N-BODY CODE TO DEMONSTRATE ELECTRON COOLING

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Abstract

In the Electron Ion Collider (EIC), the collision between the electron beam and the proton, or heavy ion, beam results in emittance growth of the proton beam. Electron cooling, where an electron beam and the proton beam co-propagate, is the desired cooling method to cool or mitigate the emittance growth of the proton beam. The pre-booster, the larger booster, and the collider ring in EIC are the major components that require electron cooling. To study the cooling effect, we previously proposed Particles' High order Adaptive Dynamics (PHAD) code that uses the Fast Multiple Method (FMM) to calculate the Coulomb interactions among charged particles. We further used the Strang splitting technique to improve the code's efficiency and used Picard iteration-based novel integrators to maintain very high accuracy. In this paper we explain how this code is used to treat relativistic particle collisions. We are able calculate the transverse emittances of protons and electrons in the cooling section while still maintaining high accuracy. This presentation will be an update on the progress with the parallelization of the code and the status of production runs.

INTRODUCTION

A set of many objects, which undergoes self-interactions and external forces, can be mathematically modeled as an N-body problem. As the number of objects, N, becomes larger, the force calculation demands remarkably high computing power.

In a charged particle beam, the mathematical formulation of the study of particles' behavior leads to an electromagnetic N-body problem. In order to calculate potential, field and Coulomb interactions among particles, we have to solve 6N differential equations (3N for x, y, z and another 3N for p_x, p_y, p_z). We have developed a code based on a novel FMM algorithm [1, 2] using certain differential algebraic techniques to solve differential equations efficiently while maintaining high accuracy.

ALGORITHM

The time taken for calculating the interaction among particles using the point-to-point or direct method exhibits quadratic growth, i.e. $O(\mathrm{N}^2)$. Even today's high-performance supercomputers cannot provide the necessary computational power to solve this problem in a reasonable amount of time. Therefore, the sheer necessity arises for finding computationally efficient and accurate methods to calculate the Coulomb interactions. From the literature review, one can find several approximate methods to calculate interactions, which are relatively efficient compared to the direct method, namely: 1. Basis function methods; 2. Particle-mesh methods; 3. Hierarchical domain decomposition methods. Due to certain drawbacks in the first and the second method, we choose the third [3]. The third group can be further divided into three sub-groups: tree, cluster and fast multipole methods (FMM). Tree and cluster groups, however, are merely special versions of the FMM. We employ a novel FMM [1] since it has many advantages over the original FMM algorithms. The FMM calculates interaction forces within a prescribed accuracy in linear time and memory usage.

In the 3D (2D) FMM, particles reside in small boxes, or octree (quadtree) nodes. The force calculation in FMM can be interpreted as the force calculation between these boxes. Due to the particle domain decomposition, the FMM enables to identify the near and far regions and calculates the far-field interactions rapidly. The near field interactions are still based on the particle–particle calculation method.

Further, we developed a code named PHAD, which is comprised of three important techniques that guarantee efficiency and accuracy. Firstly, we have shown that FMM calculates the far-field interactions with a variable but a priori guaranteed accuracy that can be adjusted by setting an appropriate FMM order. The fact that, at the optimum conditions, its calculation cost is in the order of N implies that FMM is efficient.

As the second technique, we used a variable order Picard iteration-based integrator [4, 5] to calculate the particle distributions' propagation in time. The adjustable time step size of the integrator for each particle allows investigating close encounters. In addition, the Picard integrator provides a dense output. Hence, the ability to adjust the optimum Picard order and the time step size automatically enable to calculate the near range interactions precisely, and with the appropriate number of iterations govern the efficiency.

Finally, we used a second order accurate operator splitting method, the Strang splitting, to speed up the performance of PHAD. It splits the complicated system into two simpler parts: far-range and near-range with external fields. The FMM is time-wise the most expensive procedure, thus we need to reduce the number of FMM calls. Each particle in the beam undergoes fast varying forces and slow varying forces. The fast varying forces are a result of the close encounters between the particle and its neighbors. The collective interaction due to the far away particles can be expressed as the mean field and the slow varying forces are due to this mean field. We need to select the appropriate time step size such that the slow forces stay approximately unchanged. However, in order to calculate the fast varying forces, this time interval should be split into smaller time steps. Due to this fact, we

