EMITTANCE GROWTH FROM MODULATED FOCUSING AND BUNCHED ELECTRON BEAM COOLING*

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Abstract

The low energy RHIC electron cooling (LEReC) project at Brookhaven employs a linac to supply electrons with kinetic energies from 1.6 to 2.6 MeV. Along with cooling the stored ion beam the electron bunches create a coherent space charge field which can cause emittance growth. This process is investigated both analytically and through simulation.

INTRODUCTION AND THEORY

The low energy RHIC electron cooling project is currently under construction at BNL. We are using an electron linac with bunch lengths of a few centimeters to cool gold beams with lengths of several meters. Let γ be the Lorentz factor of the ions, $\alpha_{\rm p}$ be the momentum compaction factor, $\sigma_{\rm p}$ be the rms fractional momentum spread, $\eta = 1/\gamma_t^2 - 1/\gamma^2$, and T_0 be the revolution period. The rms longitudinal slip per turn is $\sigma_{\rm slip} = T_0|\eta|\sigma_{\rm p}$. Table 1 shows this and other RHIC parameters.

Table 1: Gold Beam Parameters

parameter	$\gamma = 4.1$ value	$\gamma = 6.0$ value
$\sigma_{\rm tg}$(ns)	11.7	9.6
$\sigma_{\rm p}$	3.5×10^{-4}	3.8×10^{-4}
$N_{\rm ion}$	6×10^8	1×10^9
emittance μm	2.5	2.5
f_0 (kHz)	75.8	77.2
$\sigma_{\rm slip}$(ps)	280	127

Table 2: Electron Beam Parameters

parameter	$\gamma = 4.1$ value	$\gamma = 6.0$ value
$\sigma_{\rm te}$(ps)	100	67
$\sigma_{\rm p}$	$4-8 \times 10^{-4}$	$4-8 \times 10^{-4}$
$Q_{\rm e}$(pC)	$65-130$	78-156
emittance μm	1-2	1-2
bunch spacing (ns)	1.42	1.42
bunches per train	31	25

The electron parameters are still under discussion but ranges are shown in Table 2. In the tables $\sigma_{\rm tg}$ and $\sigma_{\rm te}$ are the root mean square (rms) bunch durations, $Q_{\rm e}$ is the electron bunch charge, and $N_{\rm ion}$ is the number of ions per bunch.The emittance is the rms normalized emittance. There is a train of electron bunches of length $\sim 4\sigma_{\rm tg}$ as illustrated in Figure 1. For all cases one has $\sigma_{\rm te} < \sigma_{\rm slip}$ which means that if an ion is subjected to a maximal space charge force on one turn it will not be subject to a significant force on the next turn. Unpublished work by Gang Wang and Vladimir Litvinenko has shown that it is critical that the electron bunches not slip with respect to the ion bunches. We assume this is the case but this still leaves the possibility of synchrobetatron resonances.

To study these resonances assume the cooling section is centered on β^* with $\alpha^* = 0$ and take the transverse ion coordinates to be x and $p = \beta^* x'$ so that the one turn matrix is just a rotation with phase advance $\psi_0 = 2\pi Q_x$. As a first approximation assume a single electron bunch centered on the ion bunch so that an ion interacts with it twice per synchrotron oscillation. Assuming the electron bunch has focusing strength k the map for half a synchrotron oscillation is

$$\begin{bmatrix} x_{n+1} \\ p_{n+1} \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi Q_x}{Q_s} & \sin\frac{\pi Q_x}{Q_s} \\ -\sin\frac{\pi Q_x}{Q_s} & \cos\frac{\pi Q_x}{Q_s} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta^* k & 1 \end{bmatrix} = \begin{bmatrix} x_n \\ p_n \end{bmatrix}, \tag{1}$$

