ADVANCED HESR LATTICE WITH NON-SIMILAR ARCS FOR IMPROVED STOCHASTIC COOLING

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Abstract

The advanced HESR lattice with two arcs of identical layout and different slip factors has been developed. The conception of arcs with three families of quadrupoles makes it easy to adjust the imaginary transition energy in one arc and the real transition energy in another arc with the absolute value close to the beam energy in the whole required region from 3.0 GeV to 14 GeV. The arcs have the special feature that the high order non-linearities are fully compensated inside each arc, and therefore the dynamic aperture of the whole machine is conserved. We consider and compare two lattices with the same absolute value of transition energy: the current lattice with a negative momentum compaction factor in both arcs and correspondingly the lattice with negative and positive momentum compaction factors in different arcs. Simultaneously, we analyzed the 4- and 6- fold symmetry arc machine. Thus allows us to conclude that the 4-fold symmetry lattice is more suitable for acquiring slip factors. At the lowest energy 3 GeV, this is $\eta_{imag}/\eta_{real} \approx 4 \div 5$ in the imaginary and the real arc, respectively. For the higher beam energy this ratio is much bigger.

INTRODUCTION

To intensify the stochastic cooling process it is desirable to have the mixing factor between the pick-up and kicker as large as possible, and, on the contrary, in the case of mixing between the kicker and pick-up we should try to make it smaller. This option can be realized if the lattice has different local optical features between pick-up – kicker and kicker – pick-up.

The idea with different slip factors was first proposed by Möhl [1,2]. Later many authors tried to design such a lattice, for instance [3, 4]. However, this involved a more complicated lattice with a large number of quadrupole and sextupole families and the need to have different optical settings at different energies. As result the dynamic aperture in such lattices is usually unacceptably small, and it has very difficult tuning. Therefore the compromise was to scarify some of the desired re-randomization in order to avoid too much unwanted mixing. In the classical lattice the slip factors between pick-up and kicker η_{pk}, kicker and pick-up η_{kp} are similar, and by Möhl's definition [2] the mixing factors are approximately equal. In paper [5] the comprehensive analysis of the stochastic cooling was done in the HESR lattice with similar arcs and the negative momentum compaction factor ($\gamma_{tr} = 6.5i$) [6]. In this paper, we consider the advanced HESR lattice with different slip factors η_{pk}, η_{kp} in two arcs.

ARCS WITH DIFFERENT SLIP FACTOR

The HESR lattice consists of two arcs and two straight sections for the target and cooling facilities with a circumference ~500÷600 m. The arcs have 6-fold (or 4-fold) symmetry with superperiodicity S=6 (or 4). The phase advance per arc is $\nu_{x,y} = 5.0$ (or $\nu_{x,y} = 3.0$) in both planes. Each superperiod consists of three FODO cells with 4 superconducting bending magnets (B=3.6T) and superconducting quadrupoles with G<60T/m (see fig. 1).

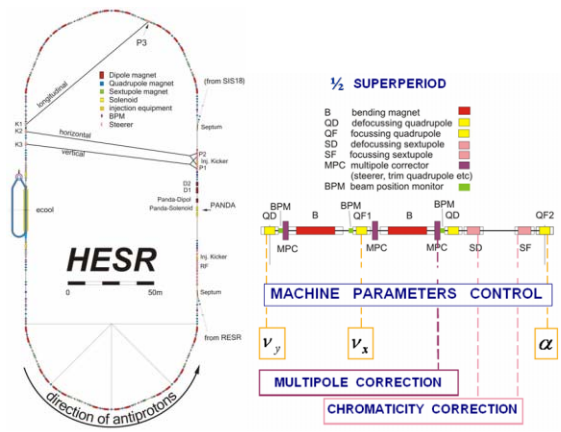

Figure 1: HESR layout and half super-period.

The momentum compaction factor is one of the most important characteristics of any accelerator, and defines its transition energy. The slip factor, $\eta = 1/\gamma^2 - 1/\gamma_{tr}^2$ determined by transition γ_{tr} and current γ energy, should be as high as possible in order to increase the microwave stability threshold.