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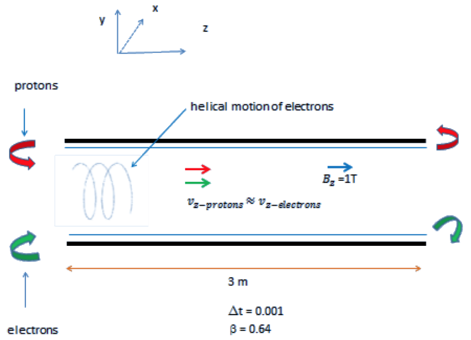

y
x
z
protons
helical motion of electrons
B_z =1T
$v_{z-protons} \approx v_{z-electrons}$
3 m
Δt = 0.001
β = 0.64
electrons

Figure 1: Cooling section in the pre-booster.

can identify two time step sizes for fast and slow varying forces. The smaller time step size depends on the distance between the target and the source particle and their relative speed, and hence we have individualized adaptive time steps. We call FMM only for the larger time steps and it implies that we can reduce the number of FMM calls. As a result, the computational cost can be substantially minimized in PHAD [3, 6].

We validated the accuracy of PHAD by comparing its results with the stand-alone N-body code, and they show an excellent agreement [3, 6].

PERFORMANCE ANALYSIS

In the Medium Energy Ion Collider, protons or ions are generated in the ion source and accelerated by the linac before being injected into the pre-booster. At the assumed injection, the kinetic energy of a proton beam is about 280 MeV and that of a lead ion beam is about 112 MeV per nucleon.

In this paper, our simulation is restricted to a proton beam. In the pre-booster ring, the average velocity of 280 MeV protons in the longitudinal direction is 0.64c ms^{-1}. The pre-booster, which also functions as the accumulator ring, accumulates protons and cools them to raise the beam current. The straight section of the pre-booster accommodates a ~3 m long cooling section and the 'hot' proton beam co-propagates with the 'cold' electron beam in this cooling section (Fig. 1). The electron beam takes 3.6 ns to pass the cooling section. Since the cooling section is immersed in a longitudinal magnetic field of ~1 T the electrons make a spiraling motion around the magnetic field and the estimated period for one revolution is about 35.77 ps. Therefore, in the 3 m long cooling section, the electrons make about 100 revolutions.

To evaluate the cooling motion, we studied the revolutions in increments of time. Even though large time steps can be prized as being computationally efficient, they might mask important physics such as close encounters of particles. To avoid this, we further divide the 35.77 ps period into ~10 small time steps of 3.57 ps each. Hence, the total number of such small time steps that the electron beam undergoes in the cooling section before exiting is ~1000. According to the design Medium-energy Electron Ion Collider (MEIC) parameters [7], the cooling time in pre-booster ring is in the millisecond range. This suggests that the proton beam has to pass the cooling section about 10^6 times before it is cooled.

Table 1: Simulation Parameters

Time step) size(m)	FMM order	Picard order	Picard iterations
0.001	9	16	1

In our proof-of-principle simulation, we used 100 protons and 1000 electrons as the two particle beams. Since they must co-propagate in the cooling section, the initial momenta of the particles in the longitudinal direction, p_z, were chosen such that the average longitudinal velocities of both beams are 0.64c, where c is the speed of light. Therefore, the corresponding relativistic gamma factor in the pre-booster is 1.3. In addition, the transverse momenta, p_x and p_y, were chosen as 1% of the longitudinal momenta. The initial configuration of protons and electrons were randomly generated such that they lie on the x-y plane and they are spatially close mimicking a high-density particle beam.

The proton beam and the electron beam were launched in the cooling section and data was gathered for 150k time steps of size 3.6 ps each. In our simulation, we used a scaling factor for the time and the transferred time step size has units of 1/c, where c is the speed of light. Therefore, the time step size is equivalent to 0.001 m. The simulation parameters are shown in Table 1.

The Figures 2 and 3 show the transverse emittance plots for protons and electrons, respectively. In the longitudinal direction, the average velocity of both protons and electrons stay around 0.64c and it implies that all particles are commoving in the cooling section. Table 2 shows the average velocities of protons and electrons and their difference. The velocity difference diminishes as time passes (Fig. 4).