where Q_s is the synchrotron tune. When Q_x/Q_s is close to an integer the map is unstable. Taking $\sin(\pi Q_x/Q_s) = \epsilon$ and assuming an eigenvalue $\lambda = 1 + \delta$ one finds $\delta \approx \sqrt{\beta^* k\epsilon - \epsilon^2}$. The resonances for LEReC are typically very weak with $\beta^* k \sim 10^{-5}$. When coupled with the small fraction of time the ions interact with the electrons one expects a very small fraction of the beam would be harmed by these resonances. However there is another important dynamical effect. Longitudinal intrabeam scattering causes the longitudinal action to wander and with it the synchrotron tune. This causes individual particles to wander back and forth through resonances, usually increasing betatron amplitude with each passage. If we look at it in terms of statitical averages the average increase in amplitude will be proportional to the maximum growth and the fraction of time growing is proportional to the resonance width. Since both terms are linear in the charge of the electron bunch one expects the emittance growth rate to scale as the square of the electron bunch charge.

A better model can be obtained using perturbation theory. Since we only consider a matrix and a thin lens cooling region we take $|Q_x| < 1/2$. The equations of motion are generated by the hamiltonian

$$H(x,p;\theta) = \frac{Q_x}{2}\left(p^2 + x^2\right) + \delta_p(\theta)F_e(\tau)\ln\left(1 + x^2/a^2\right), \tag{2}$$

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where we use azimuth θ as the time-like variable, $\tau = \tau(\theta)$ is the arrival time of the ion relative to the synchronous particle, a characterizes the radius of the electron beam,

$$\delta_p(\theta) = \sum_{k=-\infty}^{\infty} \delta(\theta - 2\pi k) = \sum_{m=-\infty}^{\infty} \frac{e^{2\pi i m\theta}}{2\pi}$$

and

$$F_e(\tau) = \frac{-\beta^* Z_0 I_e(\tau)\ell}{4\pi\beta^3\gamma^3 mc^2/q}.$$

In F_e we have a cooling section of length ℓ, $\beta = v/c$, $Z_0 = 377\Omega$, $I_e(\tau)$ is the electron current, and the ion has charge q and mass m. In the simplest approximation take $\ln(1 + x^2/a^2) \approx x^2/a^2$ and $\delta_p(\theta) \approx 1/2\pi$. This leaves only the slow variation associated with $\tau(\theta)$. Assuming interaction with a single synchrotron harmonic we have

$$\frac{d^2x}{d\theta^2} = -Q_x^2 x - 2Q_x \hat{C}\cos(pQ_s\theta), \tag{3}$$

where $pQ_s = 2Q_x + \delta$ with $|\delta| \ll 1$ and

$$\hat{C} = \left| \int_0^{2\pi} \frac{d\psi_s}{2\pi} e^{ip\psi_s} \frac{F_e(\hat{\tau}\cos\psi_s)}{\pi a^2} \right|, \tag{4}$$

where $\hat{\tau}$ is the amplitude of the synchrotron oscillation. Equation (4) leads to parametric resonances [1]. The amplitude of oscillation grows as $e^{s\theta}$ with

$$s = \frac{1}{2}\sqrt{\hat{C}^2 - \delta^2}.$$

As with the previous analysis the strength and width of the resonance are both proportional to the electron bunch charge. When synchrotron tune wander is included it follows that this analysis also predicts the emittance growth rate should scale as the square of the electron bunch charge.

The previous analyses assumed a linear restoring force for the electrons. While a detailed non-linear analysis has not been obtained a few general comments are in order. First we use action angle variables with $x = \sqrt{2J}\cos\psi$ and the slow approximation on (2) yielding.

$$H(J,\psi;\theta) = Q_x J + \frac{F_e(\tau)}{2\pi}\ln\left(1 + \frac{2J\cos^2\psi}{a^2}\right) \tag{5}$$

$$= Q_x J + \frac{F_e(\tau)}{2\pi}\sum_{m=0}^{\infty} a_n(J/a^2)\cos(2m\psi). \tag{6}$$

Define $b = J/a^2$. For $m = 0$ we find [2]

$$a_0(b) = \ln\left(\frac{1 + b + \sqrt{1+2b}}{2}\right). \tag{7}$$

For $m > 0$

$$a_m(b) = \frac{-2}{m}\left(\frac{-b}{1 + b + \sqrt{1+2b}}\right)^m. \tag{8}$$

The detuning term in the Hamiltonian increases without bound as b increases but the change in tune will be quite small. The other terms $a_m(b)$ are bounded by $2/m$ even as $b \to \infty$ so the driving terms saturate with betatron amplitude. For our parameters it is likely the single resonance approximation will hold at any given $\hat{\tau}$ but the important resonance could change with $\hat{\tau}$. It is also likely that other sources of detuning will dominate a_0 but these are easily added.