The most successful solution for the control of the momentum compaction factor has achieved in [7] by simultaneously correlated curvature and gradient modulations. This lattice was used in the following projects: the Moscow Kaon Factory, the TRIUMF Kaon Factory, the SSC Low Energy Booster, the CERN Neutrino Factory and in the Main Ring of the Japan Proton Accelerator Research Complex facility constructed now [8]. In the HESR lattice the same idea was used [6].

In the advanced HESR lattice for the stochastic cooling we propose modifying the conception to provide different slip factors in two arcs, but with conservation of sequence of all bending, focusing elements and drift between them.

The proposed lattices meet the following requirements:

- momentum compaction factor is about $1/\nu^2$ in one arc (the slip factor close to zero, isochronous structure) and it is negative in the other arc $-1/\nu^2$; the total slip factor is enough high to provide a minimum spread in incoherent frequencies for the longitudinal motion stability requirements;
- dispersion-free straight sections;
- convenient method of correcting the chromaticity by the sextupoles;
- sufficiently large dynamic aperture after chromaticity correction.

The momentum compaction factor is usually determined from the integral

$$\alpha = \frac{1}{\gamma_t^2} = \frac{1}{2\pi}\int_0^{2\pi} \frac{D(\phi)}{\rho(\phi)} d\phi, \tag{1}$$

where $D(s)$ is the dispersion function and $\rho(s)$ is the curvature radius of the equilibrium trajectory.

To achieve the required momentum compaction factor we make a correlated modulation of the quadrupole gradients

$$\varepsilon \cdot k(\phi) = \sum_{k=0}^{\infty} g_k \cos k\phi \tag{2}$$

and the orbit curvature

$$\frac{1}{\rho(\phi)} = \frac{1}{\overline{R}}\left(1 + \sum_{k=1}^{\infty} r_k \cos k\phi\right) \tag{3}$$

with superperiodicity S_{arc}.

The radius curvature modulation r_n is provided by the missing magnet and it is performed once and is then fixed. However, the gradient modulation is the variable parameter. Due to the FODO structure with mirror symmetry we realize:

$$\begin{aligned} &\frac{\partial \alpha}{\partial G_{QF2}} >> \frac{\partial \alpha}{\partial G_{QF1}} \approx \frac{\partial \alpha}{\partial G_{QD}} \\ &\frac{\partial \nu_x}{\partial G_{QF1}} > \frac{\partial \nu_x}{\partial G_{QF2}} >> \frac{\partial \nu_x}{\partial G_{QD}} \\ &\frac{\partial \nu_y}{\partial G_{QD}} >> \frac{\partial \nu_y}{\partial G_{QF1}} \approx \frac{\partial \nu_y}{\partial G_{QF2}} \end{aligned} \tag{4}$$

Therefore, the lattice provides independent control of α, ν_x, ν_y by gradients of quadrupoles QF2, QF1 and QD, respectively.

In paper [7], the dispersion equation was solved for the case of both the quadrupole gradients and the orbit curvature modulation:

$$\alpha_s = \frac{1}{\nu^2}\left\{1 + \frac{1}{4(1-kS/\nu)} \cdot \left[\left(\frac{\overline{R}}{\nu}\right)^2 \frac{g_k}{\left[1-(1-kS/\nu)^2\right]} - r_k\right]^2\right\} \tag{5}$$

where $\overline{R}$ is the average radius of machine, and ν is the horizontal tune. We can see that the sign of the momentum compaction factor depends on the term $1-kS/\nu$. The negative momentum compaction factor is achieved in lattice with superperiodicity S and ν, when $1-kS/\nu < 0$ and it is determined by the kS-th harmonic.

It can be seen that this lattice has a remarkable feature: the gradient and the curvature modulation amplify each other if they have opposite signs $g_k \cdot r_k < 0$, and, on the contrary, they can compensate each other when they have the same sign. We use this feature to make arcs with different slip factors. Hereinafter, we will call the arc between the pick-up and the kicker in the line of beam the real arc $\alpha = 1/\gamma_t^2 > 0$. Correspondingly, the arc between the kicker and the pick-up will be called the imaginary arc $\alpha = 1/\gamma_t^2 < 0$, because the transition energy is imaginary.

