that can be estimated as $I = 16 \cdot Z^{0.9}\ eV$, ΔE_{BB} is the mean energy loss after crossing the target calculated in accordance with the Bethe-Bloch equation and Δx is the target thickness. The number of ionization events n after single crossing the target is

$$n = \Sigma \Delta x . \quad (11)$$

The energy loss due to ionization is calculated as

$$\Delta E_{ion} = \sum_{i=1}^{n} \frac{I}{1 - h\xi_i} . \quad (12)$$

n is the integer number sampled from the Poisson distribution, ξ_i are random numbers uniformly distributed in 0 to 1, $h = \frac{E_{\max}}{E_{\max} + I}$.

The deviation of the longitudinal component of the particle momentum is calculated as:

$$\delta\frac{\Delta p}{p} = \frac{\gamma}{1+\gamma}\frac{\Delta E_{total}}{AE} , \quad (13)$$

where E is the particle kinetic energy per nucleon, A is the particle atomic number. The expectation of the energy loss ΔE_{BB} is calculated in accordance with the equation:

$$\frac{\Delta E_{BB}}{\rho \Delta x} = -KZ_P^2 \frac{Z_T}{A_T}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_e c^2 \beta^2 \gamma^2 E_{\max}}{I^2} - \beta^2 - \frac{\delta}{2}\right] .$$

where ρ is the target density, Z_P and Z_T are the charge numbers of the projectile and target atoms, respectively, A_T is the target atomic number, δ is the density correction factor. K is a constant determined by the following expression:

$$\frac{K}{A} = \frac{4\pi N_A r_e^2 m_e c^2}{A} = 0.307075\ \text{MeV}\cdot\text{g}^{-1}\cdot\text{cm}^2,$$

r_e is the electron classic radius and N_A is the Avogadro number.

HIERARCHY OF IBS MODELS

Another physical task requiring direct simulation of a scattering process is the investigation of the beam ordering process. For this goal the simulation of the IBS process through Coulomb interaction between particles was realized in the tracking algorithm in the Betacool program. The Fokker-Plank approach is used in local and simplified kinetic models of the IBS process. To speed up the calculation a few detailed models of the IBS based on analytical expressions for the diffusion power were developed. In this chapter a brief description of the IBS simulation methods is presented.

Tracking Algorithm

A tracking algorithm is used for the simulation of the IBS process through Coulomb interaction between ions. One of the goals of this algorithm development is to simulate a formation of a crystalline state of the ion beam. In the crystalline state of the ion beam the IBS process cannot be treated in the frame of analytical models which are based on the assumption of Gaussian shape of the ion distribution function. To speed up the calculations in the tracking algorithm the IBS simulations are performed using Molecular Dynamics technique. In this case the equations of motion are solved for a small number of particles located inside a short cell. The influence of all other particles is taken into account through periodic boundary conditions in the longitudinal direction for the particle distribution function and use of Ewald's sum for the calculation of Coulomb forces. Therefore, this algorithm can be used for coasting beams only.

In the frame of the tracking algorithm the equations of motion of the particles are integrated in the real structure of the ring. The ring structure is imported from an input MAD file. Each cooling or heating effect involved in the calculations together with IBS is located in some optic element. Calculation of the variation of the particle

coordinates due to the action of an object is provided using the map of this object. The position of the object in the ring is described in the input MAD file using special markers.

Local Model of IBS and Electron Cooling

Calculation of the friction force and diffusion tensor components related with the problem of Coulomb scattering of a test particle of a mass m_t and velocity $\vec{V}$ proceeds in an array of N_{loc} field particles of mass m_f and velocities $\vec{v}_i$. The solution of this problem is well known from plasma physics. For a given distribution function $f(v)$ of the field particles in velocity space the friction force is equal to

$$\vec{F} = -\frac{4\pi n e^4 Z_t^2 Z_f^2}{\left(\frac{m_f m_t}{m_f + m_t}\right)} \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{\vec{U}}{U^3} f(v) d^3 v$$

and the components of the diffusion tensor are

$$D_{\alpha,\beta} = 4\pi n e^4 Z_t^2 Z_f^2 \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{U^2 \delta_{\alpha,\beta} - U_\alpha U_\beta}{U^3} f(v) dv \cdot$$

