# INTRODUCTION TO THE SESSION ON LATTICE OPTIMIZATION FOR STOCHASTIC COOLING

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#### Abstract

Lattices that circumvent the 'mixing dilemma' for stochastic cooling have repeatedly been considered but were not adopted in the original design of existing cooling rings. Recently new interest has arisen to modify existing machines and to design future 'optimum mixing rings'. This talk is meant to summarize the advantages and disadvantages with the aim to introduce the discussion.

# **INTRODUCTION**

For efficient stochastic cooling a small dispersion ( $\eta_{PK}$ ) in the time of flight is desirable on the beam-path from pickup to kicker and a large dispersion ( $\eta_{KP}$ ) on the way kicker to pickup. For a regular lattice one has (at least approximately)

$$\eta_{PK} = \eta_{KP} = \eta = \gamma_{tr}^{-2} - \gamma^{-2}$$

i.e. the local  $\eta$ -factors are equal to each other and given by the off-momentum factor of the whole ring. Then the spread of the flight times  $\Delta T_{PK}$  (leading to undesired mixing) and  $\Delta T_{KP}$  (desired mixing) are related by the corresponding lengths  $L_{PK}$  and  $L_{KP}$  along the circumference

$$\Delta T_{_{PK}} = \eta \left( L_{_{PK}} / \beta c \right) \left( \Delta p / p \right)$$
  
$$\Delta T_{_{KP}} = \eta \left( L_{_{KP}} / \beta c \right) \left( \Delta p / p \right)$$

Thus in the special case of a regular lattice and a cooling loop that cuts diagonally through the ring one has  $\Delta T_{PK} = \Delta T_{KP}$ . One can however design an 'asymmetric' (also called 'split ring-' or 'optimum mixing-') lattice [1], which combines sections with small local  $\eta$  in one part with large  $\eta$ -sections in the other part. In this way  $\Delta T_{PK}$ and  $\Delta T_{KP}$  can be adjusted independent of each other. In addition if the local momentum compaction factors  $\alpha = \gamma_{tr}^{-2}$  are tuneable, then optimum mixing can be envisaged for different energies and one can even envisage  $\eta$ -tuning dynamically during cooling at fixed energy. The potentially large gain in cooling speed has to be balanced against difficulties such as complexity of the lattice, and 'single particle' and collective beam stability.

# GAIN WITH AN ASYMMETRIC LATTICE

It can be concluded from [1] that by optimising the mixing one can gain a factor of  $\sim 3.4$  in the initial cooling rate. This is when the system noise is negligible and the

cooling loop cuts diagonally through the ring. To ease the discussion this 'standard case' will mostly be assumed in the following. For low energy rings where the distance  $L_{PK}$  can be made considerably smaller than  $L_{KP}$  and also for cooling systems with poor signal to noise ratio, the gain is less pronounced. On the other hand for momentum the cooling the improvement factor can be larger than 3.4 because with a regular lattice the mixing situation degrades as the  $\Delta p$  decreases. For momentum spread reduction by  $e^{-1}$  ( $e^{-2}$ ) the overall improvement turns out to be 4.4 (5.8) in our standard case.

The gain concerns transverse cooling and longitudinal cooling by the "Palmer method" [2] where the momentum error is detected via the transverse displacement of the particle. For the filter method of Thorndahl [3] where the in essence the momentum error is deduced from the change in time of flight for a whole revolution, the "split lattice" is not helpful. However for the further momentum cooling methods, that use the time of flight over part of the circumference [4,5], the advantage remains. In this case one has to provide a well chosen finite, and if possible even tuneable  $\eta$  (instead of  $\eta=0$ ) over the distance where the flight time is observed, and again large  $\eta$  for the section kicker to pickup. This can be achieved, at least in principle, by placing the observation interval partly into the low mixing and partly into the strong mixing branch of the lattice.

In summary: a factor of three to six in cooling speed can be gained with an optimum mixing lattice. The gain concerns transverse cooling as well as longitudinal cooling by the Palmer and local time of flight approaches but not the filter method.

#### LATTICE MODULES

Small  $\eta_{PK}$  requires a local momentum compaction  $\alpha_{PK} = \gamma_{trPK}^{-2}$  close to the beam's  $\gamma^{-2}$ . Big  $\eta_{KP}$  can be realized by large negative  $\alpha_{KP}$ . There is a long list of references that deal with adjusting the momentum compaction (starting with the 1955 paper of Vladimirski and Tarasov [6] who proposed reverse bend dipoles to make the momentum compaction negative). The original aim was to avoid crossing of transition energy by making  $\gamma_{tr}$  large or even imaginary ( $\alpha$  negative). In the 1970s the additional task of performing a jump of  $\gamma_{tr}$  without a too large change of the betatron tune [7-9] came up. The aim of the jump is to cross transition rapidly and this was achieved successfully, first in 1969 and operationally since 1974 in the CERN PS [7]. Later  $\gamma_{tr}$ -jumps were incorporated in the Booster and the Main Injector at

FERMILAB, as well as in the AGS and RHIC at BNL. Examples of synchrotrons that worked with imaginary transition energy are LEAR and STURN II.

In these applications negative closed orbit dispersion (D(s)<0) in bending magnets is of prime importance. Basically two methods, the 'harmonic-' and 'modular approach', are used to enhance D. In the harmonic method one excites a 'dispersion oscillation' by introducing a super-periodicity S in the bending or the focussing with S close to the betatron tune. In the modular approach local 'dispersion bumps' are generated via quadrupoles or bends. Both methods affect not only the dispersion but also the betatron functions, the tunes and other lattice properties. It is the aim of the designer to keep the unwanted perturbations small. This is more difficult in the existing machines (the PS, AGS, FNALbooster) where, to maintain constant tune, a large number of  $\gamma_r$ -jump quadrupoles has to be used and the maximum excursions of the dispersion and the  $\beta$ -functions are large. In designs, where flexible momentum compaction is included from the start, these problems are alleviated, although not completely removed.