First of all, in both arcs we create the resonant curvature modulation by the usual method of the "missing magnet" in the center of the superperiod. Then the quadrupole gradient is modulated with the opposite sign and the value determined by the gradient modulation when the ratio between them is:

$$|r_k| \le \left(\frac{\overline{R}}{\nu}\right)^2 \left|\frac{g_k}{1-(1-kS)^2}\right| \tag{6}$$

In principle, the curvature modulation can be made much higher, but since in the real arc we need full compensation of the curvature modulation by the gradient modulation, and we would like to have an identical sequence of magneto-optic elements in both arcs, it is not desirable to increase the first arc. At the same time, the gradient modulation is restricted by the parametric resonance of the envelope, when the second harmonic $kS/\nu = 2$.

Therefore, from this point of view it is desirable to have such g_k when the ratio has value:

$$\frac{1}{4(kS/\nu - 1)} \cdot \left(\frac{g_k}{[1-(1-kS/\nu)^2]} - r_k\right)^2 \approx 2 \tag{7}$$

Then the momentum compaction factor in the imaginary arc takes the form $\alpha_{kp} \approx -1/\nu^2$, and the momentum compaction in the real arc is $\alpha_{pk} \approx 1/\nu^2$. Thus, in such a lattice, we can make two arcs with different slip factors: $\eta_{pk} = 1/\gamma^2 - 1/\gamma_{tr}^2$; $\eta_{kp} = 1/\gamma^2 + 1/\gamma_{tr}^2$. In case $\gamma \approx \nu$, one of the arcs is isochronous when the slip factor is $\eta_{pk} \approx 0$, and the other slip factor is $\eta_{kp} \approx 2/\nu^2$. However, together with the advantage of two different arcs for stochastic cooling we lose the lattice mirror symmetry, which makes the probability of the structure resonance excitation higher.

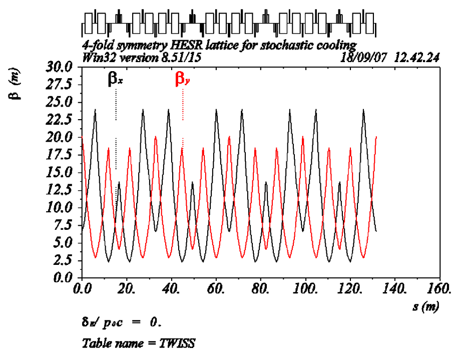

Figure 2: $\beta_{x,y}$-functions on the imaginary arc.

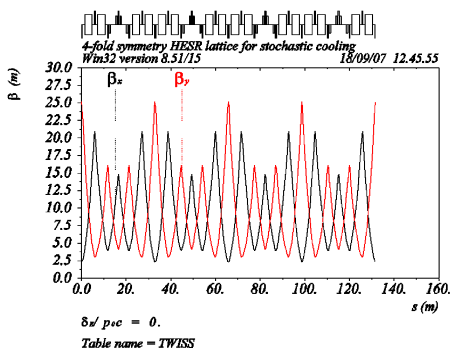

Figure 3: $\beta_{x,y}$-functions on the real arc.

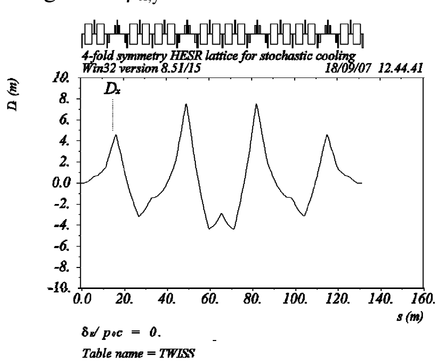

Figure 4: Dispersion-functions on the imaginary arc.

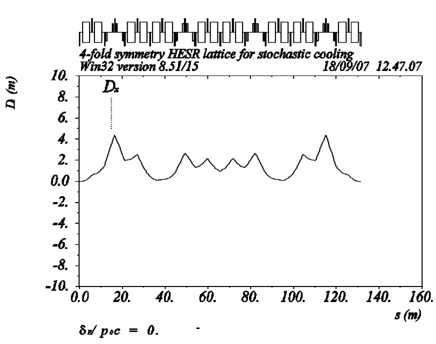

Figure 5: Dispersion-functions on the real arc.