Here $\alpha, \beta = x, y, s$, the angular brackets mean averaging over the field particles, Z_t, Z_f are the charge numbers of the test and field particle, n is the mean local density of the field particles and $\vec{U} = \vec{V} - \vec{v}$ is the relative velocity of the test and field particle. The minimum and maximum impact parameters are determined as in the simulation of electron cooling.

For a particle array the distribution function of the field particles in the velocity space is given as a series of δ-functions:

$$f(v) = \frac{1}{N_{loc}} \sum_{j=1}^{N_{loc}} \delta(\vec{v} - \vec{v}_j) \cdot$$

The minimum impact parameter in the Coulomb logarithm is calculated as

$$\rho_{\min} = \frac{Z_t Z_f e^2}{\left(\frac{m_f m_t}{m_f + m_t}\right)} \frac{1}{\left|\vec{V} - \vec{v}_j\right|^2} .$$

The value of the dynamic shielding radius required for the maximum impact parameter determination is calculated using the *rms* velocity spread of the field particles.

The algorithm described above can be used for IBS as well as for electron cooling simulations. In the case of electron cooling simulation the test particle is the ion and the field particles are the electrons. In the case of IBS the test and the field particles are the same ions. For IBS simulation the friction and diffusion components have to be calculated in each optic element of the ring. Therefore the algorithm is very slow and it is suited only for large computers.

Simplified Kinetic Model

To speed up the calculations a simplified kinetic model of the IBS process was proposed in [5]. It is based on the assumption that the friction force is proportional to the particle momentum and to the diffusion constant. The friction and diffusion components can be calculated in accordance with one of the analytical IBS models. In the Betacool program the kinetic model was realized for the ring optical structure with non-zero vertical dispersion in accordance with [6].

Detailed Models

The analytical models are used as a basis for a few detailed models of the IBS process realized in the Betacool program. In Burov's model [7] the diffusion power for each ion is calculated as a function of its action variables. In the "core-tail" models the diffusion is calculated separately for the particles from a dense core of the distribution function and from low intensive tails.

Analytical Models for IBS Growth Time Calculation

For the calculation of the IBS growth rates a few analytical models are available in the Betacool program. Their description can be found in [8].

ELECTRON COOLING SIMULATION

In order to solve all the problems related with the simulation of the electron cooling process a hierarchy of objects was developed in the BETACOOL program. The cooling simulation is based on a friction force calculation in the particle rest frame. The friction force can be calculated in accordance with one of the analytical models from a library or by using results of numerical calculations imported from an external file. The next layer of the simulation is related with a cooler representation as a map, transforming particle coordinates from entrance to exit of the cooling section and calculating the ion loss probability due to recombination with electrons. Calculation of the cooler map is based on a model of the electron beam that provides transformation of the ion velocity to the frame related with the electron beam and takes into account the real geometry of the cooler. The main models of the electron beam used in the Betacool program and a description of the friction force formulae are given in [8]. The cooler model takes into account the variation of the magnetic field in the cooling section. For this purpose the coordinates of the electron beam trajectory inside cooling section are input from an additional file and the equations of motion of the ions are solved numerically inside the cooler.

The map of the cooler can be used directly in the frame of the Molecular Dynamics algorithm or in other tracking procedures. On the basis of the map one can calculate the kick of the ion momentum after crossing the cooling section that is necessary for the simulation of the ion distribution evolution in the frame of the Model Beam algorithm. The map of the cooler is also used for the calculation of the cooling rate that is necessary for the simulation of the *rms* dynamics. The calculation of the cooling rate can be performed using two different models

of the ion beam, either the cooling rates for "*rms*" particle dynamics or for the ion beam with Gaussian distributions in all degrees of freedom.

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