SIMULATIONS

The simulation code is based on a simple one turn map for the ions and a thin lens treatment of the electron-ion interaction. The one turn map is defined by betatron tunes, coupling, chromaticities, detuning coefficients and sine wave RF. Also we include longitudinal IBS with total growth rate given by Piwinski's [3]coasting beam formula and Zenkevich's [4] viscous force. Transverse IBS is not included because the model assumes a uniform focusing lattice which yields negative growth rates. Actual rates are about 10% of the longitudinal rates [5].Transverse space charge is implemented as a phase shift that is a function of betatron amplitude and longitudinal position within the bunch.

The electron ion interaction consists of a coherent space charge kick where the electron bunch is taken to be a 3 D gaussian. Electron cooling is non-magnetized and treated with the Coloumb logarithm outside the integral. The local density is multiplied by a cooling force that has the same form as the electrostatic force [6]. The electron beam is assumed round and the cooling force is calculated at the start of the simulation and stored in a two dimensional array. A version where only one transverse variable is tracked has also been developed.

We begin by determining what parameters are relevant to the dynamics. Figure 2 shows results for $\gamma = 4.1$ but with 10 times the nominal electron bunch charge to speed things up. We can draw several conclusions. First, the two dimensional (2D) simulation in red with chromaticity $\xi = -2$ is quite similar to the one dimensional version shown in blue. We conclude the second transverse dimension is not fundamental to the emittance growth, justifying our earlier 1D analysis. The magenta and green curves in Figure2 show the nihl effect of changing chromaticity. The purple and navy lines show the effect of reducing the longitudinal IBS by factors of 10 and 100, respectively. There is clearly an effect but it is weak. For no IBS the blue line shows no growth, hence some IBS is necessary for emittance growth. Finally the yellow curve shows the effect of linear RF. Clearly the growth is much reduced when the synchrotron tune does not depend on synchrotron amplitude.

Figure 3 shows the effect of 5 different initial random seeds with 1000 and 10,000 simulation particles. The slopes of all the curves are very simular showing that the emittance growth does not depend on microscopic details. Figure 4 shows the growth rate of the emittance for 2D simulations as the betatron tunes vary for 1000 and 10,000

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macroparticles. The growth rates change by factors of two in a nonuniform way with tune, verifying that emittance growth is a resonant phenomena.

Figures 5 and 6 show emittance growth rates as a function of electron bunch charge for 1D and 2D beams respectively. For each curve we used linear least squares to fit

$$\ln\left[\frac{d\ln\epsilon}{dn}\right] = a + bQ_\mathrm{e} + \text{error}, \tag{9}$$

with parameters a and b where Q_e is the electron bunch charge. The curves in Figures 5 and 6 are labeled by the betatron tune and the fitted value of b. For 1D we have $1.8 \le b \le 2.16$ and for 2D $1.68 \le b \le 2.14$ which agrees with the value of 2 obtained by our earlier analysis.

Figures 7 and 8 show best guess results for the situation in RHIC. For both cases the smaller emittance and lower intensity gives the best transverse cooling.

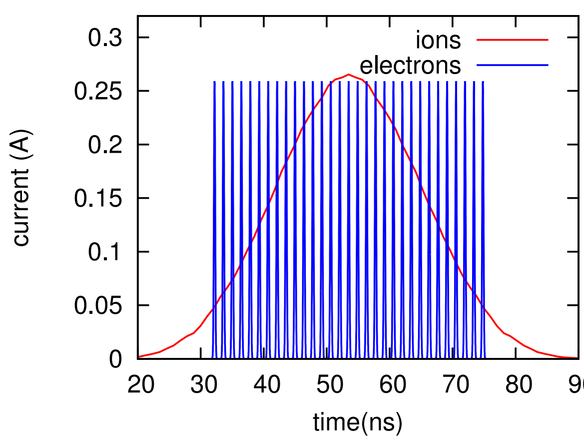

Figure 1: Ion and electron currents for $\gamma = 4.1$ with 65 pC electron bunches.