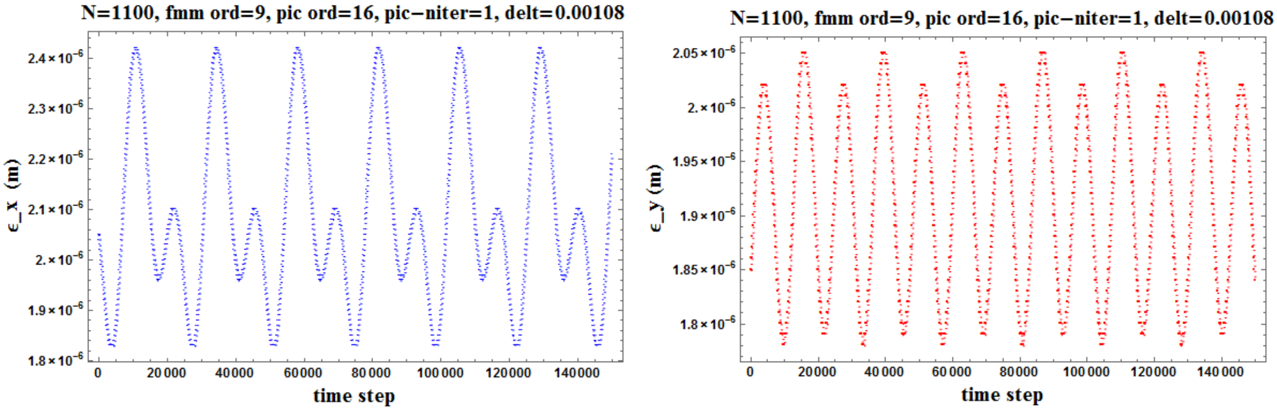
In addition, the 'frozen snapshots' of the cross sections of the beam in the x-z plane and y-z plane (Fig. 5) show that all protons and electrons have traveled the same distance, on average, in the longitudinal or z direction.

Even After 150k time steps the particles have traversed a very short distance in the cooling section. According to the estimated values, the ion beam has to pass the cooling section about 10^6 times and it may take longer time. Currently, we have a serial version of the PHAD code and in order to see the long-term effects it is inevitable to have the parallelized version. We are in the process of developing the parallelized version of the PHAD code.

The FMM piece of PHAD includes two parts: data structuring and calculation of Coulomb interactions. The former is implemented in C++ while the latter is in COSY INFINITY [8]. The remaining parts of PHAD are written in COSYScript.

The rms emittance, $\epsilon_{x,y}$, is given by:

$$\epsilon_{x,rms} = \sqrt{< x^2 > < x'^2 > \ - < xx > '^2} \qquad (1)$$



(a)

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Figure 2: Transverse emittances of protons.

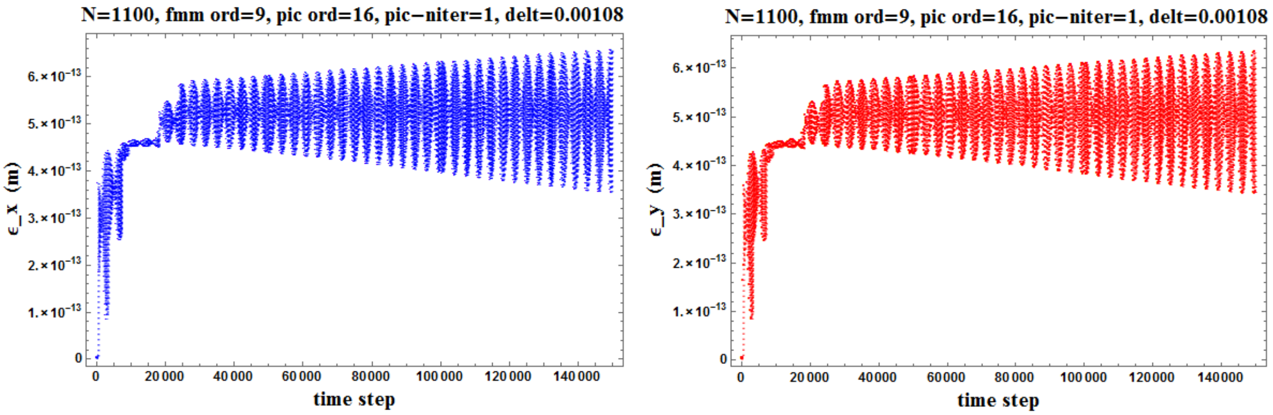

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Figure 3: Transverse emittances of electrons.