However, the lattice developed here has the fundamental feature, since the both arcs have the same tunes and the similar phase advance between all elements located on arc. Since any order resonance strength is determined by the integral

$$\left\langle h_{k_x,k_y,p}\right\rangle \propto \int_0^C \beta_x^{k_x/2}\beta_y^{k_y/2}\frac{\partial^{(k_x+k_y-1)}B_{y,x}}{\partial(x,y)^{(k_x+k_y-1)}}(s)\exp i(k_x\mu_x+k_y\mu_y)ds \quad (8)$$

and, as we can see, in the subintegral expression the $\beta_{x,y}$-functions are the multipliers of field errors, the resonance excitation probability is determined only by the difference of $\beta_{x,y}$-function behaviour in the arcs.

Figures 2-5 show the $\beta_{x,y}, D_x$ functions for the real and imaginary 4-fold symmetry arcs. In both arcs the dispersion is suppressed to have the zero-dispersion straight sections. We can see from these figures that the different momentum compaction factors are reached mainly due to the dispersion function change, and that the β-function itself changes insignificantly.

Two families of sextupoles are used for the chromaticity correction (see fig. 1). To make the sextupoles self-compensating in the first approach we have to have an even number of arc superperiods S_{arc} and as a consequence nearest to S_{arc} the arc tune $\nu_{arc} = S_{arc} - 1$. Then the phase advance between similar sextupoles of $i-th$ and $(i + S_{arc}/2)-th$ superperiods equals $\nu_{arc}/2$. This means we have an exact condition for compensating each sextuplet's non-linear action by another one. Thus, there are the combinations: $\{S_{arc}, \nu_{arc}\} = \{4,3;\ 6,5;\ 8,7;...\}$.

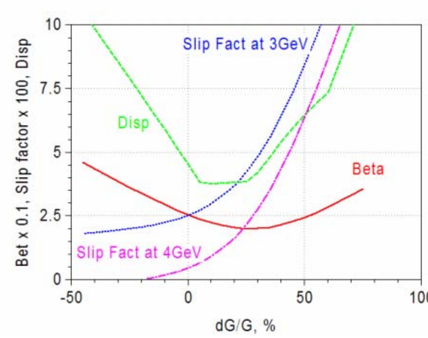

Figure 6: β_x, D_x, η vs. gradient modulation.

The optimum set for our case should be around a value $\nu = 2\nu_{arc} \approx \gamma$ and depends on the lowest energy. For instance, for energy E=3 GeV ($\gamma \approx 4.2$) the 4-fold arc with a tune of arc $\nu_{arc} = 3$ gives the best fit. Figure 6 shows Twiss parameters together with the slip factor dependence on the gradient modulation. We can see that for an energy 3 GeV at acceptable Twiss parameter behavior the maximum ratio is $\eta_{imag} / \eta_{real} \approx 4 \div 5$, while at 4 GeV this ratio can be significantly higher.

DYNAMIC APERTURE

At the end of this paper, we will discuss the numerical calculation results. Since the indicator of any structure is the dynamic aperture, we performed the tracking simulation in the lattice with non-similar arcs and compare this with the lattice where the arcs are similar. Of course, due to the loss of mirror symmetry in the whole ring lattice the dynamic aperture becomes smaller. But the significant reserve of the dynamic aperture still allows the large right value in the horizontal plane ~270 mm mrad and in the vertical plane ~500 mm mrad. Both values satisfy the required ratio between the dynamic and physical apertures very well.

CONCLUSION

We developed the advanced lattice for stochastic cooling. The lattice has two similar arcs with different mixing factors due to the different slip factors with conservation of the optic geometry. Each arc has two families of focusing quadrupoles and one family of defocusing quadrupoles. The transition energy is adjusted by the quadrupole gradient modulation. The natural chromaticity is corrected by one family of focusing and defocusing sextupoles. After the chromaticity correction the dynamic aperture remains very large. The straight section allows stochastic and electron cooling to be performed simultaneously.

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