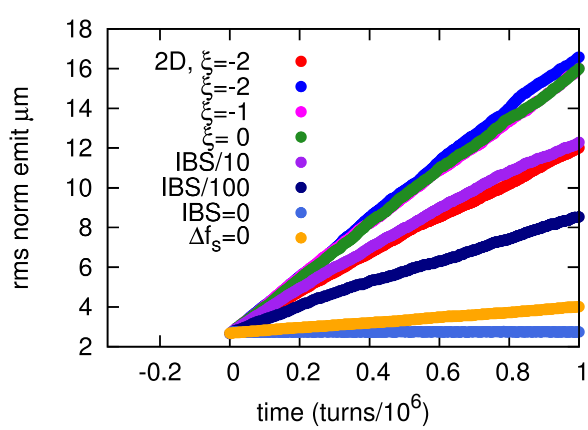

Figure 2: Simulations of emittance growth for a range of parameters, see the text for details.

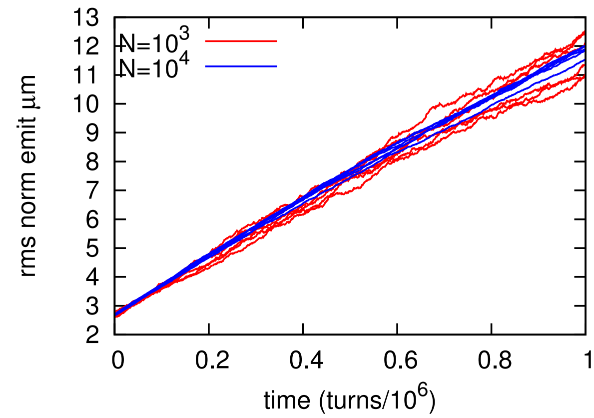

Figure 3: Emittance versus time for identical physical parameters with different random seeds and number of simulation particles.

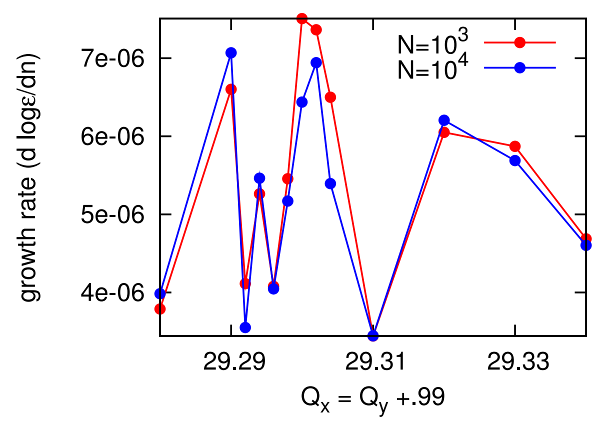

Figure 4: Growth rate as a function of betatron tune. The fine structure implies many resonances are relevant.

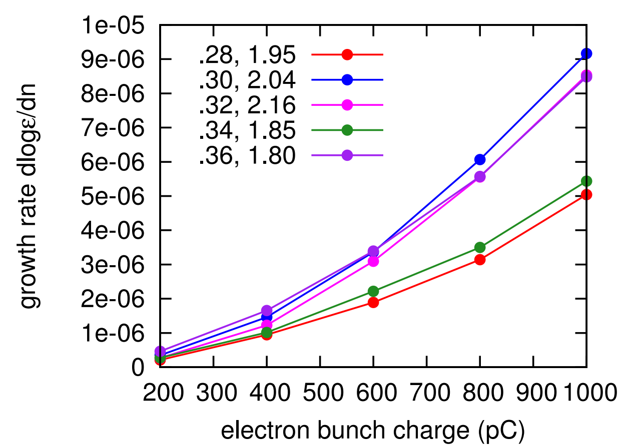

Figure 5: Growth rate versus bunch charge for 1D simulations. The curves are labeled by the non-integer part of the betatron tune and the power law for the growth rate obtained by fitting equation (9).