Figure 1: Layout of a simple Flexible Momentum Compaction (FMC) lattice module (adapted from [11]).

Already in 1972 L. Teng [10] proposed modules with a negative dispersion at dipole locations bridged by straight sections where the dispersion wave was positive (Fig. 1). Such a concept forms the basis of flexible momentum compaction (FMC) modules. More recently Trbojevic and co-workers [12-15] extended the modular approach. Their FMC sections consist of a FODO part where a negative dispersion in the dipoles provides negative  $\alpha_p$  and a matching section, where the optical functions are rematched to avoid excessive excursions.



Figure 2: Example of an advanced FMC module (adapted from [13]).

The authors of [13] find that "...the modules can be made very compact without much unwanted empty space, and at the same time, the maximum of the dispersion function can be controlled to less than that of the regular ...FODO lattice, thus overcoming ... the difficulties of Teng's original idea". The phase advance of the module can be adjusted to be an odd multiple of quarter betatron waves and modules can be positioned one after another to create long sections with small or large negative momentum compaction. Designs for large momentum compaction [12], isochronous [14] and adjustable momentum compaction [15] lattices have been established.

In summary: It appears that the modular approach is well suited, to construct a cooling ring lattice consisting of small momentum compaction modules for the (low mixing, quasi-isochronous) part pickup to kicker and negative momentum compaction modules for the (strong mixing) part kicker to pickup. For a cooling ring that has to work at different energies, the isochronous part has to be tuneable. Here techniques used for the  $\gamma_{tr}$  jump can be helpful.

# DISADVANTAGES

The FMC modules require extra quadrupoles. For example: the module of Fig. 2 needs 7 quadrupoles (on the assumption that the 2x2 adjacent quads near the centre are combined into a single lens each) compared to 6 in the corresponding regular FODO lattice. The number of different *quadrupole families* is 4 instead of 2 for the FODO structure. In fact the problems are similar to those of other lattice insertions like long straight sectors or low beta sections.

In the 'Teng structure' of Fig.1, the number of quadrupoles is the same as for the FODO lattice but the number of families is again larger, probably also 4 instead of 2 because in the 'missing magnet sections' the phase advance is specific and the horizontal defocusing of the dipoles is absent.

Moreover, a machine with *simple* FMC modules will have larger excursions of the optical function and hence reduced acceptances. With the advanced modules this drawback is absent or less pronounced. Yet there remains the problem that one will have several pieces of straight section and low betas at locations where one cannot always make use of them.

It is a question of detailed design to conceive a lattice which incorporates other basic blocks like injection/ejection, long straight sections, and locations for experimental apparatus. Lee, Ng, and Trbojevic [13] designed a complete accelerator ring using FMC modules. They found "...that this lattice is extremely tunable and is insensitive to misalignment errors. Its chromatic properties are at least comparable to that of a regular FODO lattice. ... it provides dynamical aperture as large as that of a regular ... lattice".

The 'optimum mixing lattice' will however have a basic periodicity of 1 and thus many systematic resonances will

be present. This perturbation is similar to the disturbance due to other insertions like e.g. a low beta section.

Finally the large  $\eta$  of the ring influences some RFparameters e.g. the voltage necessary to produce a bucket of given size (U proportional to  $\eta$ ) and the synchrotron oscillation frequency ( $f_s \alpha \eta^{1/2}$ )

In summary: "Optimum mixing lattices" need extra qudrupoles and extra quadrupole families. Compared to a regular FODO lattice they are more complicated, both in their design and their operation, especially when the advanced FMC modules are used. With simpler modules the acceptance will suffer. Other quantities depending on the ring  $\eta$  change.

# **COHERENT BEAM STABILITY**

Damping of longitudinal instabilities is lost close to the transition energy ( $\eta \rightarrow 0$ ). In fact the "Keil-Schnell-Boussard" stability criterion [16] requires coupling impedances Zn/n smaller than a maximum that is proportional to  $\eta$  and thus unattainably small for small  $\eta$ . However -- because the growth of the instability takes a great number of turns -- it is the  $\eta$  of the entire ring that counts. For the "split ring" (with circumference C) we have

$$\eta = \frac{L_{_{pk}}}{C}\eta_{_{pk}} + \frac{L_{_{kp}}}{C}\eta_{_{kp}} \approx \frac{L_{_{kp}}}{C}\eta_{_{kp}}$$

Then, if the mixing kicker to pickup is large ( $\eta_{kp}$  large as desired) we preserve a good margin for tolerable coupling impedance, frequently even higher than the in a regular lattice.

In summary: The "optimum mixing lattice" has automatically a large "whole ring  $\eta$ " and the longitudinal stability threshold is usually equal or even more favourable than in a regular lattice.

#### CONCLUSION

In the design of a new generation of stochastic cooling rings, the "optimum mixing concept" should be taken into consideration. FMC modules, originally developed to move up transition energy, are appropriate to construct lattices optimised for mixing. One can even think of tuning transition energy during a cooling cycle, taking advantage of concepts developed for a  $\gamma_{tr}$ -jump. The benefits have to be weighed against complexity, lager number of quadrupoles and, for simple FMC modules, larger required aperture or reduced acceptance.

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