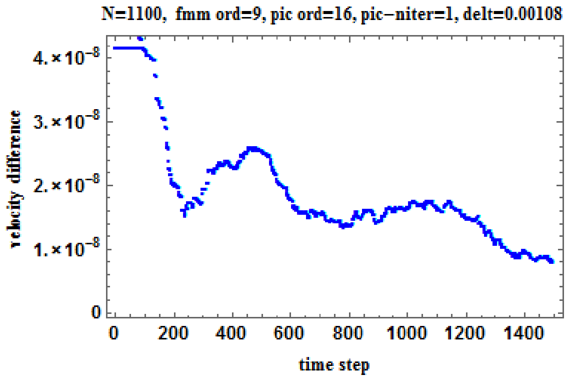


Figure 4: Average velocity difference between protons and electrons in the longitudinal direction.

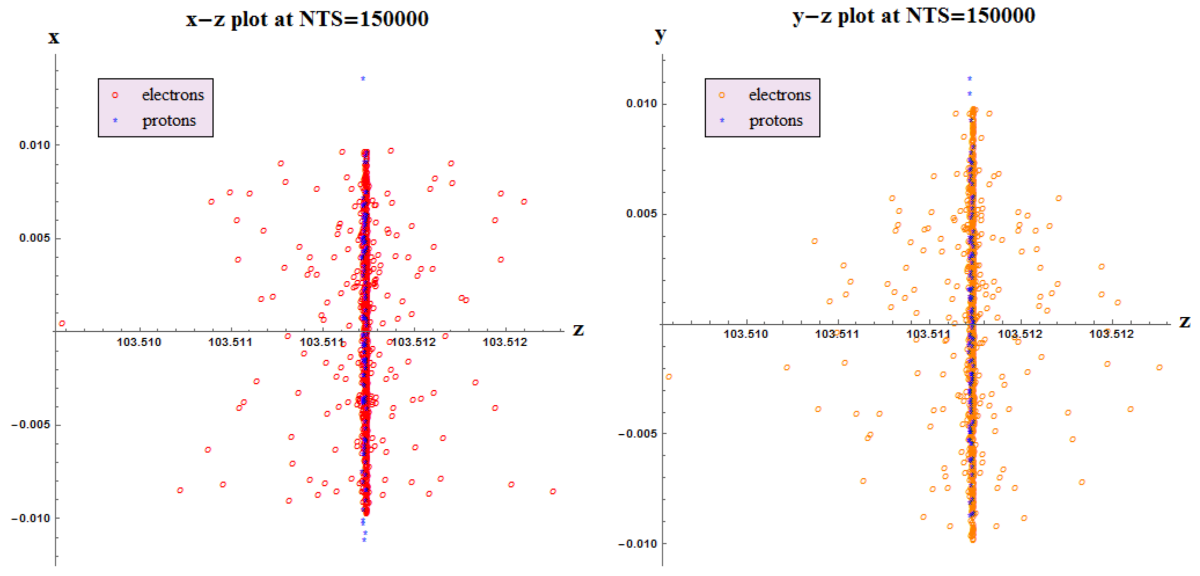
Table 2: Longitudinal Velocities of Protons and Electrons

Number of time steps	Velocity of protons×c	Velocity of electrons×c	Velocity difference ×c
1	0.638 971 004 227	0.638 971 066 378	6.21×10^{-8}
150k	0.638 971 004 521	0.638 971 012 295	7.77×10^{-9}

SUMMARY

Modelling and simulation of the electromagnetic N-body problem can be performed with the aid of suitably developed algorithms and high performance computers. In this paper we discussed a novel code developed to model and simulate the dynamic behaviour of charged particles in a beam efficiently and accurately. Fast multipole method improves the efficiency and accuracy of calculating the Coulomb interaction force. Picard iteration process is used to investigate the close encounters of particles. The number of FMM calls is reduced with Strang splitting method and the efficiency is immensely improved. Effective symplecticity is preserved with high-order Picard runs.

FMM and Strang splitting do introduce certain approximations and splitting errors, respectively. By choosing appropriate FMM orders and time step sizes we can mini-



(a)

(b)

Figure 5: Particle distribution on x-z and y-z plane at NTS=150k.

mize the influence of such errors on long term properties of the system [3].

This paper presented an update on the current status of the PHAD code. Subsequent work will give the high priority to parallelization of PHAD to expedite the time stepping.

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