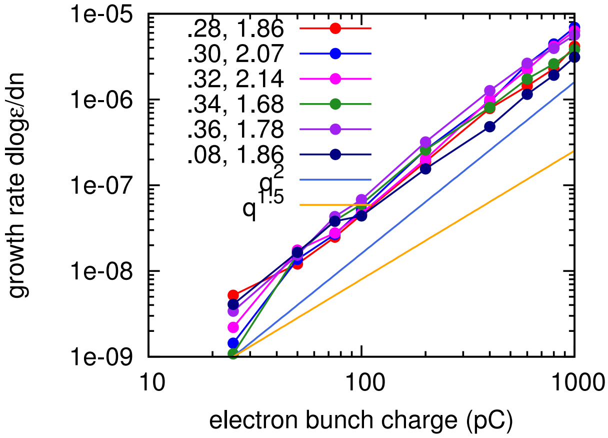

Figure 6: Growth rate versus bunch charge for 2D simulations. The curves are labeled by the non-integer part of the betatron tune and the power law for the growth rate obtained by fitting equation (9). Curves for power laws of 1.5 and 2 are shown for comparison.

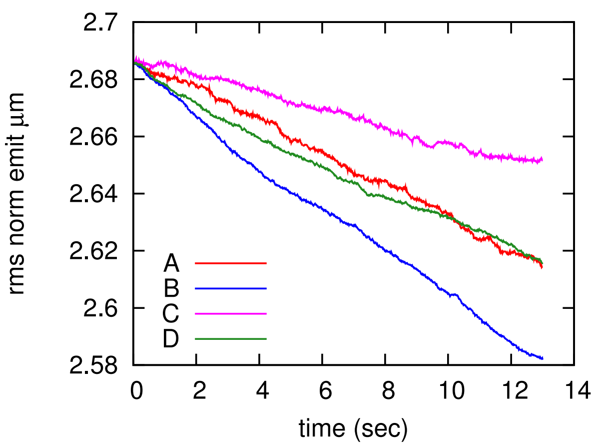

Figure 7: Ion emittance versus time for $\gamma = 4.1$ for various electron beam parameters:
A, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 2$ μm, $Q_e = 130$ pC;
B, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 1$ μm, $Q_e = 65$ pC;
C, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 2$ μm, $Q_e = 130$ pC;
D, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 1$ μm, $Q_e = 65$ pC.

CONCLUSION

Using bunched beams for electron cooling can lead to dynamically generated emittance growth. There are 3 required ingredients:

1. electron bunches that are of comparable length to the rms longitudinal slip per turn of the ions,
2. variation of the synchrotron frequency with amplitude,
3. longitudinal intrabeam scattering, although the dependence on rates is weak.

The emittance growth rate of the ions scales (approximately) like the square of the electron bunch charge. This was motivated theoretically and verified using simulations.

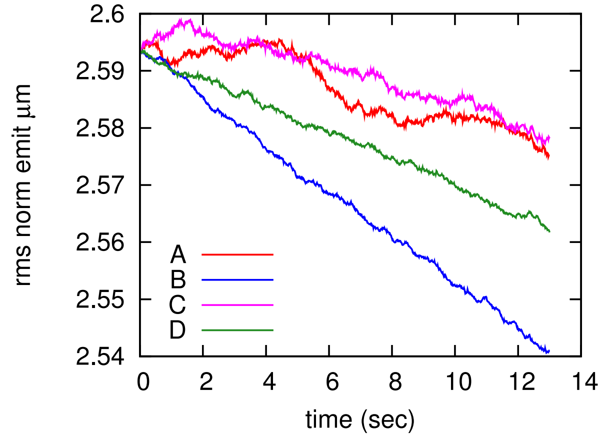

Figure 8: Ion emittance versus time for $\gamma = 6$ for various electron beam parameters:
A, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 2$ μm, $Q_e = 156$ pC;
B, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 1$ μm, $Q_e = 78$ pC;
C, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 2$ μm, $Q_e = 156$ pC;
D, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 1$ μm, $Q_e = 78$ pC.

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